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On Pseudo Regular and Pseudo Irregular Bipolar Fuzzy Graphs

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Abstract. In this paper, pseudo regular bipolar fuzzy graphs, pseudo irregular bipolar fuzzy graphs, highly pseudo irregular bipolar fuzzy graphs, totally pseudo regular bipolar fuzzy graphs, pseudo totally irregular bipolar fuzzy graphs, highly pseudo totally irregular bipolar fuzzy graphs are introduced and some properties of its properties are discussed.

Keywords: pseudo degree and total pseudo degree in bipolar fuzzy graph, pseudo regular fuzzy graph, totally pseudo regular bipolar fuzzy graph, highly pseudo irregular fuzzy graphs, highly pseudo totally irregular fuzzy graphs.

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1. **Introduction**

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by $d(v)$. A graph G is regular if all its vertices have the same degree. The 2-degree of v [4] is the sum of the degrees of the vertices adjacent to v and it is denoted by t(v). We call $\frac{t(v)}{d(v)}$, the average degree of v . A graph is called pseudo-regular if every vertex of G has equal average degree [3].

Fuzzy set theory was first introduced by Zadeh in 1965 [23]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs [8]. Now, fuzzy graphs have many applications in branches of engineering and technology.

Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Bipolar fuzzy sets whose range of membership degree is [-1,1]. In bipolar fuzzy sets, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree within (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree within [-1,0) of an element indicates the element somewhat satisfies the implicit counter property. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible [2].

2. Review of literature

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [7]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [6]. Akram and Dudek introduced the notions of regular bipolar fuzzy graphs [1] and also introduced intuitionistic fuzzy graphs [2]. Samanta and Pal introduced the concept of irregular bipolar fuzzy graphs [15]. Pal and Rashmanlou introduced irregular interval-valued fuzzy graphs [22]. Radha and Kumaravel introduced the concept of an edge degree, total edge degree in bipolar fuzzy graphs and edge regular bipolar fuzzy graphs and discussed about the degree of an edge in some bipolar fuzzy graphs [6]. Santhi Maheswari and Sekar introduced neighbourly edge irregular fuzzy graphs [10] and neighbourly edge irregular bipolar fuzzy graphs [11] and discussed some of its properties Maheswari and Sekar introduced an m-neighbourly irregular bipolar fuzzy graphs [12]. Maheswari and Sekar introduced pseudo regular fuzzy graphs and discussed its properties [13]. Maheswari and Sudha introduced pseudo irregular fuzzy graphs, highly pseudo irregular fuzzy graphs and discussed its properties [14]. For other works on fuzzy and related graphs see [16-20].

These motivates us to introduce pseudo regular bipolar fuzzy graphs, pseudo irregular bipolar fuzzy graphs and highly pseudo irregular bipolar fuzzy graphs and discussed some of their properties. Throughout this paper, the vertices take the membership value A = (m_1^+, m_1^-) and edges take the membership value B = (m_2^+, m_2^-), where (m_1^+, m_2^+) in [0,1] and (m_1^-, m_2^-) in [-1,0].

3. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 3.1. A fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V,E)$: is a pair of functions (σ, μ) where $\sigma : V \rightarrow [0; 1]$ is a fuzzy subset of a set V and $\mu : V X V \rightarrow [0; 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \Lambda \sigma(v)$ is satisfied [7].

Definition 3.2. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The 2-degree of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$. That is, $t_G(v) = \Sigma d_G(u)$, where $d_G(u)$ is the degree of the vertex u which is adjacent with the vertex v[13].

Definition 3.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. A pseudo (average) degree of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v. The total pseudo degree of a vertex v in G is denoted by $t\, d_a(v)$ and is defined as $t\, d_a(v) = d_a(v) + \sigma(v)$, for all v in G [13].

Definition 3.4. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V,E)$. If $d_a(v) = k$, for all u in V; then G is called k- pseudo regular fuzzy graph [13].

Definition 3.5. Let G : (σ, μ) be a fuzzy graph on G^* : (V,E). If $td_a(v) = k$, for all u in V; then G is called k-totally pseudo regular fuzzy graph [13].

Definition 3.6. Let G : (σ, μ) be a fuzzy graph on G^* : (V,E). Then, G is said to be pseudo irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct pseudo degrees [14].

