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The Status Gourava Indices of Middle Graphs of Some Standard Graphs

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Abstract. The sum of the shortest distance between a vertex u from all other vertices of a graph G is called the status of the vertex u and is denoted by $\sigma(u)$. In this article, we have found the precise formula for the derived graphs of a few standard graphs. We have obtained Status Gourava indices of middle graphs of some standard graphs namely cycle graph, star graph, complete graph, wheel graph and friendship graph. We have calculated ten standard status indices of middle graphs of standard graphs using this new index.

Keywords: Graphs, Gourava index, Degree-based topological index.

AMS Mathematics Subject Classification (2010): 05C31

1. Introduction

Every graph that is taken into consideration here is finite, nontrivial, undirected, free of loops and multiple edges, and without isolated vertices. For words or notations that are not defined in this work, found in Harary [1]. Vertex set is denoted by $V(G)$, edge set is denoted by $E(G)$ for a graph G. The middle graph M(G) is represented by the graph G, from which a new vertex is inserted into each edge of G, and edges are drawn between these new vertices which lie on adjacent edges of G. The length of the shortest path between two vertices u and v, denoted by $d(u,v)$ is the distance between them. The sum of distances of a vertex u from all other vertices of a graph is called the status of the vertex u with notation $\sigma(u)$. Kulli introduced some new status indices of the graph. The $(a,b)-$ status index, as

$$S_{a,b} = \sum_{uv\in E(G)} \{(\sigma(u))^a \cdot (\sigma(v))^b + (\sigma(u))^b \cdot (\sigma(v))^a\}$$

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Kulli introduced status Gourava indices of the graph. For notations and definitions, we refer [1, 5] and [6]. Kulli et al. have found the first status index $S_1(G)$, second status index $S_2(G)$, product connectivity status index $PS(G)$, reciprocal product connectivity status index $RPS(G)$, the general second status index $S_2^a(G)$, the first status Gourava index $SGO_1(G)$, the second status Gourava index $SGO_2(G)$, of middle graphs of some standard graphs namely cycle graph, star graph, complete graph, wheel graph and friendship graph.

Here we have obtained these indices for the middle graphs of cycle graph C_n, star graph $K_{1,n}$, complete graph K_n, wheel graph W_n and friendship graph F_n.

2. Main results

Theorem 1. Let $M[C_n]$ be the middle graph of the cycle graph with $2n$ vertices and $3n$ edges then the general first status Gourava index of $M[C_n]$ is

$$SG_1^a(M[C_n]) = n\left\{\frac{n^4+2n^3+5n^2+4n}{4}\right\}^a + 2n\left\{\frac{n^4+4n^3+5n^2+6n-4}{4}\right\}^a.$$

Proof: By using the definition and Table 1, we obtain

Table 1:

$(\sigma(u),\sigma(v))\ /\ uv\epsilon E(M(C_n))$	$\left(\frac{n^2+n}{2},\frac{n^2+n}{2}\right)$	$\left(\frac{n^2+n}{2},\frac{n^2+3n-2}{2}\right)$
Total number of edges	n	$2n$

$$SG_1^a(M[C_n]) = \sum_{u,v\epsilon E(C_n)}[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)]^a$$

$$SG_1^a(M[C_n]) = n\left\{\frac{n^2+n}{2}+\frac{n^2+n}{2}+(n^2+n)^2\right\}^a + 2n\left\{\frac{n^2+n}{2}+\frac{n^2+3n-2}{2}+ (\frac{n^2+n}{2})(\frac{n^2+3n-2}{2}\right\}^a$$

$$SG_1^a(M[C_n]) = n\left\{\frac{n^4+2n^3+5n^2+4n}{4}\right\}^a + 2n\left\{\frac{n^4+4n^3+5n^2+6n-4}{4}\right\}^a$$

Corollary 1.1. The first status Gourava index of $M[C_n]$ is

$$SG_1(M[C_n]) = \left\{\frac{3n^5+10n^4+15n^3+16n^2-8n}{4}\right\}$$

Proof: By taking $a = 1$ in theorem 1, we obtain

$$SG_1(M[C_n]) = n\left\{\frac{n^4+2n^3+5n^2+4n}{4}\right\} + 2n\left\{\frac{n^4+4n^3+5n^2+6n-4}{4}\right\}$$

$$SG_1(M[C_n]) = \left\{\frac{3n^5+10n^4+15n^3+16n^2-8n}{4}\right\}$$

Corollary 1.2. The first hyper status Gourava index of $M[C_n]$ is

$$HSG_1(M[C_n]) = n\left\{\frac{n^4+2n^3+5n^2+4n}{4}\right\}^2 + 2n\left\{\frac{n^4+4n^3+5n^2+6n-4}{4}\right\}^2$$

