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Influence of Lorentz Force on Casson Micropolar Nanofluid Flow over a Stretching Surface in a Fuzzy Environment

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Abstract. This study explores the influence of Lorentz force on Casson micropolar nanofluid flow over a stretching surface with in a fuzzy environment. The focus is on understanding how electromagnetic forces and uncertainties inherent in fluid properties and external conditions affect the behaviour of Casson fluids containing nano particles. By incorporating fuzzy logic, the study accounts for variability in physical parameters, leading to a more comprehensive and realistic analysis. The governing equations, incorporating the effects of Casson fluid dynamics, micro polarity, nano particle concentration, and Lorentz force, are solved numerically. The results reveal significant changes in velocity, microrotation, and temperature profiles, emphasizing the critical role of Lorentz force and fuzzy parameters in modulating the flow characteristics. This research provides valuable insights for optimizing industrial applications involving micropolar nanofluids and electromagnetic fields, enhancing heat transfer efficiency, and refining material processing techniques.

Keywords: nanoparticles, fuzzy, electromagnetic, temperature, micro polar

AMS Mathematics Subject Classification (2010): 74F10

1. Introduction

The study of fluid dynamics has been pivotal in understanding and optimizing various industrial and engineering processes. In recent years, the interest in non-Newtonian fluids, such as Casson fluids, has surged due to their unique rheological properties, which differ significantly from those of Newtonian fluids. Casson fluids, characterized by a yield stress below which no flow occurs and a nonlinear relationship between stress and strain rate, are highly relevant in applications ranging from biomedical engineering to food processing. When these fluids are combined with micropolar characteristics, where the fluid exhibits microstructure and microrotation effects, the resulting behavior becomes even more complex and intriguing. Nanofluids, which are suspensions of nanoparticles within a base fluid, have been extensively studied for their enhanced thermal properties. The inclusion of nanoparticles can significantly improve the thermal conductivity and heat transfer capabilities of the base fluid, making them ideal for advanced cooling technologies and other thermal management applications. When Casson fluids are combined with nanoparticles, the resultant Casson micropolar nanofluids exhibit both enhanced thermal properties and complex flow behaviors, making them a subject of considerable research interest. The effects of entropy generation in second-grade and third grade fuzzy hybrid nanofluids flowing over an exponentially permeable stretching/shrinking surface discussed [1-4]. Adaptive fuzzy controller design for uncertain fractional-order nonlinear systems, focusing on control theory and application of fuzzy logic in system control developed [5]. Free convection of casson nanofluid with an inclined magnetic field through porous medium simulated [6 - 8]. Concepts of vertex regularity in cubic fuzzy graph structures, partitioned neutron sophic graphs, vague graphs, complex Pythagorean fuzzy graphs, connected cubic networks and Cayley interval-valued fuzzy graphs discussed [9-19]. Properties of connectivity in vague fuzzy graphs with an application in building university networks studied [20]. The restrained-rainbow reinforcement number of graphs and topological indices in fuzzy graphs with applications in decision-making problems explored [21, 22]. The influence of electromagnetic fields on fluid flow, particularly through the Lorentz force, is another area of significant interest. The Lorentz force, resulting from the interaction between the magnetic field and the electric current, can substantially alter the flow characteristics of conducting fluids. This interaction is especially pertinent in processes involving electromagnetic pumps, magnetic drug targeting, and materials processing. In real-world applications, the fluid properties and external conditions are often subject to uncertainties. Traditional deterministic models may not adequately capture these variabilities, leading to suboptimal or inaccurate predictions. Fuzzy logic, which allows for the incorporation of uncertainties and imprecise information, provides a powerful tool to address these challenges, offering a more realistic and comprehensive modelling approach. This study aims to investigate the combined effects of Casson fluid dynamics, micropolar characteristics, nanoparticle inclusion, and Lorentz force on fluid flow over a stretching surface within a fuzzy environment. By incorporating fuzzy logic into the analysis, we aim to capture the inherent uncertainties in the system, providing a more robust and realistic understanding of the flow behaviour.

