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Brief Study on Census and Predicted Population of Bangladesh Using Logistic Population Model

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Abstract. In this article the population of Bangladesh has been predicted up to 2025 with the help of an ordinary differential equation model known as logistic population model which is parameterized by growth rate along with capacity human population. Here we used curve fitting method as solution process of the model and tried to compare the prediction between two cases- when the carrying capacity is known and when it is unknown with census population. The computer algebra software MATHEMATICA is used to carry out the computations. Finally result is described graphically.

Keywords: Logistic population model, Carrying capacity, Curve fitting method

AMS Mathematics Subject Classification (2010): 34K28, 34K50, 34K60

1. Introduction

Englishman Malthus (1798) first successfully discussed the caveats of mathematical modeling through his paper, "An Essay on the Principle of Population". One of the most basic and milestone models of population growth was the logistic model of population growth formulated by Pierre François Verhulst (1838). The logistic model takes the shape of a sigmoid curve and describes the growth of a population as exponential, followed by a decrease in growth, and bound by a carrying capacity due to environmental pressures. Hoque et al. [1] used this discrete model with the modification of the growth rate which is not constant. They showed the population of Bangladesh as projected for the period 1976-2093 and obtained a parabolic profile. Wali et al. [2] predicted the population growth of Rwanda and Kagoyire et al. [3] of Uganda using logistic model population of population growth. Here the carrying capacity and the vital coefficients governing the population growth also determined. In 2010 Afroza Polin et.al. had investigated mathematical and econometric model for the projection of ultimate population of the earth [8]. Das showed nonlinear statistical model and its applications to diffusion of mobile telephony in India in 2013 [9].

Curve fitting method for finding the unknown coefficients is also a dependable method with which we can project the future population using the present population of a given regular time interval. The prediction using curve fitting method is also more accurately near to the census population. Barun [4] in 'Differential Equations and Their Application: An Introduction to Applied Mathematics' described the mathematical calculations and its applications as mathematical modeling. Giordano and Weir [5] were used the logistic curve fitting method for predicting sample populations. Mathews [6] investigated a theoretical study on bounded population growth by a curve fitting lesson. In 2012 Minarul Haque et al. had predicted population in Bangladesh by growth rate modeling [7]. This study is to present curve fitting methods for fitting the logistic curve of Bangladesh population data. We tried to compare the prediction between two cases- when the carrying capacity is known and when it is unknown. The computer algebra software MATHEMATICA is used in this article to carry out the computations and plot graphs.

2. Mathematical Model

Let $p(t)$ denote the population at time t. Assume that $\lim_{t\to\infty} p(t)=K$

The differential equation relating $p(t)$ and $p'(t)$ is

$$p'(t)=r\,p(t)\left[1-\frac{p(t)}{K}\right] \tag{1}$$

The parameter r is the initial rate of increase when $p(t)$ is small, and K is the carrying capacity or limiting population as $t\to\infty$. The form of the solution to (1) is known to be

$$p = p(t)=\frac{K}{1+ce^{r(t-t_0)}} \tag{2}$$

3. Solution techniques

The value t_0 in (2) is usually chosen to be a convenient starting date such as 1971. We want to fit the curve of the form (2) to a given set of data (t_0, p_0), (t_1, p_1),………, (t_n, p_n). The model (2) involves three unknown parameters r, c and K. However, by judiciously selecting a value for K, we can reduce the problem to finding just two parameters r and c. This will permit us to use the technique known as "data linearization" which reduces the problem to finding a least squares line.

First, rearrange (2) in the form

$$\frac{K}{p}-1=ce^{r(t-t_0)} \tag{3}$$

Now take the logarithm of both sides of (1) and obtain

$$\ln\left(\frac{K}{p}-1\right)=\ln(c)+r(t-t_0) \tag{4}$$

Next, we introduce the change of variables

$$T=t-t_0, \qquad P=\ln\left(\frac{K}{p}-1\right), \qquad B=\ln(c) \quad \text{and} \quad A=r \tag{5}$$

This will transform equation (4) into the linear form

$$P=F(T)=AT+B \tag{6}$$

The data points to be used in (6) are the transformed pairs

$$(T_k, P_k)=\left(t_k-t_0, n\left(\frac{K}{p}-1\right)\right) \qquad \text{for} \quad k=0,1..........n \tag{7}$$

When the least squares line is fit to the data (7), the coefficients A and B in equation (6) are obtained. This computation is carried out by MATHEMATICA. Finally, the coefficients c and r are calculated.

$$c = e^B \quad \text{and} \quad r = A \tag{8}$$

We shall show how to fit data to the above model and use formula (2) to estimate the population of Bangladesh in the years 2000 through 2010. We discuss two problems; one is when the carrying capacity of a particular population is known and when it is unknown. Firstly, we assume a carrying capacity and by logistic curve fitting method determine the constant c and r. Secondly, all the three parameters c, r and K are unknown. Computer determines these values based on our arbitrary inputs.

