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Generalized Bernoulli Sub-ODE Method and its Applications

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Abstract. In this paper, by using the generalized Bernoulli sub-ODE method the Phi-four equation and Benney-Luke equation are solved analytically where some exact traveling wave solutions are established. This is the method which can be adapted to solve nonlinear partial differential equations. Solitary waves can be obtained from each traveling wave solution by setting particular values to its unknown parameters. By adjusting these parameters, one can get an internal localized mode.

Keywords: Generalized Bernoulli Sub-ODE method; Phi-four equation; Benney-Luke equation; traveling wave solutions.

AMS Mathematics Subject Classification (2010): 35Qxx

1. Introduction

The NLEEs are very much important due to its wide-ranging applications. In modern science nonlinear phenomena are one of the most impressive fields of research. Nonlinear phenomena occur in numerous branches of science and engineering, such as, plasma physics, fluid mechanics, gas dynamics, elasticity, relativity, chemical reactions, ecology, optical fiber, solid state physics, biomechanics, etc., all are essentially governed by nonlinear equations. NLEEs are frequently used to illustrate the motion of isolated waves. Since the appearance of solitary wave in natural sciences is expanding every day, it is important to seek for exact traveling wave solutions to NLEEs. The exact solutions to NLEEs help us to provide information about the structure of complex physical phenomena. Therefore, exploration of exact traveling wave solutions to NLEEs turns into an essential task in the study of nonlinear physical phenomena. It is notable to observe that there is no unique method to solve all kind of NLEEs. For this reason, a lot of methods have been established, such as, the Jacobi elliptic function method [1], the homotopy perturbation method [2], the variational method[3], the Exp-function method

[4], Enhanced (G'/G)-expansion Method [5], the modified simple equation method [6], the homogenous balance method [7] etc.

The objective of this article is to bring to bear the generalized Bernoulli sub-ODE method to extract new exact traveling wave solutions and then solitary wave solutions to the Phi-four equation [8-9] and Benney–Luke equation [10]. This application shows the simplicity, efficiency, and effectiveness of the generalized Bernoulli sub-ODE method. To the best of our knowledge the generalized Bernoulli sub-ODE method has not been applied to the above mentioned equation in the previous literature.

2. Description of generalized Bernoulli sub-ode method

Consider a nonlinear partial differential equation of the form

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \cdots\cdots\cdots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is a function of x and t , P is a polynomial in $u = u(x,t)$ and its different partial derivatives, in which the highest order derivatives and nonlinear terms are present.

The generalized Bernoulli sub-ODE method for solving the nonlinear partial differential equation (2.1) is demonstrated by the following steps:

Step 1: The traveling wave variable $u(x,t) = u(\xi)$ where $\xi = x - ct$, reduce the NPDE (2.1) to an NODE of the form

$$P(u, -cu', u', c^2u'', -cu'', u'', \cdots\cdots\cdots) = 0, \tag{2.2}$$

Step 2: Suppose that the solution of Eq. (2.2) can be expressed as a polynomial in Φ, i.e.

$$u(\xi) = \sum_{i=0}^{m} A_i \Phi^i \,, \tag{2.3}$$

where $A_m, A_{m-1}, \ldots\ldots$ are constants to be determined later and $A_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq.(2.2), and $\Phi = \Phi(\xi)$ satisfies the following equation:

$$\Phi' + \lambda\Phi = \mu\Phi^2, \tag{2.4}$$

When $\mu \neq 0$, Eq. (2.4) is the type of Bernoulli equation, and we can obtain the solution as

$$\Phi(\xi) = -\frac{\lambda}{2\mu}\left(\tanh\left(\frac{\lambda}{2}\xi\right) - 1\right). \tag{2.5}$$

$$\text{or, } \Phi(\xi) = -\frac{\lambda}{2\mu}\left(\coth\left(\frac{\lambda}{2}\xi\right) - 1\right). \tag{2.6}$$

Step 3: Substituting Eq. (2.3) into Eq. (2.2) and using Eq. (2.4), collecting all terms with the same power of Φ together, the left-hand side of Eq. (2.2) is converted into another polynomial in Φ. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations which gives $A_m, A_{m-1}, \ldots\ldots \lambda, \mu$.

Step 4: Setting the values of constants obtained in Step 3, and by using the solutions of Eq. (2.4), we can construct the traveling wave solutions of the nonlinear partial differential equation (2.1).

