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M-Polynomial and Topological Indices of Derived Graphs of Ladder Graph

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Abstract. The M-polynomial is the source of finding information about degree-based topological indices of a molecule. This polynomial will help us to predict the different properties like physiochemical properties, chemical reactivity, biological activities etc. of the chemical compounds. In this article, we establish an M-polynomial for derived graphs of Ladder graphs namely slanting ladder graph, Diagonal ladder graph and Open diagonal ladder graph. The ladder graph L_n is an undirected connected graph with $2n$ vertices and $3n-1$ edges. Also, we determine some standard degree-based topological indices for the M-polynomial of derived graphs.

Keywords: M-polynomial, Degree-based topological indices, Ladder graph, Slanting ladder graph, Diagonal ladder graph, Open diagonal ladder graph.

AMS Mathematics Subject Classification (2010): 05C07, 05C75

1. Introduction

A graph $G(V, E)$ is a set of vertices and a set of unordered pairs of edges. The cardinality of the vertex set is called the order of the graph G and the cardinality of the edge set is called the size of the graph G. The degree of a vertex $v \in V(G)$ of a graph G, denoted by d_v is the total number of edges incident on v. We request that the reader refer [1] to the notation terminologies used here.

In this article, we have considered finite, simple and connected graphs. The main graphs under consideration are the derived graphs of ladder graphs such as slanting ladder graphs, diagonal ladder graphs and open diagonal ladder graph [9, 10].

The numerical parameters of a graph which describe its topology based on the degree of a vertex are called as its topological indices. It can describe the molecular shape of the graph numerically and is applied within the advancement of qualitative structure-activity relationships (QSAR) [11] the quantitative structure-property relationship (QSPR) and also computational drugs. These numerical values correlate the structure of a graph with various physical properties, chemical reactivities and biological activities. The topological indices can be obtained in 3 types degree-based, distance-based and spectral-based. Among these, the most commonly known invariant is the degree-based topological indices.

Through the literature survey, we find that the Hosoya polynomial [2] is the key polynomial in the vast area of development in the degree-based indices. Extensive research has been done on the algebraic polynomials which can help to determine a closed formula for a given topological index and throws light on the graph properties. To demonstrate the degree-based index for a family of graphs the M-polynomial was found to be parallel to the Hosoya polynomial in the degree-based invariants.

M-polynomial was introduced by Klavzar and Deutsch [2, 3, 4, 5] in 2015. M-polynomial is rich in producing a source of many topological indices based on degree. It is the foremost progressive polynomial and determines an additionally closed formula for a given topological index as it can express the topological index as a certain derivative or integral function (or both). With the help of the M-polynomial one can get a closer idea related to the properties of the family of graphs rather than computing the several topological indices.

In this paper, we study the property of M-polynomial on derived graphs of the Ladder graph. We have derived closed formulas for some well-known degree-based topological indices like first and second Zagreb indices [6,12,14,15,16], the General Randic index and inverse Randic index, the harmonic index, the Symmetric Division index, the Augmented Zagreb index, Inverse Sum index, Atom-bomb Connectivity index and the Geometric Arithmetic index using calculus. In the next section define the M-polynomial and present the results obtained on the degree-based topological indices of the derived Ladder graphs and further, we compute the results obtained on the M-polynomial. Some 2-D and 3D graphs have been drawn to understand the graphical analysis of the polynomials.

2. Basic definitions and terminology

Definition 2.1. The ladder graph [12,13] L_n is an undirected connected graph with $2n$ vertices and $3n-1$ edges. It is the Cartesian product of path P_n with n vertices and complete graph K_2.

Definition 2.2. The slanting ladder [7, 8] graph is denoted by SL_n and is a graph obtained from two paths $P_n = \{a_1, a_2, \dots, a_n\}$ and $Q_n = \{b_1, b_2, \dots, b_n\}$ $a_i b_j$ by joining each a_i by b_{j+1} where $1 \le i \le n-1$ and $1 \le j \le n-1$.

Definition 2.3. A diagonal ladder [7, 8] graph is denoted by L_n , $n \ge 2$ is obtained from L_n by adding the edges $E(G) = \{a_i b_{j+1} : i = j, 1 \le i, j \le n-1\} \cup \{a_{i+1} b_j : i = j, 1 \le i, j \le n-1\}$.

Definition 2.4. An open diagonal ladder [7, 8] graph is denoted by ODL_n which is generated from a diagonal ladder graph by excluding the edges $a_i b_j$ for $i = 1\ and\ n$ and $j = 1\ and\ n$.

Definition 2.5. The M-polynomial of a graph G is defined as $M(G,x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G) x^i y^j$ where $\delta = \min\{d_v : v \in V(G)\}$ and $\Delta = \max\{d_v : v \in V(G)\}$ and $m_{ij}(G)$ is the number of edges $uv \in E(G)$ such that $\{d_u, d_v\} = \{i, j\}$

Lemma 2.1. For any graph G with $u, v \in V(G)$ and $e = uv \in E(G)$, then $d_e = d_u + d_v - 2$.

The definitions of the different Topological indices derived for the derived graphs of L_n with the formulae to derive them using the M-polynomial computed for the derived graphs [2,3] are shown below in Table 2.1.

