Annals of Pure and Applied Mathematics
Vol. 1, No. 1, 2012, 32-43
ISSN: 2279-087X (P),2279-0888(online)
Published on 5 September 2012
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

An EOQ Model with Parabolic Demand Rate and Time Varying Selling Price

Monika Mandal[1] and Sujit Kumar De[2]

[1]Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Medinipur-721102, India
[2]Department of Mathematics, Midnapore College, Midnapur-721101, India,
E-mail: skdemamo2008.com@gmail.com

Received 9 August 2012; accepted 28 August 2012

Abstract. This paper deals with the perishable items of seasonal product where the demand rate follows a parabolic path. This path is symmetric about the time axis in which the selling price takes time varying linear decreasing function in one part and the increasing function for the other part. However, we have assumed the gradient of the selling price line is known for the first part and for the other the slope is unknown. Considering constant deterioration rate the average profit function is developed. Solutions are made through analytical method. Graphical interpretations, numerical examples with sensitivity analysis have also been made to illustrate the model.

AMS Mathematics Subject Classification (2010): **90B05**

Keywords: Inventory, parabolic demand function, time varying selling price, deterioration

1. Introduction

The classical EOQ model was developed by assuming the demand rate as constant. But in practice, it does not convey the reality. The actual fact is, for the case of decaying items like fruits, vegetables etc. the demand rate is growing and it reaches to a pick time then it began to decrease within a stipulated period. The traditional concept on demand rate was first modified by Silver and Meal [18]. Then Resh *et al.* [14] and Donaldson [6] studied with linear trend demand in their models. Ghare and Schrader [7] initially proposed the model with exponentially decaying inventory. Subsequently, Covert and Phillip [5] and Tadikamalla [20] developed an EOQ model with Weibull and Gamma distribution respectively. After that, a numerous research papers were developed by several researchers considering the demand rate as time varying continuous function. Lot-size model with shortages and fluctuating demand under inflation was developed by Yang *et al* [21]. Recently, Bose *et al* [2], Giri *et al*. [9], Jamal *et al.* [12], Ghosh and Chaudhuri [8] developed inventory models with time dependent deterioration rate. Moreover, Aggoun *et al.* [1] was analyzed a stochastic process with random characteristic over varying deterioration. Chang and Dye [4], Mehta and Saha [13], Roy [17] and Boukhel *et al.* [3] are some others researchers who considered the inventory-level dependent or price dependent demand as a whole. Very recent, an economic production lot size (EPLS) model with random price sensitive demand was developed by Roy *et al.* [15] and that with stock-price sensitive demand and deterioration was explained by Roy and Chaudhuri [16].

Researchers like Goswami and Choudhuri [11], Giri *et al*. [10] have developed an inventory model considering time varying demand, shortages, and deterioration. Skouri *et al.* [19] have analyzed a model with weibull deterioration rate, partial backlogging and ramp type demand rate.

In our paper, we have assumed the deterioration rate as constant when the demand rate follows a parabolic path within a specific season. However in a super market we observe that the sellers are usually unwilling to give a rebate / price discount over an item for their customers to have more and more profit. Observation tells that, up to optimum demand the selling price behaves like decreasing function and after that it becomes increasing function in time with unknown gradient. Since, till date no papers were published along this direction, so considering above assumptions we have developed a model as well. Solution is made analytically, graphical interpretations; numerical examples with sensitivity analysis are done to illustrate the model.

Table-1 Major characteristics of inventory models on selected areas

Author(s) and publication year	Decision Criteria	Deterioration	Varying Demand	Varying Cost/Price	Backlogged Allowed
Bose et. al. (1995)	Total Cost	Exponential	Timevarying	Yes	Yes
Giri et al. (1996)	Total Cost	Linear	Time varying	Yes	Yes
Jamal et al. (1997)	Total Cost	Exponential	Constant	No	Yes
Chang & Dye (1999)	Total Cost	Constant	Time Varying	No	Yes
Aggoun et.al. (2001)	Total Cost	Stochastic	Stochastic	No	No
Mehta & Shah (2003)	Total Cost	Constant	Exponential	Yes	No
Ghosh &Chaudhuri (2004)	Total Cost	Weibull	Time Quadratic	Constant	No
Boukhel et.al. (2005)	Total Cost	Constant	Inventory Level	No	Yes

Roy (2008)	**Total Profit**	**Constant**	**Selling Price dependent**	**Yes**	**No**
Skouri et al. (2009)	**Total Cost**	**Weibull**	**Ramp type**	**No**	**Partial**
Roy et.al. (2010)	**Total Profit**	**No**	**Stochastic Price Dependent**	**Decreasing function**	**Yes**
Roy & Chaudhuri (2012)	**Total Profit**	**Constant**	**Stock-Price Sensitive**	**Constant**	**No**
Present Paper (2012)	**Total Profit**	**Constant**	**Time-Parabolic**	**Time -Linear**	**No**

Assumptions and Notations

The following notations and assumptions are used to develop the model.