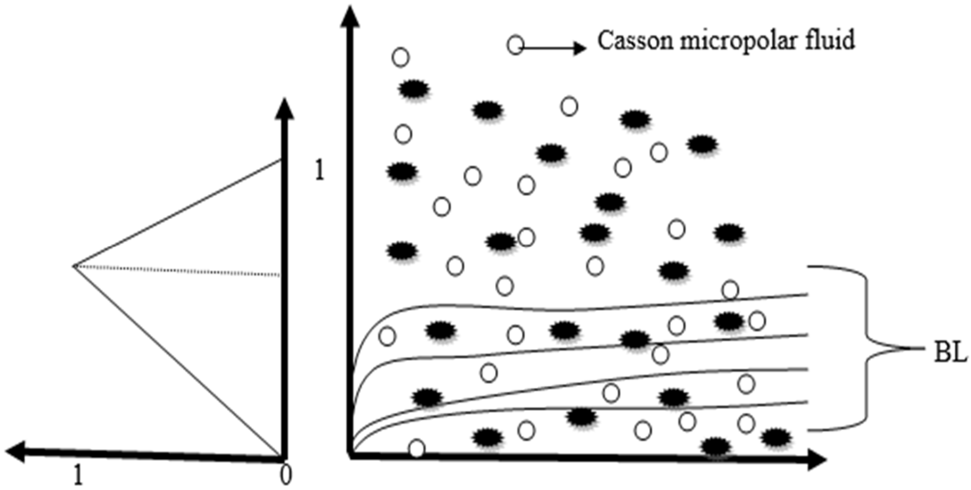


Figure 1: TFM and Flow Representation

The governing equations, which account for the complex interplay of these factors, are solved numerically to explore the impact on velocity, microrotation, and temperature profiles. The findings of this research have significant implications for various industrial applications, including enhanced heat transfer systems, improved material processing techniques, and more efficient electromagnetic devices. By providing new insights into the modulation of fluid flow under the influence of multiple complex factors, this study contributes to the advancement of fluid dynamics and thermal management technologies.

2. Mathematical analysis

Casson micropolar nanofluid flow over a stretching surface, considering the influence of the Lorentz force, the governing equations include the continuity, momentum, microrotation, and energy equations. Below are the equations in their general form

$$\frac{\partial u^*}{\partial x^*} = - \frac{\partial v^*}{\partial y^*}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} - \nu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right) - \left(\frac{k_1}{\rho}\right)\frac{\partial N^*}{\partial y^*} = -\frac{\sigma B_0^2\, u^*}{\rho}$$

$$u^* \frac{\partial N^*}{\partial x^*} + v^* \frac{\partial N^*}{\partial y^*} - \frac{\gamma}{\rho}\frac{\partial^2 N^*}{\partial y^{*2}} = \left(\frac{k_1}{\rho}\right)\left(2N^* + \frac{\partial u^*}{\partial y^*}\right)$$

$$\rho C_p \left[u^* \frac{\partial T^*}{\partial x^*} + v \frac{\partial T^*}{\partial y^*}\right] - k \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y} - q''' - \nu \rho \left[\frac{\partial u^*}{\partial y^*}\right]^2 = \frac{\rho D_m k_T}{C_s}\frac{\partial^2 C^*}{\partial y^{*2}}$$

$$u \frac{\partial C^*}{\partial x^*} + v \frac{\partial C^*}{\partial y^*} - D_m \frac{\partial^2 C^*}{\partial y^{*2}} - \frac{D_m\, k_T}{T_m}\frac{\partial^2 T^*}{\partial y^{*2}} = - k^* (C - C_\infty)$$

where (u, v) Velocity components, T - Fluid temperature, C - Concentration fluid, q''' -

Irregular heat parameter, Sc - Schmidt number, Du - Dufour number, D_m Mass diffusivity, B_{iT} Thermal Biot Number, B_{iC} Mass Biot Number.