Topological Index	**Definition**	**Formula to derive index by applying on M-Polynomial With $\boldsymbol{x = y = 1}$**
First Zagreb Index	$M_1(D(F_n)) = \sum_{uv \in E(D(F_n))} (d_u + d_v)$	$(D_x + D_y)f(x,y)$
Second Zagreb Index	$M_1(D(F_n)) = \sum_{uv \in E(D(F_n))} (d_u \cdot d_v)$	$(D_x . D_y)f(x,y)$
Modified Second Zagreb Index	$m_{M_2}(D(F_n)) = \sum_{uv \in E(D(F_n))} \left(\frac{1}{d_u . d_v}\right)$	$(S_x . S_y)f(x,y)$
General Randic Index	$R_\alpha(D(F_n)) = \sum_{uv \in E(D(F_n))} \left(\frac{1}{d_u . d_v}\right)^\alpha$	$(D_x^a . D_y^a)f(x,y)$
Inverse Randic Index	$RR_\alpha(D(F_n)) = \sum_{uv \in E(D(F_n))} \left(\frac{1}{d_u . d_v}\right)^\alpha$	$(S_x^a . S_y^a)f(x,y)$
Harmonic Index	$H(D(F_n)) = \sum_{uv \in E(D(F_n))} \left(\frac{2}{d_u . d_v}\right)$	$2S_x J(f(x,y))$
Symmetric Division Index	$SSD(D(F_n)) = \sum_{uv \in E(D(F_n))} \left(\frac{d_u}{d_v} + \frac{d_v}{d_u}\right)$	$(D_x S_y + S_x D_y)f(x,y)$

Augmented Zagreb Index	$A(D(F_n)) = \sum_{uv\in E(D(F_n))} \left(\frac{d_u . d_v}{d_u + d_v - 2}\right)^3$	$(S_x^3 Q_{-2} J D_x^3 D_y^3) f(x,y)$
Inverse Sum Index	$I(D(F_n)) = \sum_{uv\in E(D(F_n))} \left(\frac{d_u . d_v}{d_u + d_v}\right)$	$(S_x J D_x D_y) f(x,y)$
Atom-bond Connectivity Index	$ABC(D(F_n)) = \sum_{uv\in E(D(F_n))} \sqrt{\frac{d_u + d_v - 2}{d_u . d_v}}$	$\left(D_x^{\frac{1}{2}} Q_{-2} J S_x^{\frac{1}{2}} S_x^{\frac{1}{2}}\right) f(x,y)$
Geometric Arithmetic Index	$GA(D(F_n)) = \sum_{uv\in E(D(F_n))} \left(\frac{2\sqrt{d_u . d_v}}{d_{u+} d_v}\right)$	$\left(2 S_x J D_x^{1/2} D_y^{1/2}\right) f(x,y)$

Table 2.1: Definitions of different Topological Indices

Different notations used in the formulae[2, 3] are explained in table 2.2

$D_x = x\frac{\partial f}{\partial x}$	$J = f(x,x)$	$D_x^{\frac{1}{2}} = \sqrt{x\frac{\partial f}{\partial x}} . \sqrt{f(x,y)}$
$D_y = y\frac{\partial f}{\partial y}$	$Q_\alpha = x^\alpha f(x,y)$	$D_y^{\frac{1}{2}} = \sqrt{y\frac{\partial f}{\partial y}} . \sqrt{f(x,y)}$
$L_x = f(x^2, x)$	$S_x = \int_0^x \frac{f(t,y)}{t} dt$	$S_x^{\frac{1}{2}} = \sqrt{\int_0^x \frac{f(t,y)}{t}} . \sqrt{f(x,y)}$
$L_x = f(x, x^2)$	$S_y = \int_0^y \frac{f(x,t)}{t} dt$	$S_y^{\frac{1}{2}} = \sqrt{\int_0^y \frac{f(x,t)}{t}} . \sqrt{f(x,y)}$

Table 2.2: Notations used in computing indices

3. Main results

Theorem 3.1. Let SL_n be the Slanting ladder graph then,

$$g(x,y) = 2xy^3 + 4x^2y^3 + (3n-3)x^3y^3.$$

Proof: By definition of the ladder graph and the slanting ladder graph, we find that SL_n has $2n$ vertices and $3(n-1)$ edges. SL_n has 2 vertices of degree 1, 2 vertices of degree 2 and $(2n-4)$ vertices of degree 3.
Based on the above degrees of the end vertices, the edges set of SL_n can be put in the below format:

(dv_1, dv_2)	1,3	2,3	3,3
Total number of edges	2	4	$3(n-3)$

Thus, the M-polynomial of the SL_n is,

$$\mathrm{M}(SL_n; x, y) = \mathrm{g}(x, y) = \sum_{\delta \le i \le j \le \Delta} mij(G)x^i y^j$$