Proof: By taking $a = 2$ in theorem 1, we obtain

$$HSG_1(M[C_n]) = n\left\{\frac{n^4+2n^3+5n^2+4n}{4}\right\}^2 + 2n\left\{\frac{n^4+4n^3+5n^2+6n-4}{4}\right\}^2$$

Corollary 1.3. The sum connectivity status Gourava index of $M[C_n]$ is

$$SSG(M[C_n]) = \frac{2n}{\sqrt{n^4+2n^3+5n^2+4n}} + \frac{4n}{\sqrt{n^4+4n^3+5n^2+6n-4}}.$$

Proof: By taking $a = \frac{-1}{2}$ in theorem 1, we obtain

$$SSG(M[C_n]) = \frac{2n}{\sqrt{n^4+2n^3+5n^2+4n}} + \frac{4n}{\sqrt{n^4+4n^3+5n^2+6n-4}}.$$

Theorem 2. The general second status Gourava index of $M[C_n]$ is

$SG_2^a(M[C_n]) = n\left\{\frac{n^6+3n^5+3n^4+n^3}{4}\right\}^a + 2n\left\{\frac{2n^6+12n^5+16n^4-8n^3-10n^2+4n}{8}\right\}^a.$

Proof: $SG_2^a(M[C_n]) = \sum_{u,v\in E(C_n)}\{[\sigma(u)+\sigma(v)].\sigma(u)\sigma(v)\}^a.$

$SG_2^a(M[C_n]) = n\left\{\left[\frac{n^2+n}{2}+\frac{n^2+n}{2}\right]\left(\frac{n^2+n}{2}\right)^2\right\}^a + 2n\left\{\left[\frac{n^2+n}{2}+\frac{n^2+3n-2}{2}\right]\left(\frac{n^2+n}{2}\right)\left(\frac{n^2+3n-2}{2}\right)\right\}^a$

$SG_2^a(M[C_n]) = n\left\{\frac{n^6+3n^5+3n^4+n^3}{4}\right\}^a + 2n\left\{\frac{2n^6+12n^5+16n^4-8n^3-10n^2+4n}{8}\right\}^a.$

Corollary 2.1. The second status Gourava index of $M[C_n]$ is

$SG_2(M[C_n]) = \left\{\frac{3n^7+15n^6+19n^5-7n^4-10n^3+4n^2}{4}\right\}$

Proof: By taking $a = 1$ in theorem 2, we obtain

$$SG_2(M[C_n]) = \left\{\frac{3n^7+15n^6+19n^5-7n^4-10n^3+4n^2}{4}\right\}$$

Corollary 2.2. The second hyper status Gourava index of $M[C_n]$ is

$HSG_2(M[C_n]) = \frac{n^7}{16}\{n^3+3n^2+3n+1\}^2 + \frac{n^3}{32}\{2n^5+12n^4+16n^3-8n^2-10n+4\}^2.$

Proof: By taking $a = 2$ in theorem 2

$HSG_2(M[C_n]) = \frac{n^7}{16}\{n^3+3n^2+3n+1\}^2 + \frac{n^3}{32}\{2n^5+12n^4+16n^3-8n^2-10n+4\}^2.$

Corollary 2.3. The product connectivity of status Gourava index of $M[C_n]$ is

$PSG(M[C_n]) = \frac{2n}{\sqrt{n^6+3n^5+3n^4+n^3}} + \frac{4\sqrt{2}n}{\sqrt{2n^6+12n^5+16n^4-8n^3-10n^2+4n}}$

Proof: By taking $a = -\frac{1}{2}$ in theorem 2.

$PSG(M[C_n]) = \frac{2n}{\sqrt{n^6+3n^5+3n^4+n^3}} + \frac{4\sqrt{2}n}{\sqrt{2n^6+12n^5+16n^4-8n^3-10n^2+4n}}$

Corollary 2.4. The reciprocal product connectivity of status Gourava index of $M[C_n]$ is

$RPSG(M[C_n]) = \frac{n}{2\sqrt{n^6+3n^5+3n^4+n^3}} + \frac{n}{\sqrt{2n^6+12n^5+16n^4-8n^3-10n^2+4n}}.$

Proof: By taking $a = \frac{1}{2}$ in Theorem 2.