Definition 3.7. Let G : (σ, μ) be a fuzzy graph on G^* : (V,E). Then, G is said to be pseudo totally irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total pseudo degrees [14].

Definition 3.8. Let G : (σ, μ) be a fuzzy graph on G^* : (V, E). Then, G is said to be highly pseudo irregular fuzzy graph if every vertex is adjacent to the vertices having distinct pseudo degrees [14].

Definition 3.5. Let G : (σ, μ) be a fuzzy graph on G^* : (V, E). Then, G is said to be highly pseudo totally irregular fuzzy graph if every vertex is adjacent to the vertices having distinct total pseudo degrees [14].

Definition 3.9. A bipolar fuzzy graph with an underlying set V is defined to be a pair (A, B), where A = (m_1^+, m_1^-) is a bipolar fuzzy set on V and B = = (m_2^+, m_2^-) is a bipolar fuzzy set on E such that $m_2^+(x, y) \leq \min\{(m_1^+(x), m_1^+(y)\}$ and $m_2^+(x, y) \geq \max\{(m_1^-(x), m_1^-(y)\}$ for all (x,y) in E. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E [1,15].

Definition 3.10. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). The positive degree of a vertex u in G is defined as $d^+(u) = \sum m_2^+(u,v)$, for uv in E. The negative degree of a vertex u in G is defined as $d^-(u) = \sum m_2^+(u,v)$,, for uv in E and $\sum m_2^+(u,v)$, $= \sum m_2^-(u,v)$, = 0 if uv not in E. The degree of a vertex u is defined as $d(u)=(d^+(u),d^-(u))$ [1,15].

Definition 3.11. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). The positive total degree of a vertex u in G is defined as $td^+(u) = \sum m_2^+(u,v)+ m_1^+(u)$, for uv in E. The negative total degree of a vertex u in G is defined as $td^-(u) = \sum m_2^-(u,v)+ m_1^-(u)$, for uv in E [1,15].

4. Pseudo degree and total pseudo degree in bipolar fuzzy graphs

In this section, we define pseudo degree in bipolar fuzzy graphs, total pseudo degree in bipolar fuzzy graphs.

Definition 4.1. Let G : (A, B) be a fuzzy graph on G^* : (V, E). The 2-degree of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$ $=(t_G + (v), t_G - (v))$. That is, the positive 2-degree of v is $t_G^+(v) = \Sigma d_G^+(u)$, where

$d_G^+(u)$ is the positive degree of the vertex u which is adjacent with the vertex v and the negative 2-degree of v is $t_G^-(v) = \Sigma d_G^-(u)$, where $d_G^-(u)$ is the negative degree of the vertex u which is adjacent with the vertex v.

Definition 4.2. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). The positive pseudo (average) degree of a vertex v in bipolar fuzzy graph G is denoted by $d_a^+(v)$ and is defined by $d_a^+(v) = \frac{t_G+(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v. The negative pseudo (average) degree of a vertex v in bipolar fuzzy graph G is denoted by $d_a^-(v)$ and is defined by $d_a^-(v) = \frac{t_G-(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v. The pseudo degree of a vertex u in bipolar fuzzy graph G is defined as

$d_a(v) = (d_a^+(v), d_a^+(v))$.

Definition 4.3. The positive total pseudo degree of a vertex v in G is denoted by $t\, d_a^+(v)$ and is defined as $t\, d_a^+(v) = d_a^+(v) + m_1^+(v)$, for all v in G. The negative total pseudo degree of a vertex v in G is denoted by $t\, d_a^-(v)$ and is defined as $t\, d_a^-(v) = d_a^-(v) + m_1^-(v)$, for all v in G.The total pseudo degree of a vertex v in bipolar fuzzy graph G is defined by $t\, d_a(v) = (t\, d_a^+(v), t\, d_a^-(v))$.

5. Pseudo regular bipolar fuzzy graphs and totally pseudo regular bipolar fuzzy graphs

In this section, pseudo regular bipolar fuzzy graph and totally pseudo regular bipolar fuzzy graph and discussed about its properties.

Definition 5.1. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). If $d_a(v) = (f_1, f_2)$, for all v *in* V, then G is called (f_1, f_2) - pseudo regular bipolar fuzzy graph.