$$g(x, y) = 2xy^3 + 4x^2y^3 + (3n-3)x^3y^3.$$

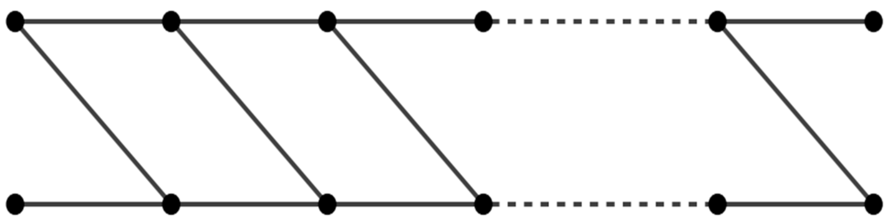

Figure 3.1: Slanting ladder graph

Theorem 3.1.2. The topological indices of SL_n are given by the following

1. $M_1(SL_n) = 18n - 26$
2. $M_2(SL_n) = 27n - 51$
3. $m_{M_2}(SL_n) = \frac{n+1}{3}$
4. $R_\alpha(SL_n) = 2.3^\alpha + 2^{\alpha+2}.3^\alpha + 3^{3\alpha}(x-3)$
5. $RR_\alpha(SL_n) = \frac{2}{3^\alpha} + \frac{4}{2^\alpha 3^\alpha} + \frac{3(n-3)}{3^{2\alpha}}$
6. $H(SL_n) = \frac{5n-2}{5}$
7. $SSD(SL_n) = \frac{18n-8}{3}$
8. $A(SL_n) = \frac{3^3}{2^2} + 2^5 + \frac{3^7(n-3)}{4^3}$
9. $I(SL_n) = \frac{45n-72}{10}$
10. $ABC(SL_n) = \frac{2\sqrt{2}}{\sqrt{3}} + 2\sqrt{2} + 2(n-3) = \frac{2\sqrt{2}}{\sqrt{3}}(1+\sqrt{3}) + 2(n-3)$
11. $GA(SL_n) = \frac{5\sqrt{3}+8\sqrt{6}}{10} + 3(\mathrm{n}-3)$

Proof: Let $\mathrm{M}(SL_n; x, y) = g(x, y) = 2xy^3 + 4x^2y^3 + (3n-3)x^3y^3$. Then we have,

1. The first Zagreb index is computed as,

$$D_x = 2xy^3 + 8x^2y^3 + 9(n-3)x^3y^3$$

$$D_y = 6xy^3 + 12x^2y^3 + 9(n-3)x^3y^3$$

Thus $M_1(SL_n) = (D_x + D_y)g(x,y)\ at\ x = y = 1$ gives,

$$M_1(SL_n) = 18n - 26.$$

2. We find, the second Zagreb index as,

$$D_x = 6xy^3 + 12x^2y^3 + 9(n-3)x^3y^3$$
$$D_xD_y = 6xy^3 + 24x^2y^3 + 27(n-3)x^3y^3$$

Thus $M_2(SL_n) = (D_x.D_y)g(x,y)\ at\ x = y = 1$ gives,

$$M_2(SL_n) = 27n - 51$$

The modified second Zagreb index is computed as,

$$S_y(f(x,y)) = \frac{2xy^3}{3} + \frac{4x^2y^3}{3} + \frac{3(n-3)x^3y^3}{3}$$

$$(S_x.S_y)(g(x,y)) = \frac{2xy^3}{3} + \frac{4x^2y^3}{6} + \frac{3(n-3)x^3y^3}{9}$$

Thus, $mM_2(SL_n) = (S_x.S_y)(g(x,y))\ at\ x = y = 1$ gives,

$$mM_2(SL_n) = \frac{n+1}{3}$$

3. We calculate the general Randic index as,

$$D_y^\alpha(g(x,y)) = 2\,.3^\alpha xy^3 + 4\,.3^\alpha x^2y^3 + 3(n-3)3^\alpha x^3y^3$$
$$(D_x^\alpha.D_y^\alpha)(g(x,y)) = 2\,.3^\alpha xy^3 + 4\,.2^\alpha.3^\alpha x^2y^2 + 3(n-3)3^{2\alpha}x^3y^3$$

Thus, $R_\alpha(SL_n) = (D_x^\alpha.D_y^\alpha)(g(x,y))\ at\ x = y = 1$ gives

$$R_\alpha(SL_n) = 2\,.3^\alpha + 2^{\alpha+2}3^\alpha + 3^{2\alpha+1}(n-3)$$

4. We find the inverse Randic index here as,

$$S_y^\alpha(g(x,y)) = \frac{2xy^3}{3^\alpha} + \frac{4x^2y^3}{3^\alpha} + \frac{3(n-3)x^3y^3}{3^\alpha}$$
$$(S_x^\alpha.S_y^\alpha)(g(x,y)) = \frac{2xy^3}{3^\alpha} + \frac{4x^2y^3}{2^\alpha 3^\alpha} + \frac{3(n-3)x^3y^3}{3^\alpha.3^\alpha}$$

Thus, $RR_\alpha(SL_n) = (S_x^\alpha.S_y^\alpha)(g(x,y))\ at\ x = y = 1$ gives,

$$\frac{2}{3^\alpha} + \frac{4}{2^\alpha 3^\alpha} + \frac{3(n-3)}{3^{2\alpha}}$$

5. We compute the Harmonic index,

$$J(g(x,y)) = 2x^4 + 4x^5 + 3(n-3)x^6$$
$$2S_xJ(g(x,y)) = x^4 + \frac{8x^5}{5} + (n-3)x^6$$