Theorem 3. Let $M[K_{1,n}]$ be the middle graph of star graph with $n+1$ vertices and $\frac{n^2+3n}{2}$

$(\sigma(u),\sigma(v))\ /\ uv\epsilon E\left(M(K_{1,n})\right)$	$(3n-1,3n-1)$	$(3n,3n-1)$	$(3n-1,5n-2)$
Total number of edges	$\frac{n(n-1)}{2}$	n	n

edges then the general first status Gourava index of $M[K_{1,n}]$ is

$SG_1^a(M[K_{1,n}]) = \frac{n(n-1)}{2}\{9n^2-1\}^a + n\{9n^2+3n-1\}^a + n\{15n^2-3n-1\}^a$

Proof: By using the definition and Table 2, we obtain

$SG_1^a(M[C_n]) = \sum_{u,v \in E(K_{1,n})}[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^a$

$SG_1^a(M[C_n]) = \frac{n(n-1)}{2}\{3n - 1 + 3n - 1 + (3n-1)^2\}^a + n\{3n + 3n - 1 + 3n(3n-1)\}^a$

$+n\{3n - 1 + 5n - 2 + (3n-1)(5n-2)\}^a$

$SG_1^a(M[K_{1,n}]) = \frac{n(n-1)}{2}\{9n^2 - 1\}^a + n\{\{9n^2 + 3n - 1\}\}^a + n\{15n^2 - 3n - 1\}^a$

Corollary 3.1. The first status Gourava index of $M[K_{1,n}]$ is

$$SG_1(M[K_{1,n}]) = \left(\frac{9n^4 + 39n^3 - n^2 - 3n}{2}\right)$$

Proof: By taking $a = 1$ in theorem 3, we obtain

$$SG_1(M[K_{1,n}]) = \left(\frac{9n^4 + 39n^3 - n^2 - 3n}{2}\right)$$

Corollary 3.2: The firsthyper status Gourava index of $M[K_{1,n}]$ is

$$HSG_1(M[K_{1,n}]) = \frac{81n^6 + 531n^5 - 90n^4 - 42n^3 + n^2 + 3n}{2}$$

Proof. By taking $a = 2$ in theorem 3, we obtain

$$HSG_1(M[K_{1,n}]) = \frac{81n^6 + 531n^5 - 90n^4 - 42n^3 + n^2 + 3n}{2}.$$

Corollary 3.3. The sum connectivity status Gourava index of $M[K_{1,n}]$ is

$$SSG(M[K_{1,n}]) = \frac{n(n-1)}{2\sqrt{9n^2 - 1}} + \frac{n}{\sqrt{9n^2 + 3n - 1}} + \frac{n}{\sqrt{15n^2 - 3n - 1}}$$

Proof: By taking $a = -\frac{1}{2}$ in theorem 3, we obtain

$$SSG(M[K_{1,n}]) = \frac{n(n-1)}{2\sqrt{9n^2 - 1}} + \frac{n}{\sqrt{9n^2 + 3n - 1}} + \frac{n}{\sqrt{15n^2 - 3n - 1}}$$

Theorem 4. The general second status Gourava index of $M[K_{1,n}]$ is

$$SG_2^a(M[K_{1,n}]) = \frac{n(n-1)}{2}\{54n^3 - 54n^2 + 18n - 2\}^a + n\{54n^3 - 27n^2 + 3n\}^a + n\{120n^3 - 133n^2 + 49n - 6\}^a$$

Proof: $SG_2^a(M[K_{1,n}]) = \sum_{u,v \in E(K_{1,n})}\{[\sigma(u) + \sigma(v)].\sigma(u)\sigma(v)\}^a$

$SG_2^a(M[K_{1,n}]) = \frac{n(n-1)}{2}\{(3n - 1 + 3n - 1)(3n-1)^2\}^a + n\{(3n + 3n - 1)3n(3n-1)\}^a$

$+n\{(3n - 1 + 5n - 2)(3n-1)(5n-2)\}^a$

$$SG_2^a(M[K_{1,n}]) = \frac{n(n-1)}{2}\{54n^3 - 54n^2 + 18n - 2\}^a + n\{54n^3 - 27n^2 + 3n\}^a + n\{120n^3 - 133n^2 + 49n - 6\}^a$$