The irregular heat parameter is defined by the non-uniform heat source/sink in the system. The temperature difference between T and the free stream temperature T_∞ is small, and by neglecting higher-order terms, Taylor's series expansion about T_∞ simplifies the temperature dependency. Using Roseland's approximation for thermal radiation. The following equation is the result of using Rosland's approximation for thermal radiation.

$$q_r = -\frac{16}{3}\frac{\sigma^*}{k^*}T^3\frac{\partial T^*}{\partial y^*}$$

The boundary conditions are specified for the flow and temperature fields

$$u = u_w + L\left[\frac{\partial u^*}{\partial y^*}\right], -k\left[\frac{\partial T^*}{\partial y^*}\right] = h_1\,[T_\infty - T], -N = -m$$

$$\left[\frac{\partial u^*}{\partial y^*}\right], D_m\left[\frac{\partial C^*}{\partial y^*}\right] = h_2\,[C_w - C], \quad \text{as } y \to 0$$

$$u \to 0, T \to T_\infty, C \to C_\infty \quad \text{as } y \to \infty$$

The stream function $\psi = \psi(x, y)$ is defined such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \eta = \sqrt{\frac{a}{v}}y$$

The velocity components are

$$\varphi = \sqrt{av}\,xf(\eta) \quad , \quad u = \frac{\partial \varphi}{\partial y} = axf'(\eta) \quad , \quad N = ax\left(\sqrt{\frac{a}{v}}\right)h(\eta) \quad , \quad v = -\frac{\partial \varphi}{\partial x} = \sqrt{avf(\eta)}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad T = T_\infty(1 + (\theta_w - 1)\theta)$$

Here, $f(\eta), h(\eta), \theta(\eta)$ and $\phi(\eta)$ are dimensionless functions representing the velocity, microrotation, temperature, and concentration profiles, respectively. The parameters T_w and C_w denote the temperature and concentration at the wall.

Using similarity Equation and the governing Equations are transformed as

$$\left[1 + \frac{1}{\beta}\right]f''' + ff' - (f')^2 + Kh' - Mf' = 0$$

$$\left[1 + \frac{K}{2}\right]h'' + fh' - f'h - K(2h + f'') = 0$$

$$[1 + N_r\{1 + 3(\theta_w - 1) + 3(\theta_w - 1)^2\theta^2 + (\theta_w - 1)^3\theta^3\}]\theta'' + N_r[(\theta_w - 1)(\theta')^2 + 6(\theta_w - 1)^2\theta(\theta')^2 + 3(\theta_w - 1)^3\theta^2(\theta')^2] + P_r f\theta' - P_r f'\theta + A^* f' + B^*\theta + P_r Du\varphi'' = 0$$

$$\varphi'' + Scf\varphi' - k_r Sc\varphi + ScSr\theta'' = 0$$

here

$$Pr = \frac{\upsilon\rho C_p}{k} = \frac{\mu C_p}{k}, M = \frac{\sigma B_0^2}{a\rho}, \text{Sr} = \frac{D_m K_T}{\upsilon T_m}\left(\frac{T_w - T_\infty}{C_w - C_\infty}\right), \text{Sc} = \frac{\upsilon}{D_m}, \text{Du} = \frac{D_m K_T(C_w - C_\infty)}{C_s C_p \upsilon(T_w - T_\infty)},$$

The transformed boundary settings are as follows,

$$f' = 1 + \gamma f'', \ f = 0, \ \theta' = Bi_T(1 - \theta), \ \phi' = Bi_c(1 - \phi) \text{ as } \eta \to 0,$$

$$f'(\eta) = 0, \quad \theta(\eta) = 0, \quad \phi(\eta) = 0 \text{ as } \eta \to \infty$$

It is discovered that the nanofluid model lacks micropolar effects when vertex viscosity or $K = 0$ is eliminated.