Thus, $H(SL_n) = 25_xJ(g(x,y))at\ x = 1\ gives,$

$$H(SL_n) = \frac{5n-2}{5}$$

6. The Symmetric division index is calculated by,

$$(D_xS_y)g(x,y) = \frac{2xy^3}{3} + \frac{8x^2y^3}{3} + 3(n-3)x^3y^3$$

$$(S_xD_y)g(x,y) = 6xy^3 + 6x^2y^3 + 3(n-3)x^3y^3$$

$$(D_xS_y + S_xD_y)g(x,y) = \frac{20xy^3}{3} + \frac{26x^2y^3}{3} + 6(n-3)x^3y^3$$

Thus, $SSD(SL_n) = \big(D_xS_y + S_xD_y\big)g(x,y) at\ x = y = 1$ gives

$$SSD(SL_n) = \frac{18n-8}{3}$$

7. We obtain the Augmented Zagreb index by finding,

$$D_y^3 = 2x\,3^3y^3 + 4x^2 3^3 y^3 + 3(n-3)x^3 3^3 y^3$$

$$D_x^3D_y^3 = 2\,.\,3^3xy^3 + 4z^3x^2 3^3y^3 + 3(n-3)x^3 3^3y^3$$

$$JD_x^3D_y^3(g(x,y)) = 2\,.\,3^3x^4 + 2^5.\,3^3x^5 + 3^7(n-3)x^6$$

$$S_xQ_{-2}JD_x^3D_y^3(g(x,y)) = \frac{2\,.\,3^3x^2}{2^3} + \frac{2^5.\,3^3x^3}{3^3} + \frac{3^7(n-3)x^4}{4^3}$$

Thus, $A(SL_n) = S_xQ_{-2}JD_x^3D_y^3\big(g(x,y)\big)with\ x = 1$ gives

$$A(SL_n) = \frac{3^3}{2^2} + 2^5 + \frac{3^7(n-3)}{4^3}$$

$$= \frac{2^2 3^3 + 2^{11} + 3^7(n-3)}{2^6}$$

8. We find here the Inverse Sum index as,

$$(D_xD_y)g(x,y) = 6xy^3 + 24x^2y^3 + 27(n-3)x^3y^3$$

$$(JD_xD_y)g(x,y) = 6x^4 + 24x^5 + 27(n-3)x^6$$

$$(S_xJD_xD_y)g(x,y) = \frac{6x^4}{4} + \frac{24x^5}{5} + \frac{27(n-3)x^6}{6}$$

Thus, $I(SL_n) = (S_xJD_xD_y)g(x,y)\ with\ x = 1$ gives

$$I(SL_n) = \frac{45n-72}{10}$$

9. The Atom bomb connectivity index is computed as,

$$(S_y^{\frac{1}{2}})g(x,y) = \frac{2xy^3}{\sqrt{3}} + \frac{4x^2y^3}{\sqrt{3}} + \frac{3(n-3)x^3y^3}{\sqrt{3}}$$

$$(S_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2xy^3}{\sqrt{3}} + \frac{4x^2y^3}{\sqrt{2}.\sqrt{3}} + \frac{3(n-3)x^3y^3}{\sqrt{3}.\sqrt{3}}$$

$$(JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2x^4}{\sqrt{3}} + \frac{4x^5}{\sqrt{2}.\sqrt{3}} + \frac{3(n-3)x^6}{3}$$

$$(Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2x^2}{\sqrt{3}} + \frac{4x^3}{\sqrt{2}.\sqrt{3}} + (n-3)x^4$$

$$(D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2\sqrt{2}x^2}{\sqrt{3}} + \frac{4\sqrt{3}x^3}{\sqrt{2}.\sqrt{3}} + \sqrt{4}(n-3)x^4$$

Thus, $ABC(SL_n) = (D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y)\ with\ x = 1$ gives

$$\frac{2\sqrt{2}}{\sqrt{3}} + 2\sqrt{2} + 2(n-3)$$

$$= \frac{2\sqrt{2}(1+\sqrt{3})}{\sqrt{3}} + 2(n-3)$$

10. We find the Geometric Arithmetic index here as,

$$(D_y^{\frac{1}{2}})g(x,y) = 2x\sqrt{3}y^3 + 4x^2\sqrt{3}y^3 + 3(n-3)x^3\sqrt{3}y^3$$

$$(D_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 2x\sqrt{3}y^3 + 4\sqrt{2}\sqrt{3}x^2y^3 + 3(n-3)\sqrt{3}\sqrt{3}x^3y^3$$

$$(JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 2\sqrt{3}x^4 + 4\sqrt{6}x^5 + 9(n-3)x^6$$

$$(2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = \frac{4\sqrt{3}x^4}{4} + \frac{8\sqrt{6}x^5}{5} + \frac{18(n-3)}{6}x^6$$

Thus, $GA(SL_n) = (2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y)\ at\ x = 1$ gives

$$GA(SL_n) = \sqrt{3} + \frac{8\sqrt{6}}{5} + 3(\mathrm{n}-3)$$

$$= \frac{5\sqrt{3}+8\sqrt{6}}{10} + 3(\mathrm{n}-3)$$

Theorem 3.2. Let DL_n be the diagonal ladder graph then,

$$g(x,y) = 2x^3y^3 + 8x^3y^5 + (5n-14)x^5y^5$$

Proof: From the definition of the diagonal ladder graph and computation we obtain/ find that DL_n has 2n vertices and $(5n-4)$ edges, where 4 vertices are of degree 3 and the remaining $(2n-4)$ vertices are of degree 5.