Corollary 4.1. The second status Gourava index of $M[K_{1,n}]$ is

$$SG_2(M[K_{1,n}]) = \frac{54n^5 + 240n^4 - 248n^3 + 84n^2 - 10n}{2}$$

Proof: By taking $a = 1$ in theorem 4

$$SG_2(M[K_{1,n}]) = \frac{54n^5 + 240n^4 - 248n^3 + 84n^2 - 10n}{2}$$

Corollary 4.2. The second hyper status Gourava index of $M[K_{1,n}]$ is

$$HSG_2(M[K_{1,n}]) = \frac{n(n-1)}{2}\{54n^3 - 54n^2 + 18n - 2\}^2 + n\{54n^3 - 27n^2 + 3n\}^2 + n\{120n^3 - 133n^2 + 49n - 6\}^2$$

Proof: By taking $a = 2$ in theorem 4

Corollary 4.3. The product connectivity of status Gourava index of $M[K_{1,n}]$ is

$$PSG(M[K_{1,n}]) = \frac{n(n-1)}{2\sqrt{54n^3 - 54n^2 + 18n - 2}} + \frac{n}{\sqrt{54n^3 - 27n^2 + 3n}} + \frac{n}{\sqrt{120n^3 - 133n^2 + 49n - 6}}$$

Proof: By taking $a = -\frac{1}{2}$ in theorem 4.

Corollary 4.4. The reciprocal product connectivity of status Gourava index of $M[K_{1,n}]$ is

$$RPSG(M[K_{1,n}]) = \frac{n(n-1)\sqrt{54n^3 - 54n^2 + 18n - 2}}{2} + n\sqrt{54n^3 - 27n^2 + 3n} + n\sqrt{120n^3 - 133n^2 + 49n - 6}$$

Proof: By taking $a = \frac{1}{2}$ in theorem 4.

Theorem 5. Let $M[W_n]$ be the middle graph of wheel graph with $\frac{n(n+1)}{2}$ vertices and $\frac{n^2+5n}{2}$ edges then the general first status Gourava index of $M[W_n]$ is

$\begin{pmatrix}\sigma(u),\\ \sigma(v)\end{pmatrix}$ / $uv\epsilon E(M(W_n))$	$\begin{pmatrix}5n,\\ 5n-3\end{pmatrix}$	$\begin{pmatrix}5n-3,\\ \frac{n^2+7n}{2}\end{pmatrix}$	$\left(\frac{n^2+5n}{2}, \frac{n^2+5n}{2}\right)$	$\begin{pmatrix}\frac{n^2+5n}{2},\\ \frac{n^2+7n}{2}\end{pmatrix}$	$\begin{pmatrix}5n-3,\\ \frac{n^2+5n}{2}\end{pmatrix}$	$\begin{pmatrix}5n-3,\\ 5n-3\end{pmatrix}$
Total number of edges	n	n	n	$2n$	$2n$	$\frac{n(n-1)}{2}$

$$SG_1^a(M[W_n]) = n\{25n^2 - 5n - 3\}^a + n\left\{\frac{1}{2}(5n^3 + 33n^2 - 4n - 6)\right\}^a + n\left\{\frac{1}{4}(n^4 + 10n^3 + 29n^2 + 20n)\right\}^a + 2n\left\{\frac{1}{4}(n^4 + 12n^3 + 39n^2 + 24n)\right\}^a + 2n\left\{\frac{1}{2}(5n^3 + 23n^2 - 6)\right\}^a + \frac{n(n-1)}{2}\{25n^2 - 20n + 3\}^a$$

Proof: By using the definition and table 3, we obtain

$$SG_1^a(M[W_n]) = \sum_{u,v \in E(K_{1,n})}[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^a$$

$$SG_1^a(M[W_n]) = n\{(5n-3) + 5n + 5n(5n-3)\}^a + n\left\{(5n-3) + \left(\frac{n^2+7n}{2}\right) + (5n-3)\left(\frac{n^2+7n}{2}\right)\right\}^a + n\left\{\left(\frac{n^2+7n}{2}\right) + \left(\frac{n^2+7n}{2}\right) + \left(\frac{n^2+7n}{2}\right)^2\right\}^a + 2n\left\{\left(\frac{n^2+5n}{2}\right) + \left(\frac{n^2+7n}{2}\right) +\right.$$