3. Application of the method

3.1. Generalized Bernoulli sub-ODE method for Phi-four equation

In this section, we will apply the generalized Bernoulli sub-ODE method to find the traveling wave solutions of Phi-four equation

$$u_{tt} - au_{xx} - u + u^3 = 0, \quad a > 0 \tag{3.1}$$

Making the wave transformation $u(x,t) = u(\xi)$, $\xi = x - ct$, Eq.(3.1) becomes the following ODE:

$$(c^2 - a)u'' - u + u^3 = 0 \ , \tag{3.2}$$

Balancing the order of u'' and u^3 appearing in Eq.(3.2), we have $m = 1$.
So, according to the rule of sub-ODE method, the solution of Eq.(3.2) takes the following form

$$u(\xi) = A_1\Phi + A_0, \ \ A_1 \neq 0 \tag{3.3}$$

where A_1, A_0 are constants to be determined later.
Substituting Eq.(3.3) along with Eq.(2.4) into Eq.(3.2) and collecting all the terms with the same power of Φ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$3A_0^2 A_1 - aA_1\lambda^2 + c^2 A_1\lambda^2 - A_1 = 0$$
$$3c^2 A_1\lambda\mu + 3A_0 A_1^2 + 3aA_1\lambda\mu = 0$$
$$-2aA_1\mu^2 + 2c^2 A_1\mu^2 3 + A_1^3 = 0$$
$$A_0^3 - A_0 = 0$$

Solving the algebraic equations above, we obtain two sets of values for the constants:

Set A: $A_0 = 1, \quad A_1 = -\dfrac{2\mu}{\lambda}, \quad c = \sqrt{a - \dfrac{2}{\lambda^2}}$ (3.4)

Set B: $A_0 = -1, \quad A_1 = \dfrac{2\mu}{\lambda}, \quad c = \sqrt{a - \dfrac{2}{\lambda^2}}$ (3.5)

Substituting Eq. (3.4) and Eq. (3.5) into Eq. (3.3) along with Eq. (2.5) and Eq. (2.6), we obtain the solution of Eq.(3.1) as:

$$\left.\begin{aligned} u_{1A}(x,t) &= \tanh\left(\frac{\lambda \mathrm{x} - \mathrm{t}\sqrt{\mathrm{a}\lambda^2 - 2}}{2}\right) \\ or, \quad u_{1A}(x,t) &= \coth\left(\frac{\lambda \mathrm{x} - \mathrm{t}\sqrt{\mathrm{a}\lambda^2 - 2}}{2}\right) \end{aligned}\right\} \tag{3.6}$$

$$\left.\begin{aligned} &and \quad u_{1B}(x,t) = -\tanh\left(\frac{\lambda x - t\sqrt{a\lambda^2 - 2}}{2}\right) \\ &or, \quad u_{1B}(x,t) = -\coth\left(\frac{\lambda x - t\sqrt{a\lambda^2 - 2}}{2}\right) \end{aligned}\right\} \tag{3.7}$$

3.2. Generalized Bernoulli sub-ODE method for Benney-Luke equation

We know that the Beneey-Luke equation is

$$u_{tt} - u_{xx} + a u_{xxxx} - b u_{ttxx} + u_t u_{xx} + 2u_x u_{xt} = 0, \tag{3.8}$$

The transformation $u(x,t) = u(\xi)$, $\xi = x - ct$, Eq.(3.8) becomes the following ODE:

$$(c^2 - 1)u'' + (a - bc^2) + u'''' - 3cu'u'' = 0 \quad , \tag{3.9}$$

Here m=1 and the same procedure described above yields a set of algebraic equations of constants.

$$c^2 A_1 \lambda^2 - a A_1 \lambda^4 - A_1 \lambda^2 + bc^2 A_1 \lambda^4 = 0$$

$$3c^2 A_1 \lambda\mu + 3A_1 \lambda\mu - 15bc^2 A_1 \lambda^3 \mu + 15ac^2 A_1 \lambda^3 \mu + 3cA_1^{\,2} \lambda^3 = 0$$

$$-2A_1 \mu^2 + 2c^2 A_1 \mu^2 + 50bc^2 A_1 \lambda^2 \mu^2 - 12cA_1^{\,2} \lambda^2 \mu - 50aA_1 \lambda^2 \mu^2 = 0$$

$$60bc^2 A_1 \lambda\mu^3 + 15cA_1^{\,2} \lambda\mu^2 + 60A_1 \lambda\mu^3 = 0$$

$$24bc^2 A_1 \mu^4 - 24aA_1 \mu^4 - 6cA_1^{\,2} \mu^3 = 0$$

Solving, we get

$$A_0 = A_0, \quad A_1 = -\sqrt{(1 + a\lambda^2)(1 + b\lambda^2)}, \quad c = \sqrt{\frac{1 + a\lambda^2}{1 + b\lambda^2}} \tag{3.10}$$