Definition 5.2. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). If $td_a(v) = d_a(v) + A(v) = (k_1, k_2)$, for all v *in* V, then G is called (k_1, k_2) – totally pseudo regular bipolar fuzzy graph.

Remark 5.3. A pseudo regular bipolar fuzzy graph need not be a totally pseudo regular bipolar fuzzy graph.

Remark 5.4. A totally pseudo regular bipolar fuzzy graph need not be a pseudo regular bipolar fuzzy graph.

Theorem 5.5. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). Then $A(u) = (m_1^+(u), m_1^-(u))$, *for all* $u \in V$ is a constant function if and only if the following are equivalent.

(i) G is a pseudo regular bipolar fuzzy graph.

(ii) G is a totally pseudo regular bipolar fuzzy graph.

Proof. Assume that $A = (m_1^+, m_1^-)$ is a constant function. Let $A(u) = (m_1^+(u), m_1^-(u)) = (c_1,c_2)$, for all u ϵ V. Suppose G is a pseudo regular bipolar fuzzy graph. Then $d_a(u) = (f_1, f_2)$, for all uϵV. Now, $td_a(u) = d_a(u) + A(u) = (f_1,f_2) + (m_1^+(u), m_1^-(u)) = (f_1,f_2) + (c_1,c_2)$

$=(f_1+c_1, f_2+c_2)$, for all uϵV. Hence G is a totally pseudo regular bipolar fuzzy graph. Thus (i) $\Longrightarrow$ (ii) is proved.

Suppose G is a totally pseudo regular bipolar fuzzy graph. Then $td_a(u) = (k_1, k_2)$, for all uϵV $\Longrightarrow d_a(u)+A(u) = (k_1, k_2)$, for all u$\epsilon$V $\Longrightarrow d_a(u)+ (c_1,c_2) = (k_1, k_2)$, for all u$\epsilon$V $\Longrightarrow d_a(u) = (k_1, k_2) - (c_1, c_2) = (k_1-c_1, k_2-c_2)$, all u ϵ V. Hence G is a pseudo regular bipolar fuzzy graph. Thus (ii) $\Longrightarrow$ (i) is proved. Hence (i) and (ii) are equivalent.

Conversely, suppose (i) and (ii) are equivalent. Let G be a pseudo regular bipolar fuzzy graph and a totally pseudo regular bipolar fuzzy graph. Then $d_a(u) =(f_1,f_2)$ and $td_a(u) = (k_1, k_2)$, for all u ϵ V. Now $td_a(u) = (k_1, k_2)$, for all uϵV $\Longrightarrow d_a(u)+A(u) = =(k_1, k_2)$, for all u$\epsilon$V $\Longrightarrow (f_1, f_2)+ A(u) =(k_1, k_2)$, for all u$\epsilon$V $\Longrightarrow A(u) =(k_1-f_1, k_2-f_2)$, for all u$\epsilon$V. Hence A is a constant function.

Theorem 5.6. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E). If G is both pseudo regular and totally pseudo regular bipolar fuzzy graph then A is a constant function.

Proof. Assume that G is both pseudo regular and totally pseudo regular bipolar fuzzy graph. Then $d_a(u) = (f_1, f_2)$ and $td_a(u) = (k_1, k_2)$, for all uϵV. Now, $td_a(u) = (k_1, k_2) \Longrightarrow d_a(u)+A(u) = (k_1, k_2) \Longrightarrow (f_1, f_2) +A(u) = (k_1, k_2) \Longrightarrow A(u) = (k_1 - f_1, k_2 - f_2)$ = constant. Hence A is a constant function.

Theorem 5.7. Let G : (A, B) be a bipolar fuzzy graph on G*(V, E), a cycle of length n. If B is a constant function, then G is a pseudo regular bipolar fuzzy graph.

Proof. If B is a constant function say $B(uv) = (m_2^+(uv), m_2^-(uv))= (c_1, c_2)$, for all uv ϵ E. Then $d_a(u) = (2c_1, 2c_2)$, for all u ϵ V. Hence G is a $(2c_1, 2c_2)$ - pseudo regular bipolar fuzzy graph.

Remark 5.8. *Converse of the above theorem 5.7 need not be true.*

Example 5.9. Consider a bipolar fuzzy graph on G*(V, E).