Assumptions

1. Replenishments are instantaneous
2. Lead time is zero
3. Shortages are not allowed
4. Demand rate is a time dependent parabolic function $D(t) = d + bt - at^2 \;\; for\; 0 < t < t_2$

   where a, b >0, $t_1 (= \sqrt{b^2 + 4ad} / 2a)$ the axis of symmetry of the demand function and $t_2 = t_1 + b/2a \;, a > 0$ is the time of zero demand.
5. The selling price functions below and above the axis $t = t_1$ of symmetric demand are given by $p_1(t) = s_0 - \lambda t \quad for\; 0 < t \le t_1$ and $p_2(t) = s_1 + \mu t \quad for\; t_1 \le t \le t_2$ respectively. Also for continuity we have $p_1(t_1) = p_2(t_1)$.
6. Deterioration is allowed and it is constant

Notations

i) q : The instantaneous inventory at time t.

ii) D(t) : Instantaneous demand rate

iii) Q : The order quantity per cycle

iv) d : the demand rate at time zero

v) s : Unit selling price ($)

vi) h_c : Inventory holding cost per unit quantity per week ($)

vii) d_c : The deterioration cost

viii) p : Purchasing price of unit item ($)

ix) c : ordering cost per order ($)

x) t_1 : Growth period of demand

xi) θ : The deterioration rate

xii) T : Cycle time in weeks

xiii) W : Average Profit of the inventory ($)

2. Model formulation

The inventory starts at time t = 0 with maximum order level Q and depletes with deterioration and parabolic demand rate $D(t)$ for the period $[0,\ t_1]$. After time t_1 the demand rate began to decrease up to time T. Also, the selling price is a known time linear decreasing function within $[0,\ t_1]$ and for the period $[t_1, \mathrm{T}]$ it began to increase with unknown slope of linear time price line. The governing differential equation of the inventory function is given by

$$\frac{dq}{dt} + \theta q = -D(t) \qquad for \quad 0 < t \le T \tag{1}$$

Subject to the conditions

$$q(0) = Q \qquad for \quad 0 < t \le t_1 \tag{2}$$

and

$$q(T) = 0 \qquad for \quad t_1 \le t \le T \tag{3}$$

Now, solving (1) using (2) and (3) we get

$$q(t) = Qe^{-\theta t} - at^3/3 - bt^2/2 - dt + \theta t^2(d/2 + bt/6 - at^2/12) \quad 0 < t \le T \tag{4}$$

and
$$Q = T\left[d + \frac{bT}{2} - \frac{aT^2}{3}\right] + \frac{\theta T^2}{12}\left[6d + 4bT - 3aT^2\right] \tag{5}$$

(Assuming $\theta << 1$)

Now, using (5), the inventory holding cost per cycle is given by

$$HC = h_c \int_0^T q(t)\,dt$$

$$= h_c \left[\int_0^T \left\{Qe^{-\theta t} - \frac{t}{6}\left(2at^2 + 3bt + 6d\right) + \frac{\theta t^2}{12}\left(6d + 2bt - at^2\right)\right\} dt\right]$$

$$= h_c \left[\frac{T^2}{12}\left(6d + 4bT - 5aT^2\right) + \frac{\theta T^3}{120}\left(20d + 15bT - 12aT^2\right)\right] \tag{6}$$

The deterioration cost per cycle can be found as

$$DC = d_c\left\{Q - \int_0^T D(t)\,dt\right\}$$

$$= d_c\left[Q - \int_0^T \left\{d + bt - at^2\right\} dt\right] \tag{7}$$

$$= \frac{\theta d_c T^2}{12}\left(6d + 4bT - 3aT^2\right) \quad \text{(using(5))}$$

Purchasing cost of the ordered quantity per cycle is given by $PC = pQ$ (8)
and the set up cost per order = $OC = c$ (9)

