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The Generalized Kudryshov Method Implemented to the Nonlinear Conformable Time-Fractional PHI-Four Equation

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Abstract. This research explores the new exact solutions of the nonlinear conformable time-fractional PHI-four equation through the generalized Kudryshov method with conformable fractional derivative. The got new exact solutions are designed in styles of the rational and exponential functions designate that the studied procedure is serviceable to study the fractional nonlinear evolution equations in mathematical physics and engineering.

Keywords: Generalized Kudryshov method, nonlinear conformable time-fractional PHI-four equation, Solitary wave solutions.

AMS Mathematics Subject Classification (2010): 26A33

1. Introduction

Nonlinear systems of the conformable time-fractional partial differential equations (CTFPDEs) can explain mathematical models of various phenomena in distinct categories of applied physical sciences. Soliton structures to nonlinear systems of CTFPDEs are essential for investigating natural aspects in broad sectors of applied physical sciences. The performance of a symbolic computation package will obtain it functional to introduce numerous analytical techniques, for example, the extended rational sinh-cosh method [1], modified (G'/G)-expansion method [2, 3], Fractional SineGordon Equation Approach [4], (G'/G)-expansion method [5, 6, 7], New extended direct algebraic method [8], the $\exp(-\phi(\xi))$-expansion method [9, 10], the new extended direct algebraic method [11], iterative reproducing kernel Hilbert space method [13], the first integral method [14], modified khater method [15], Shifted Jacobi spectral collocation method [16], residual power series method [17], the generalized Kudryshov method [18], reproducing kernel Hilbert space method [19], and many more.

The paper applied the generalized Kudryshov method [18] to derive the different type of soliton structures for nonlinear conformable time-fractional PHI-four equation [11].

Let us consider that nonlinear conformable time-fractional PHI-four equation [11]:

$$W_t^{(\mu)} - W_{xx} + a^2 W + bW^3(\xi) = 0\,,\ t \le 0, 0 < \mu < 1. \tag{1}$$

where, *a* and *b* are constants. The Klein-Gordon (KG) equation has worked a significant function in mathematical physics [21], as well as the KG equation, which has drawn much application in investigating solitons in condensed matter physics, in examining the interaction of solitons in a collisionless plasma and the recurrence of initial states [22]. The PHI-four equation can be studied as a particular form of the KG equation that models the phenomenon in particle physics where kink and anti-kink solitary waves interact [23]. The PHI-four equation has executed an essential role in nuclear and particle physics over the decades. Traveling wave solutions for a nonlinear variant of the Phi-4 equation are analyzed through the Weierstrass elliptic function method in [23].

2. Fractional derivative

We consider that $\Phi : (0\ ,\ \infty) \to$ R, so the fractional derivative of Φ of order μ [20]:

$$\frac{\partial^\mu \Phi}{\partial t^\mu} = \lim_{\varepsilon \to 0} \frac{\Phi(t + \varepsilon t^{1-\mu}) - \Phi(t)}{\varepsilon}, t > 0, \mu \in (0,1) \tag{2}$$

Some notable highlights of the fractional derivative are as follows:

i. $\frac{\partial^\mu}{\partial t^\mu}(a\Phi + b\Phi) = a\frac{\partial^\mu}{\partial t^\mu}(\Phi) + b\frac{\partial^\mu}{\partial t^\mu}(\Phi),\ \forall a,b \in R$

ii. $\frac{\partial^\mu}{\partial t^\mu}(t^\mu) = \mu t^{\beta-\mu},\ \forall \beta \in R$

iii. $\frac{\partial^\mu}{\partial t^\mu}(c) = 0,\ \mathrm{c} = const.$

iv. $\frac{\partial^\mu}{\partial t^\mu}(\Phi 0 \Phi) = t^{1-\mu}\Phi'(t)\Phi'(\Phi(t)).$

3. Glimpse of the generalized Kudryshov method

Step 1: We consider that a frcational NLEE for *W*(*x*, *t*):

$$K(\frac{\partial^\mu W}{\partial t^\mu}, \frac{\partial W}{\partial t}, \frac{\partial^{2\mu} W}{\partial t^{2\mu}}, \frac{\partial^2 W}{\partial t^2}, ...) = 0 \tag{3}$$

where, K represents a polynomial in *W* and $\frac{\partial^\alpha W}{\partial t^\alpha}$ and $\frac{\partial^{2\alpha} W}{\partial t^{2\alpha}}$ are fractional derivatives of *W*. To locate the transformation of equation (3):

$$W = W(x,t) = W(\xi),\quad \xi = x - \frac{ct^\mu}{\mu} \tag{4}$$

From equation (3) and equation (4) we locate the following ODE:

$$L(W, W', W'', W''', ...) = 0 \tag{5}$$

Step 2: Calculate *M* and *N* through the balance rule on equation (5).