The Sherwood coefficient, Nusselt number, and skin friction coefficient are key dimensionless quantities used to characterize mass transfer, heat transfer, and fluid flow, respectively

Skin friction coefficient (C_f)

The skin friction coefficient is a dimensionless number representing the ratio of the shear stress at the wall to the dynamic pressure of the fluid flow. It is given by $C_{f_x} Re_x^{\frac{1}{2}} = \left(1 + \frac{1}{\beta}\right) f''(0)$

Nusselt number(Nu)

The Nusselt number is a dimensionless number representing the ratio of convective to conductive heat transfer across the boundary layer. It is given by $NuRe_x^{\frac{1}{2}} = -(1 + N_r(\theta_w)^3)\theta'(0)$

Sherwood coefficient(Sh)

The Sherwood coefficient is a dimensionless number used to describe mass transfer at the surface. It is given by $Sh\ Re_x^{-\frac{1}{2}} = -\emptyset'(0)$.

Fuzzy membership function

The triangular membership function is defined by three parameters a,b,c, where $a \le b \le c$,

$$\mu_\beta(x) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a} & if\ a < x \le b \\ \dfrac{c-x}{c-b} & if\ b < x \le c \\ 0 & if\ x \ge c \end{cases}$$

3. Fuzzification

The σ -cut approach for both $\phi 1$ and $\phi 2$ shows how the triangular fuzzy number can be expressed depending on the chosen level of certainty σ. For example, as σ increases, the interval of certainty $[0.05 - \sigma, 0.1 - \sigma]$, shrinks, reflecting a more precise estimate within the specified fuzzy range. Using the σ -cut approach, $\phi 1$ and $\phi 2$ can be expressed as,

$$\phi 1 = [0.05 - \sigma, 0.1 - \sigma], \phi 2 = [0.05 - \sigma, 0.1 - \sigma]\ .$$

Converting differential equations into fuzzy models involves translating deterministic relationships into fuzzy rules, which capture uncertainties and variability. This approach allows for more flexible modeling and decision-making in complex systems where precise inputs or conditions are not fully known or are subject to variation. Each step involves careful selection of fuzzy sets, formulation of rules, and consideration of defuzzification methods to ensure accurate representation and interpretation of the fuzzy model outputs.

4. Results and discussion

The equations (10) – (15) are solved by MATLAB bvp4c method. For graphical results, $\beta = 1.5, Gr_T = 0.8, M = 1.5, Nr = 0.2, A^* = B^* = 0.1, \mathrm{Pr} = 7, \theta_w = 2, Gr_c = 0.8, BiT = 2, BiC = 2, Sc = 0.6, Kr = 0.2, Sr = 0.2, Ec = 0.2$ and Du = 0.1 . The plots, $f'(\eta), \theta(\eta)$ and $\phi(\eta)$ indicate the corresponding flow fields' curves.