Now the total selling price per cycle is given by

$$SP = \int_0^{t_1} p_1(t)D(t)dt + \int_{t_1}^{T} p_2(t)D(t)dt$$

$$= \int_0^{t_1}\left(s_0 - \lambda t\right)\left(d + bt - at^2\right)dt + \int_{t_1}^{T}\left(s_1 + \mu t\right)\left(d + bt - at^2\right)dt \tag{10}$$

$$= \frac{(s_0 - s_1)t_1}{12}\left(6d + 2bt_1 - at_1^2\right) + \frac{s_1 T}{6}\left(6d + 3bT - 2aT^2\right) + \frac{\mu T^2}{12}\left(6d + 4bT - 3aT^2\right)$$

where to simplify we have used $p_1(t_1) = p_2(t_1)$ which gives

$$\lambda + \mu = \left(s_0 - s_1\right)/t_1 \tag{11}$$

Again if the selling price is independent of time then total fixed selling price is given by $FSP = s\left[Q - \frac{\theta T^2}{12}\left(6d + 4bT - 3aT^2\right)\right] = s\left(d + \frac{bT}{2} - \frac{aT^2}{3}\right)$ (12)

Therefore, total average profit per cycle is given by

W = [Selling price-purchasing cost-holding cost-deterioration cost-set up cost]/Cycle Time

Model-I: **The selling price is a time dependent linear function**

We have the total average profit W_1 = [*SP-PC-HC-DC-OC*]/ T .

So our inventory problem is

$$\begin{aligned}
\textit{Maximize } W_1 &= \frac{(s_0 - s_1)t_1}{12T}\left(6d + 2bt_1 - at_1^2\right) + \frac{s_1}{6}\left(6d + 3bT - 2aT^2\right) + \frac{\mu T}{12}\left(6d + 4bT - 3aT^2\right) \\
&\quad - pQ/T - \frac{h_c T}{12}\left(6d + 4bT - 5aT^2\right) - \frac{h_c \theta T^2}{120}\left(20d + 15bT - 12aT^2\right) \\
&\quad - \frac{d_c \theta T}{12}\left(6d + 4bT - 3aT^2\right) - c/T \\
&= \frac{(s_0 - s_1)t_1}{12T}\left(6d + 2bt_1 - at_1^2\right) + \frac{s_1 - p}{6}\left(6d + 3bT - 2aT^2\right) + \frac{T(\mu - \theta p - \theta d_c)}{12}\left(6d + 4bT - 3aT^2\right) \\
&\quad - \frac{h_c T}{12}\left(6d + 4bT - 5aT^2\right) - \frac{h_c \theta T^2}{120}\left(20d + 15bT - 12aT^2\right) - c/T \qquad (13)
\end{aligned}$$

Model-II: **The selling price is fixed throughout the season**

We have the total average profit $W_2 = [\, FSP\text{-}PC\text{-}HC\text{-}DC\text{-}OC]/\, T$.

So in this case our inventory problem is

$$\begin{aligned}
\textit{Maximize} \quad W_2 &= \frac{s - p}{6}\left(6d + 3bT - 2aT^2\right) + \frac{\theta T(p + d_c)}{12}\left(6d + 4bT - 3aT^2\right) \\
&\quad - \frac{h_c T}{12}\left(6d + 4bT - 5aT^2\right) - \frac{h_c \theta T^2}{120}\left(20d + 15bT - 12aT^2\right) - c/T \qquad (14)
\end{aligned}$$

Special Cases

Case-I: In Model-I, if we put $s_0 = s_1 = s$ *and* $\lambda = 0 = \mu$ then we can easily get the Model-II.

Case-II : If $a \to 0$ *and* $b \to 0$ then Model-II reduces to the classical profit model with deterioration,

$$W = (s-p)d - \frac{dT}{2}\left[\theta\left(d_c + p\right) + h_c\left(1 + \frac{\theta T}{3}\right)\right] - \frac{c}{T} \quad \textit{and} \quad Q = dT + \frac{\theta d T^2}{2} .$$