Substituting Eq. (3.10) into Eq. (3.3) along with Eq. (2.5) and Eq. (2.6), we obtain the solution of Eq.(3.8) as:

$$\left.\begin{aligned} &u_2(x,t) = A_0 - \frac{\lambda}{2\mu}\left[\tanh\left\{\frac{\lambda}{2}\left(x - \sqrt{\frac{1 + a\lambda^2}{1 + b\lambda^2}}.t\right)\right\} - 1\right] \\ &or, \quad u_2(x,t) = A_0 - \frac{\lambda}{2\mu}\left[\coth\left\{\frac{\lambda}{2}\left(x - \sqrt{\frac{1 + a\lambda^2}{1 + b\lambda^2}}.t\right)\right\} - 1\right] \end{aligned}\right. \tag{3.11}$$

Remark: All the obtained results has been checked with Maple-13 by putting them back into the original equation and found correct.

4. Graphical representation

In this section, we will discuss the physical interpretation of the solution of the Phi-four equations and Benney–Luke equation. By applying the generalized sub-ODE method Phi-four equation and Benney–Luke equation affords the traveling wave solutions from Eqs. (3.6), (3.7) and Eq. (3.11) respectively. The solutions (3.6) and (3.7) are represented in Fig. 1-2. These show the shape of kink type traveling wave solutions. The solutions

Eq. (3.11) represented in Fig. 3 shows kink type whereas the Fig.-4 shows the shape of singular kink type traveling wave solution.

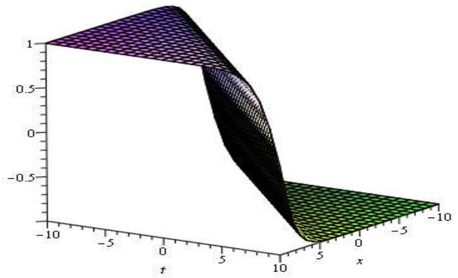

Figure 1: Profile of $u_{1A}(x,t)$ for a=3,λ=1 within the interval $-10 \le x,t \le 10$.

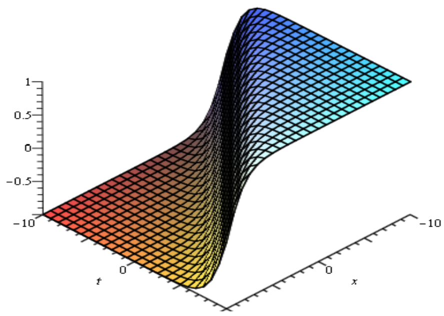

Figure 2: Profile of $u_{1B}(x,t)$ a=3,λ=1within the interval $-10 \le x,t \le 10$.

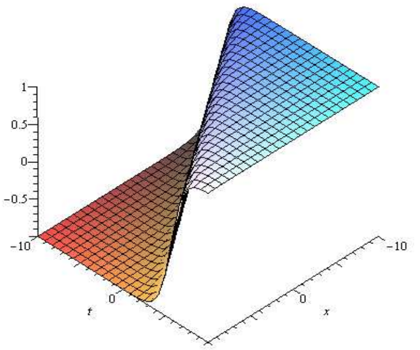

Figure 3: Periodic profile of $u_2(x,t)$ fora=3,λ=1,b=1, $\mu = 1, A_0 = 0$ within the interval $-10 \le x,t \le 10$.

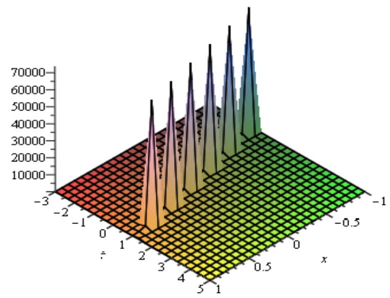

Figure4: Soliton profile of $u_2(x,t)$ for a=3,λ=1,b=1, $\mu = 1, A_0 = 0$ within the interval $-1 \le x \le 1, -3 \le t \le 5$.

5. Conclusions

In this article, the generalized sub-ODE method method has been implemented to find the exact traveling wave solutions and then the solitary wave solutions of two very important nonlinear evolution equations, namely, the Phi-four equation and the Benney–Luke equation. It is important to observe that, the currently proposed method in comparing to other methods the generalized sub-ODE method is much simpler. Here, we achieved the value of the coefficients A_1, A_0 etc. with using any symbolic computation software such as Maple-13 where the procedure is very easy, concise and straightforward. Also it is quite capable and almost well suited for finding exact solutions of other NLEEs in mathematical physics.

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