Now, according to the degree of the end vertices, we can define the edge set of DL_n as

(dv_1, dv_2)	3,3	3,5	5,5
Total number of edges	2	8	5n-14

Thus, the M-polynomial of DL_n can be defined as

$$\mathrm{M}(DL_n, x, y) = g(x,y) = \sum_{\delta \le i \le j \le \Delta} mij(G)x^iy^j$$

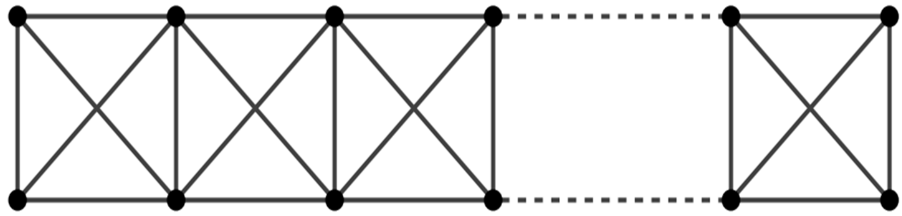

Figure 3.2: Diagonal ladder graph

Theorem 3.2.1. The topological indices of DL_n are given by the following

1. $M_1(DL_n) = 50n - 64$
2. $M_2(DL_n) = 125n - 212$
3. $m_{M_2}(DL_n) = \frac{45n+44}{225}$
4. $R_\alpha(DL_n) = 2 \times 3^{2\alpha} + 8\,.\,3^\alpha.\,5^\alpha + (5n - 14)5^{2\alpha}$
5. $RR_\alpha(DL_n) = \frac{2}{3^{2\alpha}} + \frac{8}{5^\alpha\, 3^\alpha} + \frac{(5n-14)}{5^{2\alpha}}$
6. $H(DL_n) = \frac{15n-2}{15}$
7. $SSD(DL_n) = \frac{2(75n-44)}{15}$
8. $A(DL_n) = \frac{3^6}{2^5} + 5^3 + (5n - 14)\frac{5^6}{8^3}$
9. $I(DL_n) = \frac{25n-34}{2}$
10. $ABC(DL_n) = \frac{4}{3} + \frac{8\sqrt{2}}{\sqrt{5}} + \frac{(5n-14)\sqrt{8}}{5}$
11. $GA(DL_n) = 5n + 2\sqrt{15} - 12.$

Proof: By computing the edges and their degrees (respectively)

1. We observe the first Zagreb index for the Diagonal Ladder graph is computed as,

$$D_x = 6x^3y^3 + 24x^3y^5 + 5(5n - 14)x^5y^5$$
$$D_y = 6x^3y^3 + 40x^3y^5 + 5(5n - 14)x^5y^5$$

Thus $M_1(DL_n) = (D_x + D_y)g(x, y)\ at\ x = y = 1$ gives

$$M_1(DL_n) = 50n - 64$$

2. We find the second Zagreb index as,

$$D_x = 6x^3y^3 + 40x^3y^5 + 5(5n - 14)x^5y^5$$
$$D_x.D_y = 18x^3y^3 + 120x^3y^5 + 25(5n - 14)x^5y^5$$

Thus $M_2(DL_n) = (D_x + D_y)g(x, y)\ at\ x = y = 1$ gives

$$M_2(DL_n) = 125n - 212$$

3. We find here the modified second Zagreb index,

$$S_y\big(g(x,y)\big) = \frac{2x^3y^3}{3} + \frac{8x^3y^5}{5} + \frac{(5n-14)x^5y^5}{5}$$

$$(S_x.S_y)\big(g(x,y)\big) = \frac{2x^3y^3}{9} + \frac{8x^3y^5}{15} + \frac{(5n-14)x^5y^5}{25}$$

Thus, $m_{M_2}(DL_n) = (S_x.S_y)\big(g(x,y)\big)\ at\ x = 1\ and\ y = 1$ gives

$$m_{M_2}(DL_n) = \frac{45n+44}{225}$$

4. We calculate the general Randic index as,

$$D_y^\alpha\big(g(x,y)\big) = 2.3^\alpha x^3y^3 + 8.5^\alpha x^3y^5 + (5n-14)5^{2\alpha}x^5y^5$$

$$(D_x^\alpha.D_y^\alpha)\big(g(x,y)\big) = 2.3^{2\alpha}x^3y^3 + 8.3^\alpha 5^\alpha x^3y^5 + (5n-14)5^{2\alpha}x^5y^5$$

Thus, we get $R_\alpha(DL_n) = (D_x^\alpha.D_y^\alpha)\big(g(x,y)\big)\ at\ x = y = 1$ gives

$$R_\alpha(DL_n) = 2\times 3^{2\alpha} + 8\,.3^\alpha.5^\alpha + (5n-14)5^{2\alpha}$$

5. The inverse Randic index is obtained by calculating,

$$S_y^\alpha\big(g(x,y)\big) = \frac{2x^3y^3}{3^\alpha} + \frac{8x^3y^5}{5^\alpha} + \frac{(5n-14)x^5y^5}{5^\alpha}$$

$$(S_x^\alpha.S_y^\alpha)\big(g(x,y)\big) = \frac{2x^3y^3}{3^{2\alpha}} + \frac{8x^3y^5}{3^\alpha 5^\alpha} + \frac{(5n-14)x^5y^5}{5^{2\alpha}}$$