Step 3: Let us consider that

$$W(\xi)=\frac{\sum_{i=0}^{N}A_i\psi^i}{\sum_{j=0}^{M}B_j\psi^j} \tag{6}$$

where A_i and B_j are real constants, N and M are positive integers such that $A_N, B_M \neq 0$ and Ψ satisfies the following ODE:

$$\psi'(\xi)=\psi^2(\xi)-\psi(\xi) \tag{7}$$

The general solution of equation (6) is of the form:

$$\psi(\xi)=\frac{1}{1+pe^{\xi}} \tag{8}$$

where, p is any arbitrary constant.

Step 4: Determine the positive integers N and M in equation (6) by balancing the highest order derivative term with the nonlinear term of $W(\xi)$ in equation (3) or equation (5). Moreover, we define the degree of $W(\xi)$ as $D(W(\xi))=N-M$, which gives rise to the degree of other expression as

$$D(\frac{d^qW}{d\xi^q})=N-M+q,\ D(W^p(\frac{d^qW}{d\xi^q})^s=(N-M)p+s(N-M+q) \tag{9}$$

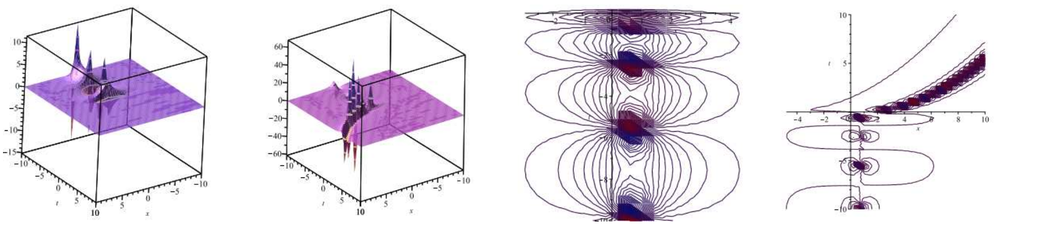
where, p, q, s are integer numbers.

Step 5: Applying equation (6) into equation (5) and equation (9), collecting all terms with the same order of Φ together. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations which can be solved to find the values of $\Phi(\xi)$ with the help of MAPLE.

4. Solitons to the nonlinear conformable time-fractional Phi-4 equation

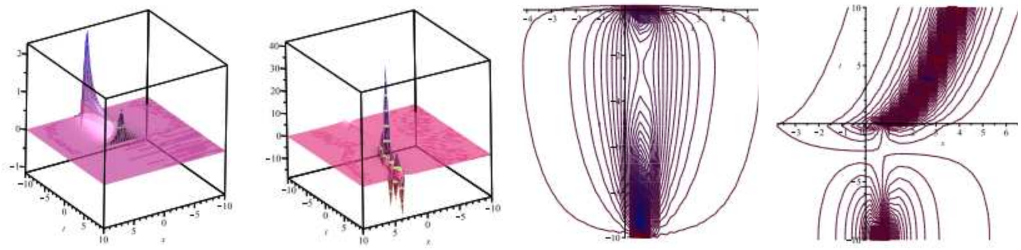
Let us consider that the nonlinear conformable time-fractional Phi-4 equation:

$$W_t^{\mu}-W_{xx}+a^2W+bW^3(\xi)=0,\quad t\leq 0, 0<\mu<1. \tag{10}$$



(a) Real 3D shape (b) Imaginary 3D shape (c) Real contour plot (d) Imaginary contour plot

Figure 1: The three-dimensional and contour shape of the solution in $W_1(x,t)$ for $p=-0.5, b=1,\quad B_0=0.6,\ c=2$ and $\alpha=0.5$.



(a) Real 3D shape (b) Imaginary 3D shape (c) Real contour plot (e) Imaginary contour (d) Plot

Figure 2: The three-dimensional and contour shape of the solution in $W_2(x,t)$ for $p = -0.5, b = -10, \quad B_0 = -5$, $c = 0.5$ and $\alpha = 0.5$.

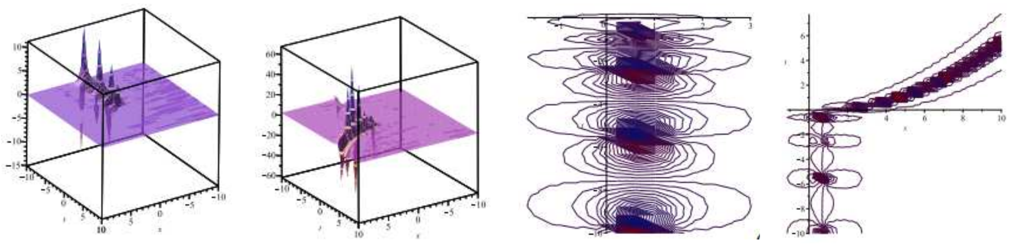
Using $W(x,t) = W(\xi)$, where $\xi = x - \frac{ct^{\mu}}{\mu}$, the equation (10) converts the following ODE:

$$(c^2 - 1)W''(\xi) + a^2 W + bW^3(\xi) = 0 \tag{11}$$

where, a, b are real parameters and c is a traveling wave variable. Applying the rule of homogeneous balance on equation (11), ($W''(\xi)$ and $W^3(\xi)$ $\Rightarrow$ $(3(N-M) = N - M + 2)$ $\Rightarrow$ $N = M + 1$. Setting $M = 1$ then $N = 2$. Therefore, we get:

$$W(\xi) = \frac{A_0 + A_1\Psi + A_2\Psi^2}{B_0 + B_1\Psi} \tag{12}$$

By equation (12) and equation (11) and then equating each coefficients of Ψ^i to zeros, we get:



(a) Real 3D shape (b) Imaginary 3D shape (c) Real contour plot (d) Imaginary contour Plot

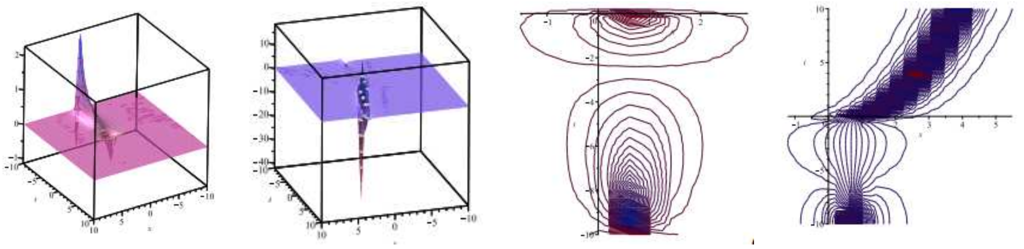
Figure 3: The three-dimensional and contour shape of the solution in $W_3(x,t)$ for $p = -0.5, b = 1, \quad B_0 = 0.6$, $c = 2$ and $\alpha = 0.5$.

The first set:

$$a=\sqrt{1-c^2}\ A_0=0,\ A_1=-B_0\sqrt{\frac{2-2c^2}{b}},\ A_2=2B_0\sqrt{\frac{2-2c^2}{b}},\ \mathrm{B}_0=B_0,\ \mathrm{B}_1=-2B_0$$

where, c, b and B_0 are constants. Using the values of the first set, equation (12) and equation (11), we have:

$$W_1(\xi)=\frac{-B_0\sqrt{\frac{2-2c^2}{b}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)+2B_0\sqrt{\frac{2-2c^2}{b}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)^2}{B_0-2B_0\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)}.$$



(a) Real 3D shape (b) Imaginary 3D shape (c) Real contour plot (d) Imaginary contour plot

Figure 4: The three-dimensional and contour shape of the solution in $W_4(x,t)$ for $p=-0.5, b=-10,\quad B_0=-5,\ c=0.5$ and $\alpha=0.5$.

The second set:

$$a=-\sqrt{1-c^2}\ A_0=0,\ A_1=B_0\sqrt{\frac{2-2c^2}{b}},\ A_2=-2B_0\sqrt{\frac{2-2c^2}{b}},\ \mathrm{B}_0=B_0,\ \mathrm{B}_1=-2B_0$$

where, c, b and B_0 are constants. Similarly, we get:

$$W_2(\xi)=\frac{B_0\sqrt{\frac{2-2c^2}{b}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)-2B_0\sqrt{\frac{2-2c^2}{b}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)^2}{B_0-2B_0\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)}$$

The third set:

$$a=\sqrt{2-2c^2},\ A_0=-B_0\frac{2(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}},\ A_1=B_0\frac{4(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}},\ A_2=2B_0\sqrt{\frac{2-2c^2}{b}},$$

$B_0=B_0, B_1=-2B_0$

where, c, b and B_0 are constants. Similarly, we get:

$$W_3(\xi)=\frac{-B_0\frac{2(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}}+B_0\frac{4(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)+2B_0\sqrt{\frac{2-2c^2}{b}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)^2}{B_0-2B_0\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)}$$

The fourth set:

$$a=-\sqrt{2-2c^2},\ A_0=B_0\frac{2(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}},\ A_1=-B_0\frac{4(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}},\ A_2=-2B_0\sqrt{\frac{2-2c^2}{b}},$$

$B_0=B_0, B_1=-2B_0$

where, c, b and B_0 are constants. Similarly, we get:

$$W_4(\xi)=\frac{B_0\frac{2(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}}-B_0\frac{4(c^2-1)}{b\sqrt{\frac{2-2c^2}{b}}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)-2B_0\sqrt{\frac{2-2c^2}{b}}\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)^2}{B_0-2B_0\left(\frac{1}{1+pe^{x-\frac{ct^{\mu}}{\mu}}}\right)}$$

5. Conclusions

In this paper, we have derived diverse kinds of exact solutions which are shown in the Figures (1- 4) of the equation (1) through the generalized Kudryshov method with conformable fractional derivative. The investigated way is sincere and reliable for producing new varieties of exact solutions of fractional NLEEs in mathematical physics and engineering.

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