We will fit the curve $p_1 = p_1(t) = \frac{K}{1+ce^{r(t-t_0)}}$ to the points

$(2000,129.19)$, $(2001,131.27)$, $(2002,133.38)$, $(2003,138.45)$, $(2004,141.34)$, $(2005,144.32)$, $(2006,147.36)$, $(2007,150.49)$, $(2008,153.55)$, $(2009,156.05)$, $(2010,156.12)$ which are the census figures for the population of the Bangladesh and also predict the future population. Where, K is the known carrying capacity, r is the initial rate of increase.

Case-I: when carrying capacity is known

Firstly assume that carrying capacity $K = 250$ (millions) and use the method of "data linearization".

From equation (4) we know, $P = AT + B$. Then the normal equations are `

$$\sum P = A\sum T + nB \tag{9}$$

$$\sum PT = A\sum T^2 + B\sum T \tag{10}$$

By using the above equations we get the following table

t	p (in millions)	$T = t - t_0$	$P = \ln\left(\frac{K}{P} - 1\right)$	PT	T^2
2000	129.19	0	−0.0670651	0	0
2001	131.27	1	−0.1004043	−0.1004043	1
2002	133.38	2	−0.1342815	−0.268563	4
2003	138.45	3	−0.2160363	−0.6481089	9
2004	141.34	4	−0.2629445	−1.051778	16
2005	144.32	5	−0.3116175	−1.5580875	25
2006	147.36	6	−0.3616509	−2.1699054	36
2007	150.49	7	−0.4136386	−2.8954702	49
2008	153.55	8	−0.4650016	−3.7200128	64
2009	156.05	9	−0.5074138	−4.5667242	81
2010	156.12	10	−0.5086075	−5.086075	100
		$\sum T = 55$	$\sum P = -3.348661625$	$\sum PT = -22.0651293$	$\sum T^2 = 385$

Table 1: List square curve fitting table

Here, $n=11$, $\sum T=55$, $\sum P=-3.348661625$, $\sum PT=-22.0651293$ and $\sum T^2=385$

Substituting these values in the normal equation (9) and (10) we get

$$A=-0.048380192$$
$$B=-0.062522821$$

Also we can write from (8), $c=e^B=0.939391624$, $r=A=-0.0483801928$

Putting the values of c, r and K in equation (2), we obtain

$$p_1=p_1(t)=\frac{250}{1+0.939391624\times e^{-0.0483801928\times(t-2000)}} \tag{11}$$

Proceeding this way we get the following prediction

Year	**Census Population**	**Predicted Population**		**Year**	**Census Population**	**Predicted Population**
2000	129.19	128.906		2013		166.573
2001	131.27	131.924		2014		169.241
2002	133.38	134.934		2015		171.863
2003	138.45	137.933		2016		174.437
2004	141.34	140.916		2017		176.964
2005	144.32	143.881		2018		179.439
2006	147.36	146.824		2019		181.864
2007	150.49	149.743		2020		184.235
2008	153.55	152.634		2021		186.553
2009	156.05	155.494		2022		188.816
2010	156.12	158.321		2023		191.024
2011	158.57	161.111		2024		193.176
2012	161.08	163.863		2025		195.272

Table 2: Predicted population $p_1(t)$ (when K is known)

Graphical representation of the predicted population

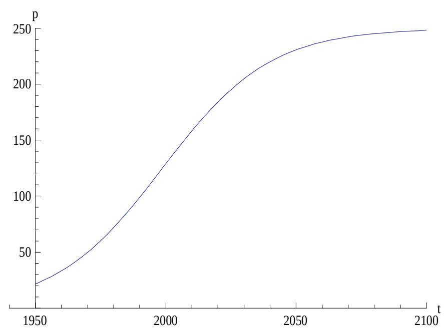

Figure 1: Predicted population curve (when K is known)

Case-II: when carrying capacity is unknown

Secondly we fit the curve $p_2(t)=\dfrac{K}{1+ce^{r(t-t_0)}}$ to the above data points by finding the parameters c, r and K.