Thus, $RR_\alpha(DL_n) = (S_x^\alpha.S_y^\alpha)\big(g(x,y)\big)\ at\ x = y = 1$ gives

$$RR_\alpha(DL_n) = \frac{2}{3^{2\alpha}} + \frac{8}{5^\alpha\, 3^\alpha} + \frac{(5n-14)}{5^{2\alpha}}$$

6. We compute the Harmonic index as,

$$J\big(g(x,y)\big) = 2x^6 + 8x^8 + (5n-14)x^{10}$$

$$2S_xJ\big(g(x,y)\big) = \frac{2x^6}{3} + 2x^8 + \frac{2(5n-14)x^{10}}{10}$$

Thus, $H(DL_n) = 2S_xJ\big(g(x,y)\big)\ at\ x = 1$ gives

$$H(DL_n) = \frac{15n-2}{15}$$

7. We obtain the Symmetric division index as,

$$(D_xS_y)g(x,y) = 2x^3y^3 + \frac{24x^3y^5}{5} + (5n-14)x^5y^5$$

$$(S_xD_y)g(x,y) = 2x^3y^3 + \frac{40x^3y^5}{3} + (5n-14)x^5y^5$$

$$(D_xS_y + S_xD_y)g(x,y) = 4x^3y^3 + \frac{272x^3y^5}{15} + 2(5n-14)x^5y^5$$

Thus, $SSD(DL_n) = (D_xS_y + S_xD_y)g(x,y)\ at\ x = y = 1$ gives

$$SSD(DL_n) = \frac{2(75n-44)}{15}$$

8. We obtain the Augmented Zagreb index as,

$$D_y^3 = 2.3^3x^3y^3 + 8.5^3x^3y^5 + 5^3(5n-14)x^5y^5$$

$$D_x^3.D_y^3 = 2.3^6x^3y^3 + 8.3^35^3x^3y^5 + 5^6(5n-14)x^5y^5$$

$$(JD_x^3.D_y^3)g(x,y) = 2.3^6x^6 + 8.3^3 5^3x^8 + 5^6(5n-14)x^{10}$$

$$(Q_{-2}JD_x^3D_y^3)g(x,y) = 2.3^6x^4 + 8.3^3 5^3x^6 + 5^6(5n-14)x^8$$

$$(S_x^3Q_{-2}JD_x^3D_y^3)g(x,y) = \frac{3^6x^4}{2^5} + 5^3x^6 + \frac{5^6(5n-14)x^8}{8^3}$$

Thus, $I(DL_n) = (S_x^3Q_{-2}JD_x^3D_y^3)g(x,y) with\ x = 1$ gives

$$I(DL_n) = \frac{3^6}{2^5} + 5^3 + (5n-14)\frac{5^6}{8^3}$$

9. We find here the Inverse Sum index as,

$$(D_xD_y)g(x,y) = 18x^3y^3 + 120x^3y^5 + 25(5n-14)x^5y^5$$

$$(JD_xD_y)g(x,y) = 18x^6 + 120x^8 + 25(5n-14)x^{10}$$

$$(S_xJD_xD_y)g(x,y) = 3x^6 + 15x^8 + \frac{5}{2}(5n-14)x^{10}$$

Thus, $I(DL_n) = (S_xJD_xD_y)g(x,y)\ with\ x = 1$ gives

$$I(DL_n) = \frac{25n-34}{2}$$

10. We compute the Atom bomb connectivity index as,

$$(S_y^{\frac{1}{2}})g(x,y) = \frac{2x^3y^3}{\sqrt{3}} + \frac{8x^3y^5}{\sqrt{5}} + \frac{(5n-14)x^5y^5}{\sqrt{5}}$$

$$(S_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2x^3y^3}{3} + \frac{8x^3y^5}{\sqrt{3}\sqrt{5}} + \frac{(5n-14)x^5y^5}{5}$$

$$(JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2x^6}{3} + \frac{8x^8}{\sqrt{3}.\sqrt{5}} + \frac{(5n-14)x^{10}}{5}$$

$$(Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{2x^4}{3} + \frac{8x^6}{\sqrt{3}.\sqrt{5}} + \frac{(5n-14)x^8}{5}$$

$$(D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{4x^4}{3} + \frac{8\sqrt{6}x^6}{\sqrt{3}.\sqrt{5}} + \frac{(5n-14)\sqrt{8}x^8}{5}$$

Thus, $ABC(DL_n) = (D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y)\ with\ x = 1$ gives

$$ABC(DL_n) = \frac{4}{3} + \frac{8\sqrt{2}}{\sqrt{5}} + \frac{(5n-14)\sqrt{8}}{5}$$

11. We find here the Geometric Arithmetic index as,

$$(D_y^{\frac{1}{2}})g(x,y) = 2\sqrt{3^3}x^3y^3 + 8\sqrt{5^3}x^3y^5 + (5n-14)\sqrt{5}x^5y^5$$

$$(D_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 6x^3y^3 + 8.\sqrt{3}.\sqrt{5}\,x^3y^5 + 5\,(5n-14)\sqrt{5}x^5y^5$$

$$(JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 6x^6 + 8\sqrt{15}\,x^8 + 5\,(5n-14)\sqrt{5}x^{10}$$

$$(2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 2x^6 + 2\sqrt{15}\,x^8 + (5n-14)\sqrt{5x}^{10}$$