$\left(\frac{n^2+5n}{2}\right)\left(\frac{n^2+7n}{2}\right)\Big\}^a + 2n\left\{(5n-3)+\left(\frac{n^2+5n}{2}\right)+(5n-3)\left(\frac{n^2+5n}{2}\right)\right\}^a + \frac{n(n-1)}{2}\{(5n-3)+(5n-3)+(5n-3)^2\}^a$

$SG_1^a(M[W_n]) = n\{25n^2-5n-3\}^a + n\left\{\frac{1}{2}(5n^3+33n^2-4n-6\right\}^a + n\left\{\frac{1}{4}(n^4+10n^3+29n^2+20n\right\}^a + 2n\left\{\frac{1}{4}(n^4+12n^3+39n^2+24n\right\}^a + 2n\left\{\frac{1}{2}(5n^3+23n^2-6\right\}^a + \frac{n(n-1)}{2}\{25n^2-20n+3\}^a$

Corollary 5.1. The first status Gourava index of $M[W_n]$ is

$SG_1(M[W_n]) = \frac{1}{4}(35n^5+114n^4+275n^3+86n^2-54n)$

Proof: By taking $a = 1$ in theorem 3, we obtain

Corollary 5.2. The first hyper status Gourava index of $(M[W_n])$is

$$HSG_1(M[W_n]) = n\{25n^2-5n-3\}^2 + n\left\{\frac{1}{2}(5n^3+33n^2-4n-6)\right\}^2 + n\left\{\frac{1}{4}(n^4+10n^3+29n^2+20n)\right\}^2 + 2n\left\{\frac{1}{4}(n^4+12n^3+39n^2+24n)\right\}^2 + 2n\left\{\frac{1}{2}(5n^3+23n^2-6)\right\}^2 + \frac{n(n-1)}{2}\{25n^2-20n+3\}^2$$

Proof: By taking $a = 2$ in theorem 5, we obtain

$$HSG_1(M[W_n]) = n\{25n^2-5n-3\}^2 + n\left\{\frac{1}{2}(5n^3+33n^2-4n-6)\right\}^2 + n\left\{\frac{1}{4}(n^4+10n^3+29n^2+20n)\right\}^2 + 2n\left\{\frac{1}{4}(n^4+12n^3+39n^2+24n)\right\}^2 + 2n\left\{\frac{1}{2}(5n^3+23n^2-6)\right\}^2 + \frac{n(n-1)}{2}\{25n^2-20n+3\}^2$$

Corollary 5.3. The sum connectivity status Gourava index of $(M[W_n])$is

$SSG(M[W_n]) = \frac{n}{\sqrt{25n^2-5n-3}} + \frac{\sqrt{2}\,n}{\sqrt{5n^3+33n^2-4n-6}} + \frac{2n}{\sqrt{n^4+10n^3+29n^2+20n}} + \frac{4n}{\sqrt{n^4+12n^3+39n^2+24n}} + \frac{2\sqrt{2}\,n}{\sqrt{5n^3+23n^2-6}} + \frac{n(n-1)}{2\sqrt{25n^2-20n+3}}$

Proof: By taking $a = -\frac{1}{2}$ in theorem 5, we obtain

$SSG(M[W_n]) = \frac{n}{\sqrt{25n^2-5n-3}} + \frac{\sqrt{2}\,n}{\sqrt{5n^3+33n^2-4n-6}} + \frac{2n}{\sqrt{n^4+10n^3+29n^2+20n}} + \frac{4n}{\sqrt{n^4+12n^3+39n^2+24n}} + \frac{2\sqrt{2}\,n}{\sqrt{5n^3+23n^2-6}} + \frac{n(n-1)}{2\sqrt{25n^2-20n+3}}$

Theorem 6. The general second status Gourava index of Middle graph of wheel $M[W_n]$ is

$SG_2^a(M[W_n]) = n\{250n^3 - 225n^2 + 45n\}^a + n\left\{\frac{1}{4}(5n^5 + 117n^4 + 493n^3 - 549n^2 + 126n)\right\}^a + n\left\{\frac{1}{4}(n^6 + 15n^5 + 75n^4 + 125n^3)\right\}^a + 2n\left\{\frac{1}{4}(5n^5 + 97n^4 + 285n^3 - 357n^2 + 90n)\right\}^a + 2n\left\{\frac{1}{8}(2n^6 + 36n^5 + 214n^4 + 420n^3)\right\}^a + \frac{1}{2}\{250n^5 - 700n^4 + 720n^3 - 324n^2 + 54n\}^a$