Case-III : If $a \to 0$, $b \to 0$ *and* $\theta \to 0$ then Model-II reduces to the classical profit model without deterioration where $W = (s-p)d - \dfrac{h_c dT}{2} - \dfrac{c}{T}$ *and* $Q = dT$ Now to optimize (13) we take, $\dfrac{dW_1}{dT} = 0$ which gives

$$\left[\frac{c-(s_0-s_1)t_1\left(6d+2bt_1-at_1^2\right)}{12T^2}\right]+\frac{s_1-p}{6}(3b-4aT)+\frac{(\mu-\theta p-\theta d_c)}{12}\left(6d+8bT-9aT^2\right)$$

$$-\frac{h_c}{12}\left(6d+8bT-15aT^2\right)-\frac{h_c\theta T}{120}\left(80d+45bT-48aT^2\right)=0 \tag{15}$$

$$\frac{d^2W_1}{dT^2}=-\left[\frac{12c-(s_0-s_1)t_1\left(6d+2bt_1-at_1^2\right)}{6T^3}\right]+\frac{6a\theta h_c T^2}{5}-\frac{1}{2}\left[3a(\mu-\theta p-\theta d_c)+\frac{h_c(10a-3b\theta)}{2}\right]$$

$$-\frac{2}{3}\left[h_c(b+\theta d)+\theta b(p+d_c)+a(s_1-p)-b\mu\right] \quad <0 \ \ \text{for any } T>0 \text{ of } (15) \qquad (16)$$

(16)

Now to obtain a solution of (15), we may use any search technique or programme.

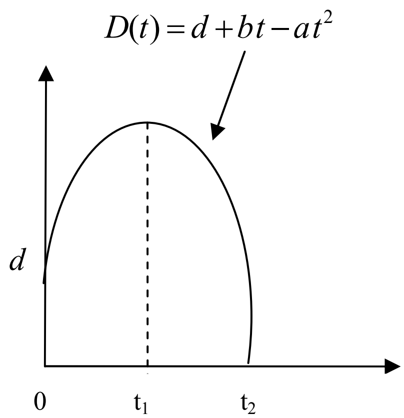

Fig.-1 Parabolic Demand

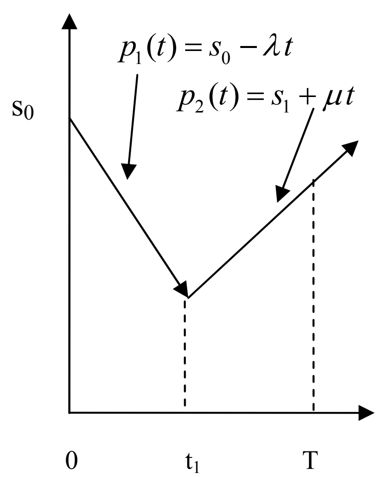

Fig.-2 Selling Price Function

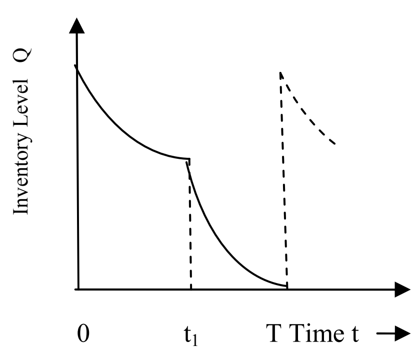

Fig.-3 EOQ model

3.1. Numerical example

If we put a =1, b = 4, d =20 in the demand curve and taking setup cost c = 500 \$, holding cost h_c =3 .5 \$, deterioration cost d_c = 1.5 \$, initial selling price s_0 = 12 \$, unit selling price s =11\$, s_1 = 3\$, unit purchasing price p = 3.5 \$,initial selling price line gradient $\lambda = 0.04$, deterioration rate $\theta = .003$ and in the Model I and Model II we get the optimum result as follows

Table-2: Results for varying and fixed selling price

	t_1	t_2	T^*	μ^*	Q^*	W^*
Model-1	4.8990	6.8990	6.7536	1.797	124.6548	80.8297
Model-II	4.8990	6.8990	6.7536	--	124.6548	61.4420

3.2. Sensitivity analysis

If we put a =1, b = 4, d =20 in the demand curve and taking setup cost c = 500 \$, holding cost h_c =3 .5 \$, deterioration cost d_c = 1.5 \$, initial selling price s_0 = 12 \$, s_1 = 3\$, unit purchasing price p = 3.5 \$,initial selling price line gradient $\lambda = 0.04$, deterioration rate $\theta = .003$ and considering the range of variations of the parameters {(a, b, c, d, s_0 , p) and (λ, s_1, h_c, d_c, θ)} from -50% to + 50% we get the following optimal Table-3 and 4.