For the "data linearization" method, presented above, we have assigned the limiting population as $\lim\limits_{t\to\infty} p_1(t)=K=250$ (millions) **.** But in this section, Let us see that computer determines the parameter K. For this we use the more sophisticated problem of minimizing the sum of the squares of the residuals and require a minimization subroutine for functions of several variables described as follows:

Here, we obtain that finally computer determines the Carrying capacity $K=189.7471$ after 2050 times iterations. Firstly we assumed that $r=-0.04$ and $c=0.94$, where computer determine the value as $r=-0.0857114$ and $c=0.4816739$.

The population function corresponding to these parameters is

$$p_2(t)=\frac{189.7471}{1+0.4816739\times e^{-0.0857114\times(t-2000)}}$$

Using this process we find the future population projection.

Year	**Census Population**	**Predicted Population**		**Year**	**Census Population**	**Predicted Population**
2000	129.19	128.063		2013		163.848
2001	131.27	131.576		2014		165.706
2002	133.38	134.975		2015		167.449
2003	138.45	138.253		2016		169.081
2004	141.34	141.405		2017		170.607
2005	144.32	144.428		2018		172.032
2006	147.36	147.318		2019		173.362
2007	150.49	150.074		2020		174.600
2008	153.55	152.697		2021		175.753
2009	156.05	155.186		2022		176.824
2010	156.12	157.543		2023		163.849
2011	158.57	159.7703		2024		165.706
2012	161.08	161.870		2025		167.449

Table 3: Predicted population $p_2(t)$ (when K unknown)

Graphical representation of the above population is shown in Figure 2.

4. Results and discussions

It is illustrated that how curve fitting is used in a simple mathematical model in this article. If the limiting value K is known, or can be determined by other means, then the first method should be used and $p_1(t)$ is used to fit the data and to extrapolate. If K is not known, but can be estimated, then the second method can be used to determine K

and the function $p_2(t)$ is used to model the data. We determined from the above study that both the two methods fitted the data smoothly for Bangladesh. Finally we can see that Method 2 predicts more nearest population figure to original population and minimize the error. A comparison of the original and projected data of Bangladesh population (in million) is given in the following table.

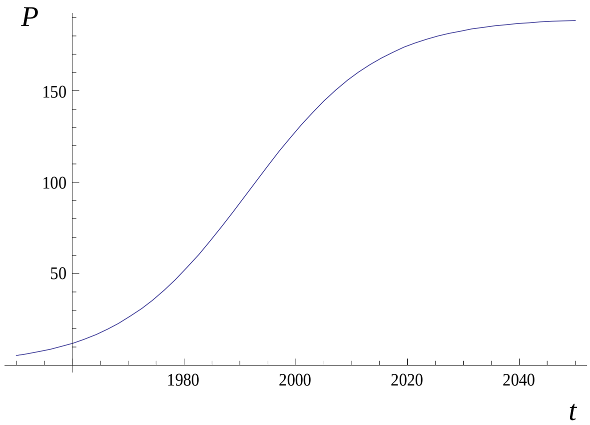

Figure 2: Predicted population curve (when K is unknown)

In this case firstly (case-I) we assume that the carrying capacity of Bangladesh is known and assumed it as 250 million. Secondly (case-II) computer determined after iteration that carrying capacity 189.75 million gives the closest prediction to the original data.

Carrying capacity known					Carrying capacity unknown	
Year t_k	**Observed Population** p_k	**Prediction Using** $p_1(t_k)$	**Percent error** $100\times\frac{p_k - p_1(t_k)}{p_k}$		**Prediction Using** $p_2(t_k)$	**Percent error** $100\times\frac{p_k - p_2(t_k)}{p_k}$
2000	129.19	128.906	0.22		128.063	0.87
2001	131.27	131.924	-0.50		131.576	-0.23
2002	133.38	134.934	-1.17		134.975	-1.18
2003	138.45	137.933	0.37		138.253	0.14
2004	141.34	140.916	0.29		141.405	-0.05
2005	144.32	143.881	0.30		144.428	-0.07
2006	147.36	146.824	0.36		147.318	0.02
2007	150.49	149.743	0.50		150.074	0.28
2008	153.55	152.634	0.60		152.697	0.56
2009	156.05	155.494	0.36		155.186	0.59
2010	156.12	158.321	-1.40		157.543	-0.91
2011	158.57	161.111	-1.60		159.7703	-0.75
2012	161.08	163.863	-1.73		161.870	-0.49

Table 4: Comparison between census and projected population of Bangladesh

5. Conclusion

Using curve fitting method we used the population of a certain period of consecutive 11 years. It gives close prediction to the real population. In its first method we can assume any possible carrying capacity and predict the future population. In second method computer can determine the suitable carrying capacity and predict more closet prediction. Second method is the best to predict data as computer define the possible carrying capacity and predict very good data.

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