Proof: $SG_2^a(M[W_n]) = \sum_{u,v\in E(K_{1,n})}\{[\sigma(u)+\sigma(v)].\sigma(u)\sigma(v)\}^a$

$SG_2^a(M[W_n]) = n\{[(5n-3)+5n]5n(5n-3)\}^a + n\left\{\left[(5n-3)+\left(\frac{n^2+7n}{2}\right)\right](5n-3)\left(\frac{n^2+7n}{2}\right)\right\}^a + n\left\{\left[\left(\frac{n^2+7n}{2}\right)+\left(\frac{n^2+7n}{2}\right)\right]\left(\frac{n^2+7n}{2}\right)^2\right\}^a + 2n\left\{\left[\left(\frac{n^2+5n}{2}\right)+\left(\frac{n^2+7n}{2}\right)\right]\left(\frac{n^2+5n}{2}\right)\left(\frac{n^2+7n}{2}\right)\right\}^a + 2n\left\{\left[(5n-3)+\left(\frac{n^2+5n}{2}\right)\right](5n-3)\left(\frac{n^2+5n}{2}\right)\right\}^a + \frac{n(n-1)}{2}\{[(5n-3)+(5n-3)](5n-3)^2\}^a$

$SG_2^a(M[W_n]) = n\{250n^3 - 225n^2 + 45n\}^a + n\left\{\frac{1}{4}(5n^5 + 117n^4 + 493n^3 - 549n^2 + 126n)\right\}^a + n\left\{\frac{1}{4}(n^6 + 15n^5 + 75n^4 + 125n^3)\right\}^a + 2n\left\{\frac{1}{4}(5n^5 + 97n^4 + 285n^3 - 357n^2 + 90n)\right\}^a + 2n\left\{\frac{1}{8}(2n^6 + 36n^5 + 214n^4 + 420n^3)\right\}^a + \frac{1}{2}\{250n^5 - 700n^4 + 720n^3 - 324n^2 + 54n\}^a$

Corollary 6.1. The second status Gourava index of $M[W_n]$ is

$SG_2(M[W_n]) = \frac{1}{4}\{3n^7 + 66n^6 + 1100n^5 + 1208n^4 - 723n^3 - 162n^2 + 108n\}$

Proof: By taking $a = 1$ in theorem 5

$SG_2(M[W_n]) = \frac{1}{4}\{3n^7 + 66n^6 + 1100n^5 + 1208n^4 - 723n^3 - 162n^2 + 108n\}$

Corollary 6.2. The second hyper status Gourava index of $M[W_n]$ is

$HSG_2(M[W_n]) =) = n\{250n^3 - 225n^2 + 45n\}^2 + n\left\{\frac{1}{4}(5n^5 + 117n^4 + 493n^3 - 549n^2 + 126n)\right\}^2 + n\left\{\frac{1}{4}(n^6 + 15n^5 + 75n^4 + 125n^3)\right\}^2 + 2n\left\{\frac{1}{4}(5n^5 + 97n^4 + 285n^3 - 357n^2 + 90n)\right\}^2 + 2n\left\{\frac{1}{8}(2n^6 + 36n^5 + 214n^4 + 420n^3)\right\}^2 + \frac{1}{2}\{250n^5 - 700n^4 + 720n^3 - 324n^2 + 54n\}^2.$

Proof: By taking $a = 2$ in theorem 6

Corollary 6.3. The product connectivity of status Gourava index of $M[W_n]$ is

$PSG(M[W_n]) = \frac{n}{\sqrt{250n^3-225n^2+45n}} + \frac{2n}{\sqrt{5n^5+117n^4+493n^3-549n^2+126n}} + \frac{2n}{\sqrt{n^6+15n^5+75n^4+125n^3}} + \frac{4n}{\sqrt{5n^5+97n^4+285n^3-357n^2+90n}} + \frac{4\sqrt{2}\,n}{\sqrt{2n^6+36n^5+214n^4+420n^3}} + \frac{\sqrt{2}}{\sqrt{250n^5-700n^4+720n^3-324n^2+54n}}$

Proof: By taking $a = -\frac{1}{2}$ in theorem 6.