Table-3. Sensitivity analysis for high sensitive parameters

Parameter	% change	μ^*	T^*	Q^*	W^*	$\frac{W^* - W_*}{W_*}100\%$
	+50	2.2752	5.2065	88.3284	46.6691	-42.26
	+30	2.0961	5.7003	99.4161	58.8868	-27.15
a	-30	1.4449	8.5714	173.3178	110.5290	36.74
	-50	1.1627	10.8315	242.9195	139.4599	72.54
	+50	1.6310	7.8553	182.5545	133.4556	65.11
	+30	1.7000	7.4022	156.8109	111.3877	37.81
b	-30	1.8800	6.1454	99.1312	53.1928	-34.19
	-50	1.9240	5.7640	85.2220	36.2880	-55.11
	+50	1.7970	6.7536	124.6548	43.8126	-45.80
	+30	1.7970	6.7536	124.6548	58.6194	-77.48
c	-30	1.7970	6.7536	124.6548	103.0399	27.48
	-50	1.7970	6.7536	124.6548	117.8468	45.80
	+50	-----	-----	----	----	----

	+30	1.6031	7.4103	168.3612	121.6490	50.50
d	-30	2.0813	6.0001	84.6480	36.4008	-54.96
	-50	2.3650	5.4160	60.2997	3.2750	-95.95
	+50	3.0220	6.7536	124.6548	192.7420	138.45
	+30	2.5320	6.7536	124.6548	147.9771	83.07
s_0	-30	1.0623	6.7536	124.6548	13.6823	-83.07
	-50	-----	-----	-----	-----	------
	+50	1.7970	6.7536	124.6548	48.5292	-39.96
	+30	1.7970	6.7536	124.6548	61.4494	-23.98
p	-30	1.7970	6.7536	124.6548	100.2100	23.98
	-50	1.7970	6.7536	124.6548	113.1302	39.96

Table-4. Sensitivity analysis for low sensitive parameters

Parameter	% change	μ^*	T^*	Q^*	W^*	$\frac{W^*-W_*}{W_*}100\%$
	+50	1.7700	6.7536	124.6548	79.8029	-1.27
	+30	1.7850	6.7536	124.6548	80.2136	-0.76
λ	-30	1.8090	6.7536	124.6548	81.4458	0.76
	-50	1.8171	6.7536	124.6548	81.8565	1.27
	+50	1.4900	6.7536	124.6548	80.3067	-0.64
	+30	1.6134	6.7536	124.6548	80.5759	-0.31
s_1	-30	1.9808	6.7536	124.6548	81.1435	0.38
	-50	2.1033	6.7536	124.6548	81.3527	0.64
	+50	1.7971	6.7536	124.6548	80.3151	-0.64
	+30	1.7971	6.7536	124.6548	80.5209	-0.38
h_c	-30	1.7971	6.7536	124.6548	81.1384	0.38
	-50	1.7971	6.7536	124.6548	81.3443	0.64
	+50	1.7971	6.7536	124.6548	80.7142	-0.14
	+30	1.7971	6.7536	124.6548	80.7604	-0.08
d_c	-30	1.7971	6.7536	124.6548	80.8989	0.08
	-50	1.7971	6.7536	124.6548	80.9452	0.14
	+50	1.7971	6.7536	125.1749	79.9300	-1.11
	+30	1.7971	6.7536	124.9668	80.2899	-0.67
θ	-30	1.7971	6.7536	124.3427	81.3695	0.67
	-50	1.7971	6.7536	124.1346	81.7293	1.11

4. Comments on sensitivity analysis

From the above Table-3, we see that, the profit function is highly sensitive with the change from -50% to +50% of the parameters of demand curve (a, b,d) and the starting selling price s_0 relative to the other parameters. The objective function is moderately sensitive whenever the setup cost c and purchasing cost p are being changed from -50% to +50%. However, from Table-4, the holding cost h_c, deterioration rate θ and its corresponding cost d_c, selling price line gradient λ and initial pick time selling price s_1 are negligible sensitive parameters over the profit function itself. Throughout the whole Table-3 we see when the initial selling price s_0 is increased to 50%, then the optimum profit is **$ 192.74** with optimum cycle time **6.75** weeks and the optimum order quantity is **124.6548** units. The other findings is that, after the pick time demand, the increase of selling price line gradient causes the increase of the average profit function. From Table-3 we see the highest gradient is **3.02** corresponds maximum average profit.

5. Conclusion and scope of future work

We have developed our model with parabolic/ time quadratic demand rate under the time varying linear selling price function and constant deterioration. From past experience we usually know the gradient of the increasing selling price function and after a pick time we usually could not predict the gradient of the selling price line. This happens due to the lack of prior knowledge and supply-consumption of commodities as well. Numerical examples shows before pick selling price line, the slow decrement of selling price line gradient causes the high average profit in the model which is a very common phenomenon in reality. For every seasonal product, the market selling price remains high at the beginning and at the end of the season, but, the purchasing cost remains constant in many cases as the seller purchase huge goods from supplier for that particular season. In our study we have shown the time dependent selling price model is more profitable than constant selling price model.