Figs 2-5 displays Effects of the magnetic terms on the velocity plot, temperature and concentration and micropolar. When the value of the magnetic parameter MMM is increased, a reduction in velocity is observed. This occurs because, when applied transversely to the flow direction, M generates a Lorentz force, which has a significant impact on fluid flow. The Lorentz force, an essential concept in electromagnetism, finds applications in hydrodynamics, plasma accelerators, magnetohydrodynamic (MHD) accelerators, and other engineering fields. In electrically conductive fluids, the Lorentz force slows down the flow. Consequently, as the magnetic parameter increases, the Lorentz force reduces the velocity and momentum boundary layer thickness. Additionally, the temperature and concentration of the fluid increase as the magnetic parameter grows. An increase in the magnetic parameter M also leads to an increase in the micropolar parameter. As M increases, the velocity decreases due to the Lorentz force acting against the flow. Lorentz force is fundamental in particle accelerators like cyclotrons and synchrotrons, where charged particles are accelerated and guided along precise paths by magnetic fields. In fusion research and plasma confinement, such as in tokamaks, Lorentz force helps control the movement and stability of the plasma, crucial for achieving controlled nuclear fusion reactions. Figs 6-9 depict casson parameter on the flow environs such as Velocity Profile, temperature, concentration and micro rotation. The decrease observed indicates that higher values of β lead to lower velocity gradients in the flow field and suggests reduced microrotation effects under increased. Velocity profile and microrotation declined for larger β while $\theta, \emptyset$ are raise β. Fig 10-13 depicts the Dufour factor on the flow fields. An increase in the Dufour number leads to an increase in temperature within the fluid. This occurs because higher Dufour values imply stronger thermal diffusion, resulting in more efficient heat transfer and thus elevated temperatures in the fluid. A decrease in concentration gradient with an increase in the Dufour number. This indicates that as Du increases, the effect of thermal diffusion becomes more pronounced compared to mass diffusion.