Corollary 6.4. The reciprocal product connectivity of the status Gourava index of $M[W_n]$ is

$$\begin{aligned} RPSG(M[W_n]) &= n\sqrt{250n^3 - 225n^2 + 45n} \\ &+ \frac{n}{2}\sqrt{5n^5 + 117n^4 + 493n^3 - 549n^2 + 126n} \\ &+ \frac{n}{2}\sqrt{n^6 + 15n^5 + 75n^4 + 125n^3} \\ &+ n\sqrt{5n^5 + 97n^4 + 285n^3 - 357n^2 + 90n} \\ &+ 4\sqrt{2}\, n\sqrt{2n^6 + 36n^5 + 214n^4 + 420n^3} \\ &+ \frac{1}{\sqrt{2}}\sqrt{250n^5 - 700n^4 + 720n^3 - 324n^2 + 54n} \end{aligned}$$

Proof: By taking $a = \frac{1}{2}$ in theorem 4.

Theorem 7. Let $M[F_n]$ be the middle graph of friendship graph with $(5n + 1)$vertices and $2n^2 + 7n$ edges. Then the general first status Gourava index of $M[F_n]$ is,

$\begin{pmatrix}\sigma(u),\\ \sigma(v)\end{pmatrix}$ / $uv\epsilon E(M($	$\begin{pmatrix}8n-2,\\ 8n-2\end{pmatrix}$	$\begin{pmatrix}8n-2\\ 8n\end{pmatrix}$	$\begin{pmatrix}8n-2,\\ 13n-7\end{pmatrix}$	$\begin{pmatrix}8n-2,\\ 13n-5\end{pmatrix}$	$\begin{pmatrix}13n-7,\\ 13n-5\end{pmatrix}$
Total number of edges	$2n^2 - n$	$2n$	$2n$	$2n$	$2n$

$SG_1^a(M[F_n]) = 2n^2 - n\,\{64n^2 - 16n\}^a + 2n\,\{64n^2 - 2\}^a + 2n\,\{104n^2 - 61n + 5\}^a + 2n\,\{104n^2 - 45n + 3\}^a + 2n\,\{169n^2 - 130n + 23\}^a$

Proof: By using the definition and table 7, we obtain

$SG_1^a(M[W_n]) = \sum_{u,v\in E(K_{1,n})}[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]^a$

$SG_1^a(M[W_n]) =]) = 2n^2 - n\,\{8n - 2 + 8n - 2 + (8n-2)^2\}^a + 2n\,\{8n - 2 + 8n + 8n(8n-2)\}^a + 2n\,\{8n - 2 + 13n - 7 + (8n-2)(13n-7)\}^a + 2n\,\{8n - 2 + 13n - 5 + (8n-2)(13n-5)\}^a + 2n\,\{13n - 7 + 13n - 5 + (13n-7)(13n-5)\}^a$

$SG_1^a(M[W_n]) = 2n^2 - n\,\{64n^2 - 16n\}^a + 2n\,\{64n^2 - 2\}^a + 2n\,\{104n^2 - 61n + 5\}^a + 2n\,\{104n^2 - 45n + 3\}^a + 2n\,\{169n^2 - 130n + 23\}^a$

Corollary 7.1. The first status Gourava index of $M[F_n]$ is

$SG_1(M[W_n]) = 128n^4 + 786n^3 - 456n^2 + 58n$

Proof: By taking $a = 1$ in theorem 7, we obtain

$SG_1(M[W_n]) = 128n^4 + 786n^3 - 456n^2 + 58n$

Corollary 7.2. The first hyper status Gourava index of $(M[F_n])$is

$$HSG_1(M[W_n]) = 2n^2 - n\,\{64n^2 - 16n\}^2 + 2n\,\{64n^2 - 2\}^2 + 2n\,\{104n^2 - 61n + 5\}^2 + 2n\,\{104n^2 - 45n + 3\}^2 + 2n\,\{169n^2 - 130n + 23\}^2$$

Proof: By taking $a = 2$ in theorem 7, we obtain

$$HSG_1(M[W_n]) = 2n^2 - n\,\{64n^2 - 16n\}^2 + 2n\,\{64n^2 - 2\}^2 + 2n\,\{104n^2 - 61n + 5\}^2 + 2n\,\{104n^2 - 45n + 3\}^2 + 2n\,\{169n^2 - 130n + 23\}^2$$