Finally, using cubic or higher powers of time in demand rate we may develop various inventory models. In future we shall develop several models incorporating various considerations like fuzzy, fuzzy stochastic, stochastic variable in the demand curve including deterioration and/or shortage in the model.

REFERENCES

1. Aggoun, L., Benkherouf, L. and Tadj, L., On a stochastic inventory model with deteriorating items, *IJMMS,* 25(3) (2001) 97-203.
2. Bose, S., Goswami, A. and Chaudhuri, K.S., An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting, *Journal of Operational Research Society*, 46 (1995) 771-782.

3. Bounkhel, M., Tadj, L. and Benhadid, Y., Optimal control of a production system with inventory-level dependent demand, *Applied Mathematics E-Notes,* 5 (2005) 36-43.
4. Chang, H. J. and Dye, C. Y., An EOQ model for deteriorating items with time varying demand and partial backlogging, *The Journal of Operational Research Society*, 50(11), (1999) 1176-1182.
5. Covert, R. P., Philip, G. C., An EOQ model for items with Weibull distribution deterioration, *AIIE Transaction*, 5(1973) 323-326.
6. Donaldson, W.A., Inventory replenishment policy for linear trend in demand: An analytic solution, *Operational Research Quarterly*, 28, (1977) 663-670.
7. Ghare, P.M., Schrader, G. F., A model for exponentially decaying inventories, *Journal of Industrial Engineering,* 14 (1963) 238-243.
8. Ghosh, S. K. and Chaudhuri, K.S., An order level inventory model for a deteriorating item with Weibull distribution deterioration, time quadratic demand and shortages, *Advanced Modeling and Optimization*, 6(1) (2004) 21-35.
9. Giri, B. C., Goswami, A. and Chaudhuri, K.S., An EOQ model for deteriorating items with time varying demand and costs, *The Journal of Operational Research Society*, 47(11) (1996) 1398-1405.
10. Giri, B.C., Jalan, A.K. and Chaudhuri, K.S., Economic order quantity model with Weibull deterioration distribution, shortages and ramp type demand, *International Journal of Systems Science,* 34(4), (2003) 237-243.
11. Goswami, A. and Choudhury K.S., An EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting, *Journal of Operational Research Society*, 46(6) (1995) 771-780.
12. Jamal, A. M. M., Sarkar, B. R. and Wang, S., An ordering policy for deteriorating items with allowable shortage and permissible delay in payment, *The Journal of Operational Research Society*, 48(8) (1997) 826-833.
13. Mehta, N. J., and Shah, N. H., An inventory model for deteriorating items with exponentially increasing demand and shortages under inflation and time discounting, *Investigacao Operational*, 23(3) (2003) 103-111.
14. Resh , M., Friedman, M. and Barbosa, L.C., On a general solution of the deterministic lot size problem with time proportional demand, *Operations Research*, 24 (1976) 718-725.
15. Roy, M. D., Sana, S.S. and Chaudhuri, K.S., A stochastic EPLS model with random price sensitive demand, *International Journal of Management Science and Engineering Management*, 5(6) (2010) 465-472.
16. Roy, T. and Chaudhuri, K.S., An EPLS model for a variable production rate with stock-price sensitive demand and deterioration, *Yugoslav Journal of Operations Research,* 22(1) (2012) 19-30.
17. Roy, A., An inventory model for deteriorating items with price dependent demand and time-varying holding cost, *Advanced Modeling and Optimization,* 10(1) (2008) 25-37.

18. Silver, E.A., and Meal, H.C., A simple modification of EOQ for the case of a varying demand rate, *Production and Inventory Management*, 10 (1969), 52-65.
19. Skouri, K., Konstantaras, I., Papachristos, S. and Ganas, I., Inventory models with ramp type demand rate, partial backlogging and weibull deterioration rate, *European Journal of Operational Research*,192(1), (2009) 79-92.
20. Tadikamalla, P. R., An EOQ inventory model for items with Gamma distribution, *AIIE Transaction,* 5 (1978) 100-103.
21. Yang, H.L., Teng, J. T. and Chern, M. S., Deterministic inventory lot size models under inflation with shortages and deterioration for fluctuating demand , *Noval Research Logistics*, 48(2001) 144-158.