Thus, $GA(DL_n) = (2S_x J D_x^{\frac{1}{2}} D_y^{\frac{1}{2}}) g(x,y)\ with\ x = 1$ gives
$GA(DL_n) = 5n + 2\sqrt{15} - 12.$

Theorem 3.3. Let ODL_n be the open diagonal ladder graph then,

$$g(x,y) = 8x^2y^5 + (5n-14)x^5y^5$$

Proof: From the definition of the open diagonal ladder graph and computation we find that ODL_n has 2n vertices and $(5n-6)$ edges, where there are 4 vertices of degree 2 and the remaining $(2n-4)$ vertices are of degree 5.
Based on the degree of the end vertices, we can determine the edge set of ODL_n as

(du, dv)	2,5	5,5
Total number of edges	8	5n-14

Thus, the M-polynomial of ODL_n is given as

$$\mathrm{M}(ODL_n, x, y) = g(x,y) = \sum_{\delta \le i \le j \le \Delta} mij(G)x^iy^j$$

$$g(x,y) = 8x^2y^5 + (5n-14)x^5y^5$$

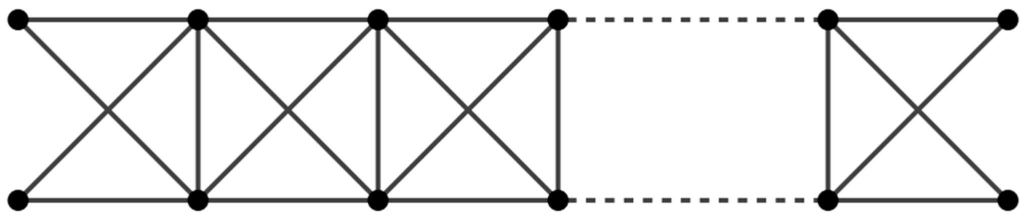

Figure 3.3: Open diagonal ladder graph

Theorem 3.3.1. The topological indices of ODL_n are given by the following

1. $M_1(ODL_n) = 50n - 84$
2. $M_2(ODL_n) = 125n - 270$
3. $m_{M_2}(ODL_n) = \frac{5n+6}{25}$
4. $R_\alpha(ODL_n) = 2^{\alpha+3}5^\alpha + (5n-14)5^{2\alpha}$
5. $RR_\alpha(ODL_n) = \frac{1}{5^\alpha 2^{\alpha-3}} + \frac{(5n-14)}{5^{2\alpha}}$
6. $H(ODL_n) = \frac{35n+64}{70}$
7. $SSD(ODL_n) = \frac{(50n-24)}{5}$
8. $A(ODL_n) = 2^6 + \frac{(5n-14)5^6}{8^3}$
9. $I(ODL_n) = \frac{875n-1650}{70}$
10. $ABC(ODL_n) = 4\sqrt{2} + \frac{(5n-14)\sqrt{8}}{5}$

11. $GA(ODL_n) = \frac{35n+16\sqrt{10}-98}{7}$

Proof: By computing the edges and their degrees respectively,

1. We find the first Zagreb index for the Open Diagonal Ladder graph as,

$$D_x = 16x^2y^5 + 5(5n-14)x^5y^5$$
$$D_y = 40x^2y^5 + 5(5n-14)x^5y^5$$

Thus, $M_1(ODL_n) = (D_x + D_y)g(x,y) at\ x = 1\ and\ y = 1$ gives

$$M_1(ODL_n) = 50n - 84$$

2. We compute the second Zagreb index as,

$$D_y = 40x^2y^5 + 5(5n-14)x^5y^5$$
$$D_x.D_y = 30x^2y^5 + 25(5n-14)x^5y^5$$

Thus, $M_2(ODL_n) = (D_x.D_y)g(x,y) at\ x = y = 1$ gives

$$M_2(ODL_n) = 125n - 270$$

3. We find here the modified second Zagreb index as,

$$S_y(g(x,y)) = \frac{80x^2y^5}{5} + \frac{(5n-14)x^5y^5}{5}$$
$$(S_x.S_y)g(x,y) = \frac{4x^2y^5}{5} + \frac{(5n-14)x^5y^5}{25}$$

Thus, $m_{M_2}(ODL_n) = (S_x.S_y)g(x,y) at\ x = y = 1$ gives

$$m_{M_2}(ODL_n) = \frac{5n+6}{25}$$

4. We compute the general Randic index as,

$$D_y^\alpha(g(x,y)) = 8\,.5^\alpha x^2y^5 + (5n-14)5^\alpha x^5y^5$$
$$(D_x^\alpha.D_y^\alpha)g(x,y) = 2^{\alpha+3}5^\alpha x^2y^5 + (5n-14)5^{2\alpha}x^5y^5$$

Thus, $R_\alpha(ODL_n) = (D_x^\alpha.D_y^\alpha)g(x,y)\ at\ x = y = 1$ gives

$$R_\alpha(ODL_n) = 2^{\alpha+3}5^\alpha + (5n-14)5^{2\alpha}$$

5. We obtain the inverse Randic index by finding,

$$S_y^\alpha(g(x,y)) = \frac{8x^2y^5}{5^\alpha} + \frac{(5n-14)x^5y^5}{5^\alpha}$$
$$(S_x^\alpha.S_y^\alpha)g(x,y) = \frac{8x^2y^5}{2^\alpha 5^\alpha} + \frac{(5n-14)x^5y^5}{5^{2\alpha}}$$

