Annals of Pure and Applied Mathematics
Vol. 29, No. 2, 2024, 103-107
ISSN: 2279-087X (P), 2279-0888(online)
Published on 21 April 2024
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v29n2a02936

Modified Elliptic Revan Index of Two Families of Nanotubes

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Received 12 March 2024; accepted 20 April 2024

Abstract. In this study, we introduce the modified elliptic Revan index and its corresponding exponential of a graph. Furthermore, we compute the elliptic Revan and modified elliptic Revan indices and their corresponding exponentials for two families of nanotubes.

Keywords: Elliptic Revan index, modified elliptic Revan index, nanotube

AMS Mathematics Subject Classification (2010): 05C07, 05C09, 05C92

1. Introduction

Let $G = (V(G), E(G))$ be a finite, simple connected graph. The degree d_u is the number of vertices adjacent to u. Let $\Delta(G)$ ($\delta(G)$) denote the maximum (minimum) degree among the vertices of G. The Revan vertex degree of a vertex u in G is defined as $r_u = \Delta(G) + \delta(G) - d_u$. We refer to the book [1] for undefined terms and notation.

A molecular graph is a graph whose vertices correspond to the atoms and the edges of the bonds. Chemical graph theory has an important effect on the development of the Chemical Sciences. A single number that can be used to characterize some properties of the graph of molecular is called a topological index. Numerous topological indices have been considered in Theoretical Chemistry see [2, 3].

The elliptic Revan index [4] of a graph G is defined as

$$ER(G) = \sum_{uv \in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}.$$

The elliptic Revan exponential [4] of a graph G is defined as

$${}^{m}ER(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{(r_u + r_v)\sqrt{r_u^2 + r_v^2}}}$$

Recently, some elliptic indices were studied in [5-9].

We put forward the modified elliptic Revan index of a graph G and it is defined as

$$ {}^{m}ER(G)=\sum_{uv\in E(G)}\frac{1}{(r_u+r_v)\sqrt{r_u^2+r_v^2}}. $$

We define the modified elliptic Revan exponential of the graph G as

$$ {}^{m}ER(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{(r_u+r_v)\sqrt{r_u^2+r_v^2}}} $$

Recently, some new graph indices were studied in [10, 11].

In this paper, we determine the modified elliptic Revan index and its exponential of two families of nanotubes.

2. Results for $HC_5C_7[p,q]$ nanotubes

We consider $HC_5C_7[p,q]$ nanotubes in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. The 2-D lattice of nanotube $HC_5C_7[8,4]$ is shown in Figure 1.

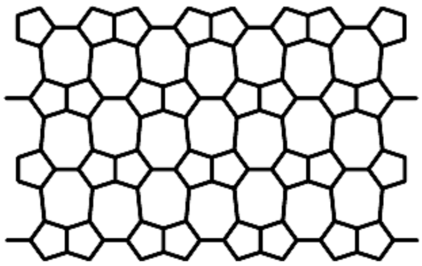

Figure 1: 2-D lattice of $HC_5C_7[8,4]$ nanotube

Let H be the graph of $HC_5C_7[p,q]$ nanotube. We obtain that H has $4pq$ vertices and $6pq - p$ edges. In H, there are two types of edges as follows:

$E_1 = \{uv\in E(H) \mid d_u = 2, d_v = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv\in E(H) \mid d_u = d_v = 3\}$, $|E_2| = 6pq - 5p$.

We have $\Delta(H) = 3$ and $\delta(H) = 2$. Thus $r_u=\Delta(H) + \delta(H) - d_u = 5 - d_u$.

Thus there are two types of Revan edges as follows:

$RE_1 = \{uv\in E(H) \mid r_u = 3, r_v = 2\}$, $|RE_1| = 4p$.

$RE_2 = \{uv\in E(H) \mid r_u = r_v = 2\}$, $|RE_2| = 6pq - 5p$.