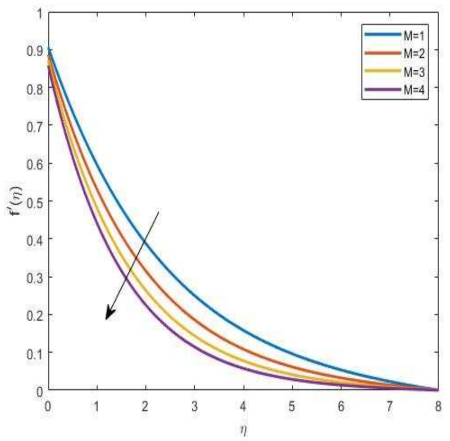


Figure 2: Magnetic field on $f'(\eta)$

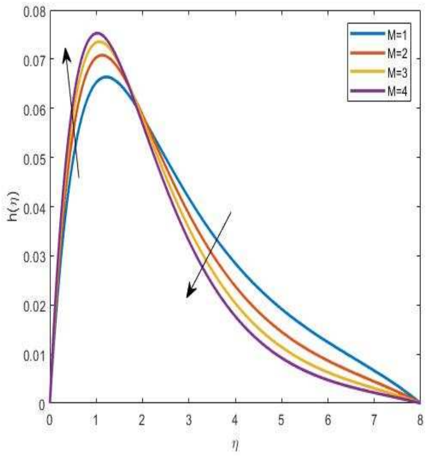


Figure 3: Magnetic field on $h(\eta)$

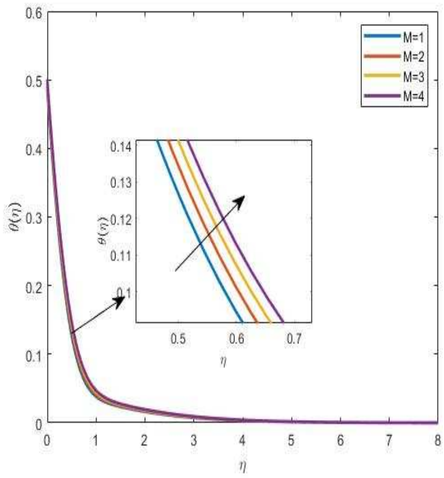


Figure 4: Magnetic field on $\theta(\eta)$

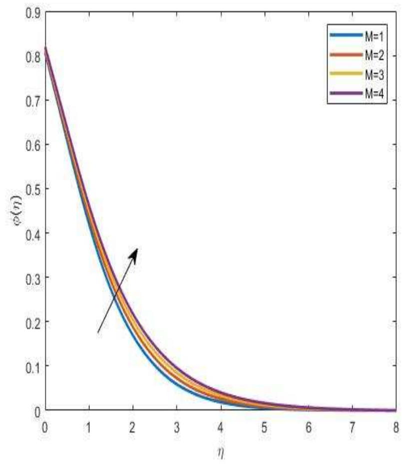


Figure 5: Magnetic field on $\varphi(\eta)$

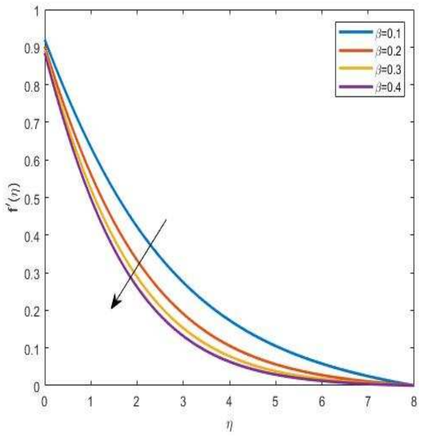


Figure 6: Casson parameter on $f'(\eta)$

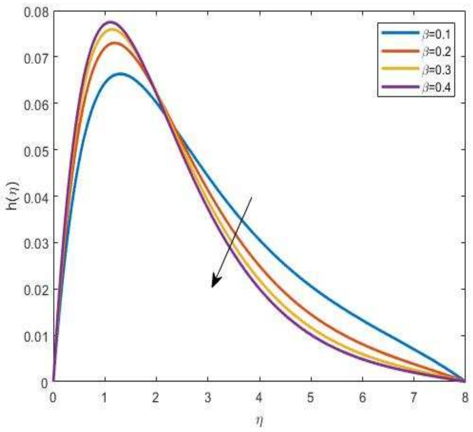


Figure 7: Casson parameter on $h(\eta)$

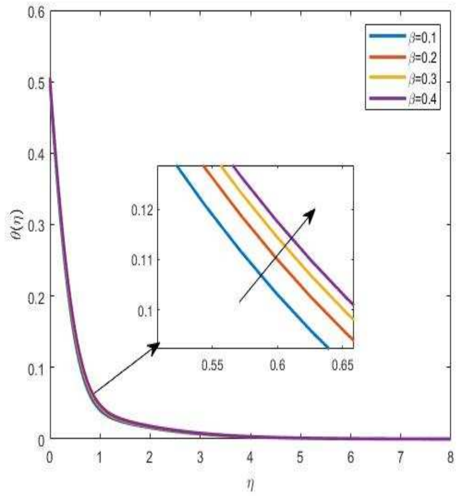


Figure 8: Casson parameter on $\theta(\eta)$

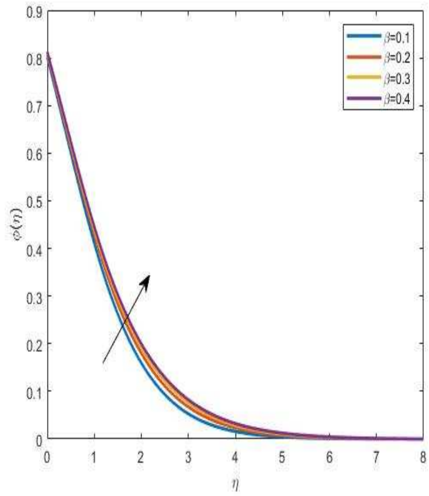
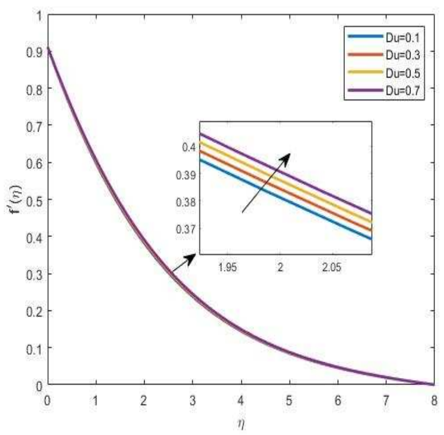