Corollary 7.3. The sum connectivity status Gourava index of$(M[F_n])$

Proof: By taking $a = -\frac{1}{2}$ in theorem 7, we obtain

$$SSG_1(M[W_n]) = \frac{2n^2 - n}{\sqrt{64n^2 - 16n}} + \frac{2n}{\sqrt{64n^2 - 2}} + \frac{2n}{\sqrt{104n^2 - 61n + 5}} + \frac{2n}{\sqrt{104n^2 - 45n + 3}} + \frac{2n}{\sqrt{169n^2 - 130n + 23}}$$

Theorem 8. The general second status Gourava index of the middle graph of friendship graph $M[W_n]$ is

$SG_2^a(M[F_n]) = (2n^2 - n)\{1024n^3 - 768n^2 - 192n - 16\}^a + 2n\{1024n^3 - 384n^2 + 32n\}^a + 2n\{2184n^3 - 2658n^2 + 1032n - 126\}^a + 2n\{2184n^3 - 2114n^2 + 672n - 70\}^a + 2n\{4394n^3 - 6084n^2 + 2782n - 420\}^a$

Proof: $SG_2^a(M[W_n]) = \sum_{u,v \in E(K_{1,n})} \{[\sigma(u) + \sigma(v)].\sigma(u)\sigma(v)\}^a$

$SG_2^a(M[F_n]) = (2n^2 - n)\{1024n^3 - 768n^2 - 192n - 16\}^a + 2n\{1024n^3 - 384n^2 + 32n\}^a + 2n\{2184n^3 - 2658n^2 + 1032n - 126\}^a + 2n\{2184n^3 - 2114n^2 + 672n - 70\}^a + 2n\{4394n^3 - 6084n^2 + 2782n - 420\}^a$

Corollary 8.1. The second status Gourava index of $M[F_n]$ is

Proof: By taking $a = 1$ in theorem 8

$$SG_2(M[F_n]) = (2n^2 - n)\{1024n^3 - 768n^2 - 192n - 16\} + 2n\{1024n^3 - 384n^2 + 32n\} + 2n\{2184n^3 - 2658n^2 + 1032n - 126\} + 2n\{2184n^3 - 2114n^2 + 672n - 70\} + 2n\{4394n^3 - 6084n^2 + 2782n - 420\}$$

Corollary 8.2. The second hyper status Gourava index of $M[F_n]$ is

Proof: By taking $a = 2$ in theorem 8

$$SG_2(M[F_n]) = (2n^2 - n)\{1024n^3 - 768n^2 - 192n - 16\}^2 + 2n\{1024n^3 - 384n^2 + 32n\}^2 + 2n\{2184n^3 - 2658n^2 + 1032n - 126\}^2 + 2n\{2184n^3 - 2114n^2 + 672n - 70\}^2 + 2n\{4394n^3 - 6084n^2 + 2782n - 420\}^2$$

Corollary 8.3. The product connectivity of status Gourava index of $M[F_n]$ is

Proof: By taking $a = -\frac{1}{2}$ in theorem 8.

$$PSG(M[F_n]) = \frac{2n^2 - n}{\sqrt{1024n^3 - 768n^2 - 192n - 16}} + \frac{2n}{\sqrt{1024n^3 - 384n^2 + 32n}} + \frac{2n}{\sqrt{2184n^3 - 2658n^2 + 1032n - 126}} + \frac{2n}{\sqrt{2184n^3 - 2114n^2 + 672n - 70}} + \frac{2n}{\sqrt{4394n^3 - 6084n^2 + 2782n - 420}}.$$

Corollary 8.4. The reciprocal product connectivity of the status Gourava index of $M[F_n]$ is

Proof: By taking $a = \frac{1}{2}$ in theorem8.

$$PSG(M[F_n]) = 2n^2 - n\sqrt{1024n^3 - 768n^2 - 192n - 16} + 2n\sqrt{1024n^3 - 384n^2 + 32n} + 2n\sqrt{2184n^3 - 2658n^2 + 1032n - 126} + 2n\sqrt{2184n^3 - 2114n^2 + 672n - 70} + 2n\sqrt{4394n^3 - 6084n^2 + 2782n - 420}.$$

3. Conclusion

In this paper, we have extended the status Gourava indices of a graph, which is introduced by Kulli, and we have obtained status Gourava indices for the derived graphs namely cycle graph, star graph, complete graph, wheel graph and friendship graph. We have calculated ten standard status indices of middle graphs of standard graphs using this new index.

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