Theorem 1. Let $HC_5C_7[p,q]$ be the nanotubes. Then

$$ ER(H)=48\sqrt{2}pq+\left(20\sqrt{13}-40\sqrt{2}\right)p\sqrt{2}. $$

Proof: We have

$$ \begin{aligned} ER(H)&=\sum_{uv\in E(H)}(r_u+r_v)\sqrt{r_u^2+r_v^2}\\ &=4p(3+2)\sqrt{3^2+2^2}+(6pq-5p)(2+2)\sqrt{2^2+2^2}\\ &=48\sqrt{2}pq+\left(20\sqrt{13}-40\sqrt{2}\right)p\sqrt{2}. \end{aligned} $$

Theorem 2. Let $HC_5C_7[p,q]$ be the nanotubes. Then

$$ER(H,x)=4px^{5\sqrt{13}}+(6pq-5p)x^{8\sqrt{2}}.$$

Proof: We have

$$ER(H,x)=\sum_{uv\in E(H)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}}=4px^{(3+2)\sqrt{3^2+2^2}}+(6pq-5p)x^{(2+2)\sqrt{2^2+2^2}}$$
$$=4px^{5\sqrt{13}}+(6pq-5p)x^{8\sqrt{2}}.$$

Theorem 3. Let $HC_5C_7[p,q]$ be the nanotubes. Then

$$^{m}ER(H)=\frac{3}{4\sqrt{2}}pq+\left(\frac{4}{5\sqrt{13}}-\frac{5}{8\sqrt{2}}\right)p.$$

Proof: We have

$$^{m}ER(H)=\sum_{uv\in E(H)}\frac{1}{(r_u+r_v)\sqrt{r_u^2+r_v^2}}=\frac{4p}{(3+2)\sqrt{3^2+2^2}}+\frac{(6pq-5p)}{(2+2)\sqrt{2^2+2^2}}$$
$$=\frac{3}{4\sqrt{2}}pq+\left(\frac{4}{5\sqrt{13}}-\frac{5}{8\sqrt{2}}\right)p.$$

Theorem 4. Let $HC_5C_7[p,q]$ be the nanotubes. Then

$$^{m}ER(H,x)=4px^{\frac{1}{5\sqrt{13}}}+(6pq-5p)x^{\frac{1}{8\sqrt{2}}}.$$

Proof: We have

$$^{m}ER(H,x)=\sum_{uv\in E(H)} x^{\frac{1}{(r_u+r_v)\sqrt{r_u^2+r_v^2}}}=4px^{\frac{1}{(3+2)\sqrt{3^2+2^2}}}+(6pq-5p)x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}}$$
$$=4px^{\frac{1}{5\sqrt{13}}}+(6pq-5p)x^{\frac{1}{8\sqrt{2}}}.$$

3. Results for $SC_5C_7[p,q]$ nanotubes

We consider $SC_5C_7[p,q]$ nanotubes in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-*D* lattice of nanotube $SC_5C_7[8,4]$ is depicted in Figure 2.

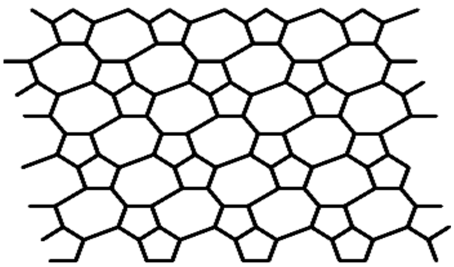

Figure 2: 2-*D* lattice of nanotube $SC_5C_7[8,4]$

Let S be the graph of $SC_5C_7[p,q]$ nanotubes. We obtain that S has $4pq$ vertices and $6pq - p$ edges. In H, there are three types of edges as follows:

$E_1 = \{uv\in E(S) \mid d_u = d_v = 2\}$, $|E_1| = q$.

$E_2 = \{uv\in E(S) \mid d_u = 2, d_v = 3\}$, $|E_2| = 6q$.

$E_3 = \{uv \in E(S) \mid d_u = d_v = 3\}$, $|E_3| = 6pq - p - 7q$.

We have $\Delta(S) = 3$ and $\delta(S) = 2$. Thus $r_u = \Delta(S) + \delta(S) - d_u = 5 - d_u$.

Thus there are three types of Revan edges as follows:

$RE_1 = \{uv \in E(S) \mid r_u = r_v = 3\}$, $|RE_1| = q$.

$RE_2 = \{uv \in E(S) \mid r_u = 3, r_v = 2\}$, $|RE_2| = 6q$.