Figure 9: Casson parameter on $\varphi(\eta)$



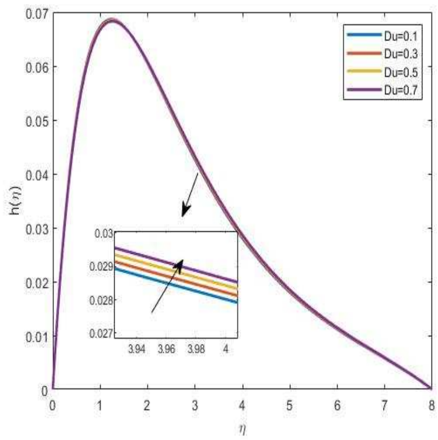
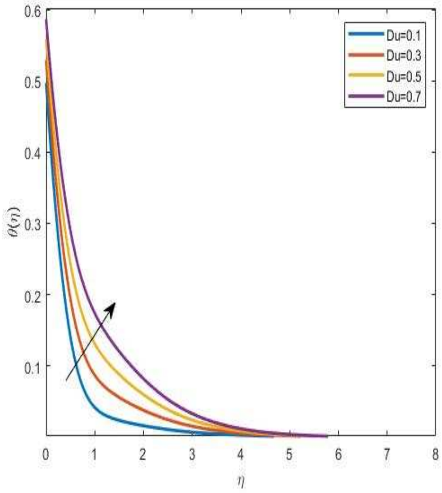


Figure 10: Dufour on $f'(\eta)$ **Figure 11:** Dufour on $h(\eta)$



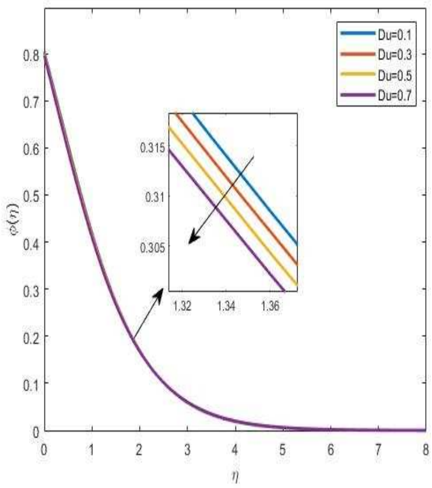


Figure 12: Dufour on $\theta(\eta)$ **Figure 13:** Dufour on $\varphi(\eta)$

5. Conclusion

The inclusion of Lorentz force significantly alters the flow characteristics. The electromagnetic force enhances the flow velocity near the stretching surface while inducing a retardation effect further away from the surface. This modulation is critical for applications involving electromagnetic control of fluid flows. By employing fuzzy logic, the study effectively captures the uncertainties in the system. The fuzzy environment representation shows that variability in fluid properties and external conditions can substantially impact the flow and thermal characteristics, underscoring the importance of considering uncertainties in real-world applications.

The major findings are

- Both decrease with higher values of the magnetic and Casson parameters. This decrease is counter balanced by an increase in temperature and concentration, highlighting the complex interplay between fluid dynamics and thermodynamic properties.
- Increase with higher M and β values, indicating enhanced thermal and mass transport effects under magnetic influence.
- A higher Dufour number results in a reduction in concentration. This effect occurs because the mass flux due to concentration gradients influences thermal transport.
- Reduces with higher Schmidt number and thermal conductivity ratio. This suggests that mass transfer efficiency decreases under these conditions.

By incorporating fuzzy logic, the study's approach can be applied to the design of robust systems that can perform reliably under uncertain conditions, making them suitable for critical applications where precision and reliability are paramount. The controlled environment provided by Lorentz force can be utilized in crystal growth processes, where uniformity and precision are essential for producing high-quality crystals for semiconductors and other applications. Future Comparing the behavior of Casson fluids with other non-Newtonian fluids, such as power-law fluids and Bingham plastics, under similar conditions and developing robust design frameworks that incorporate uncertainty quantification and fuzzy logic to ensure reliable performance under varying conditions.

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