Thus, $RR_\alpha(ODL_n) = (S_x^\alpha.S_y^\alpha)g(x,y)\ at\ x = y = 1$ gives

$$RR_\alpha(ODL_n) = \frac{1}{5^\alpha 2^{\alpha-3}} + \frac{(5n-14)}{5^{2\alpha}}$$

6. We compute the Harmonic index as,

$$Jg(x,y) = 8x^7 + (5n-14)x^{10}$$
$$2S_xg(x,y) = \frac{16x^7}{7} + \frac{(5n-14)}{5}x^{10}$$

Thus, $H(ODL_n) = (2S_xJ)g(x,y) at\ x = 1$ gives

$$H(ODL_n) = \frac{35n + 64}{70}$$

7. We find the Symmetric division index as,

$$(D_xS_y)g(x,y) = \frac{16x^2y^5}{5} + 5(5n-14)x^5y^5$$

$$(S_xD_y)g(x,y) = 20x^2y^5 + 5(5n-14)x^5y^5$$

$$(D_xS_y + S_xD_y)g(x,y) = \frac{116x^2y^5}{5} + 2(5n-14)x^5y^5$$

Thus, $SSD(ODL_n) = (D_xS_y + S_xD_y)g(x,y) at\ x = y = 1$ gives

$SSD(ODL_n) = \frac{(50n-24)}{5}$

8. We obtain the Augmented Zagreb index as,

$$D_y^3 = 8\,.5^3x^2y^5 + (5n-14)5^3x^5y^5$$

$$D_x^3.D_y^3 = 2^65^3x^2y^5 + (5n-14)5^6x^5y^5$$

$$JD_x^3D_y^3 = 2^65^3x^7 + (5n-14)5^6x^{10}$$

$$Q_{-2}JD_x^3D_y^3 = 2^65^3x^5 + (5n-14)5^6x^8$$

$$S_x^3Q_{-2}JD_x^3D_y^3 = \frac{2^65^3x^5}{5^3} + \frac{(5n-14)5^6x^8}{8^3}$$

Thus, $I(ODL_n) = (S_x^3Q_{-2}JD_x^3D_y^3)g(x,y) at\ x = 1$ gives

$$A(ODL_n) = 2^6 + \frac{(5n-14)5^6}{8^3}$$

9. We find here the Inverse Sum index as,

$$\left(D_x.D_y\right)g(x,y) = 80x^2y^5 + 25(5n-14)x^5y^5$$

$$\left(JD_xD_y\right)g(x,y) = 80x^7 + 25(5n-14)x^{10}$$

$$\left(S_xJD_xD_y\right)g(x,y) = \frac{80x^7}{7} + \frac{25(5n-14)}{10}$$

Thus, $I(ODL_n) = \left(S_xJD_xD_y\right)g(x,y) with\ x = 1$ gives

$$I(ODL_n) = \frac{875n - 1650}{70}$$

10. We compute the Atom bomb connectivity index as,

$$(S_y^{\frac{1}{2}})g(x,y) = \frac{8x^2y^5}{\sqrt{5}} + \frac{(5n-14)x^5y^5}{\sqrt{5}}$$

$$(S_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{4\sqrt{2}x^2y^5}{\sqrt{5}} + \frac{(5n-14)x^5y^5}{5}$$

$$(JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{4\sqrt{2}x^7}{\sqrt{5}} + \frac{(5n-14)x^{10}}{5}$$

$$(Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y) = \frac{4\sqrt{2}x^5}{\sqrt{5}} + \frac{(5n-14)x^8}{5}$$

$$(D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}}) = 4\sqrt{2}x^5 + \frac{(5n-14)\sqrt{8}x^8}{5}$$

Thus, $ABC(ODL_n) = (D_x^{\frac{1}{2}}Q_{-2}JS_x^{\frac{1}{2}}S_y^{\frac{1}{2}})g(x,y)\ with\ x = 1$ gives

$$ABC(ODL_n) = 4\sqrt{2} + \frac{(5n-14)\sqrt{8}}{5}$$

11. We find here the Geometric Arithmetic index as,

$$D_x^{\frac{1}{2}} = 8\sqrt{5}x^2y^5 + (5n-14)\sqrt{5}x^5y^5$$

$$(D_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 8\sqrt{2}.\sqrt{5}x^2y^5 + (5n-14)5.x^5y^5$$

$$(JD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = 8\sqrt{10}x^7 + 5(5n-14)x^{10}$$

$$(2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y) = \frac{16\sqrt{10}x^7}{7} + (5n-14)x^{10}$$

Thus, $GA(ODL_n) = (2S_xJD_x^{\frac{1}{2}}D_y^{\frac{1}{2}})g(x,y)\ with\ x = 1$ gives

$$GA(ODL_n) = \frac{35n + 16\sqrt{10} - 98}{7}$$

4. Conclusion

In this article, we have computed the degree-based topological indices of Slanting Ladder graph, Diagonal Ladder graph and Open Diagonal Ladder graph. Initially, we obtain the M-polynomial of these graphs and then find the topological indices for the same. These results can help determine the properties and uses of networks in the field of pharmacies, electronics, electrical and wireless communication.

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