$RE_3 = \{uv \in E(S) \mid r_u = r_v = 2\}$, $|RE_3| = 6pq - p - 7q$.

Theorem 5. Let $SC_5C_7[p,q]$ be the nanotubes. Then

$$ER(S) = 48\sqrt{2}pq - 8\sqrt{2}p + \left(30\sqrt{13} - 38\sqrt{2}\right)q.$$

Proof: We have

$$ER(S) = \sum_{uv \in E(S)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}$$

$$= q(3+3)\sqrt{3^2+3^2} + 6q(3+2)\sqrt{3^2+2^2} + (6pq - p - 6q)(2+2)\sqrt{2^2+2^2}$$

$$= 48\sqrt{2}pq - 8\sqrt{2}p + \left(30\sqrt{13} - 38\sqrt{2}\right)q.$$

Theorem 6. Let $SC_5C_7[p,q]$ be the nanotubes. Then

$$ER(S,x) = qx^{18\sqrt{2}} + 6qx^{5\sqrt{13}} + (6pq - p - 7q)x^{8\sqrt{2}}.$$

Proof: We have

$$ER(S,x) = \sum_{uv \in E(S)} x^{(r_u + r_v)\sqrt{r_u^2 + r_v^2}}$$

$$= qx^{(3+3)\sqrt{3^2+3^2}} + 6qx^{(3+2)\sqrt{3^2+2^2}} + (6pq - p - 7q)x^{(2+2)\sqrt{2^2+2^2}}$$

$$= qx^{18\sqrt{2}} + 6qx^{5\sqrt{13}} + (6pq - p - 7q)x^{8\sqrt{2}}.$$

Theorem 7. Let $SC_5C_7[p,q]$ be the nanotubes. Then

$$^{m}ER(S) = \frac{3}{4\sqrt{2}}pq - \frac{1}{8\sqrt{2}}p + \left(\frac{1}{18\sqrt{2}} + \frac{6}{5\sqrt{13}} - \frac{7}{8\sqrt{2}}\right)q.$$

Proof: We have

$$^{m}ER(S) = \sum_{uv \in E(S)} \frac{1}{(r_u + r_v)\sqrt{r_u^2 + r_v^2}}$$

$$= \frac{q}{(3+3)\sqrt{3^2+3^2}} + \frac{6q}{(3+2)\sqrt{3^2+2^2}} + \frac{(6pq - p - 7q)}{(2+2)\sqrt{2^2+2^2}}$$

$$= \frac{3}{4\sqrt{2}}pq - \frac{1}{8\sqrt{2}}p + \left(\frac{1}{18\sqrt{2}} + \frac{6}{5\sqrt{13}} - \frac{7}{8\sqrt{2}}\right)q.$$

Theorem 8. Let $SC_5C_7[p,q]$ be the nanotubes. Then

$$^{m}ER(S,x) = qx^{\frac{1}{18\sqrt{2}}} + 6qx^{\frac{1}{5\sqrt{13}}} + (6pq - p - 7q)x^{\frac{1}{8\sqrt{2}}}.$$

Proof: We have

$$^{m}ER(S,x)=\sum_{uv\in E(S)} x^{\frac{1}{(r_u+r_v)\sqrt{r_u^2+r_v^2}}}$$
$$=qx^{\frac{1}{(3+3)\sqrt{3^2+3^2}}}+6qx^{\frac{1}{(3+2)\sqrt{3^2+2^2}}}+(6pq-p-7q)\,x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}}$$
$$=qx^{\frac{1}{18\sqrt{2}}}+6qx^{\frac{1}{5\sqrt{13}}}+(6pq-p-7q)\,x^{\frac{1}{8\sqrt{2}}}.$$

4. Conclusion

We have introduced the modified elliptic Revan index and its exponential of a graph. Furthermore, the elliptic Revan and modified elliptic Revan indices and their corresponding exponentials for two families of nanotubes are determined.

Acknowledgements. The author is very grateful to the reviewers for their comments for updating the paper.

Conflict of interest. This is a single-author paper, so there is no conflict of interest.

Authors' Contributions. This is a single-author paper and it is fully the author's contribution.

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