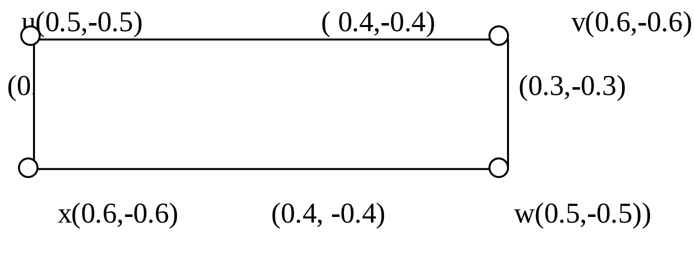

Figure 1:

Here, d(u) = (0.7,-0.7); d(v) = (0.7,-0.7); d(w) = (0.7,-0.7); d(x) = (0.7,-0.7) and $d_{G^*}(u)$ = 2, for all u ϵ V. Now, u is adjacent to v and x. So, $d_a(u) = \frac{d(v)+d(x)}{2}$ = (0.7,-0.7). Noted that $d_a(v) = (0.7,-0.7)$, for all $v \in V$. Hence G is (0.7,-0.7) - pseudo regular bipolar fuzzy graph. But B is not constant.

Theorem 5.10. Let G: (A, B) be a bipolar fuzzy graph on G*(V, E), an even cycle of length n. If the alternate edges have same membership values, then G is a pseudo regular bipolar fuzzy graph.

Proof. If the alternate edges have the same membership values, then

$$B(e_i) = \begin{cases} (c_1, c_2), & \text{if } i \text{ is odd} \\ (f_1, f_2), & \text{if } i \text{ is even} \end{cases}$$

If $(c_1, c_2) = (f_1, f_2)$; then B is a constant function. So, by above theorem G is a pseudo regular bipolar fuzzy graph. If $(c_1, c_2) \neq (f_1, f_2)$, then $d_G(v) = c_1 + c_2$, for all $v \in V$. So, $t_G(v) = (2c_1+2f_1, 2c_2+2f_2)$ and $d_{G^*}(v) = 2$. Hence G is a (c_1+f_1, c_2+f_2) - pseudo regular bipolar fuzzy graph.

Result 5.11. If v is a pendant vertex, then pseudo degree of v in bipolar fuzzy graph is the degree of the vertex in bipolar fuzzy graph which is adjacent with v. (or) If v is a pendant vertex, then $d_a(v)=d_G(u)$, where u is the vertex adjacent with v.

Theorem 5.12. If G (A, B) is a regular bipolar fuzzy graph on $G^*(V,E)$, an r-regular graph, then $d_a(v)=d_G(v)$, for all $v \in G$.

Proof. Let G is a (k_1, k_2) - regular bipolar fuzzy graph on $G^*(V,E)$, an r-regular graph. Then $d_G(v) = (k_1, k_2)$, for all $v \in G$ and $d_G^*(v) = r$, for all $v \in G$. So, $t_G(v)=\sum d_G(v_i)$, where each v_i (for i=1,2,. . . r) is adjacent with vertex $v \Longrightarrow t_G(v) =\sum d_G(v_i) = r\,(k_1, k_2)$. Also,

$$d_a(v) =\frac{t_G(v)}{d_{G^*}(v)} \Longrightarrow d_a(v) =\frac{t_G(v)}{r} \Longrightarrow d_a(v) =\frac{r\,(k1,k2)}{r} \Longrightarrow d_a(v) =(k1, k2) \Longrightarrow d_a(v) = d_G(v).$$

Theorem 5.13. Let G (A, B) be a bipolar fuzzy graph on $G^*(V, E)$, an r-regular graph. Then G is a pseudo regular bipolar fuzzy graph if G is a regular bipolar fuzzy graph.

Proof. Let G be a (k_1, k_2), -regular bipolar fuzzy graph on $G^*(V,E)$, an r-regular graph. $\Longrightarrow$ $d_a(v)= d_G(v)$, for all $v \in G$. $\Longrightarrow d_a(v)= (k_1, k_2)$, for all $v \in G \Longrightarrow$ all the vertices have same pseudo degree (k_1, k_2). Hence G is (k_1, k_2)-pseudo regular bipolar fuzzy graph.

6. Pseudo irregular bipolar fuzzy graphs and pseudo totally irregular bipolar fuzzy graphs

Definition 6.1. Let $G: (A, B)$ be a bipolar fuzzy graph on G*(V, E), where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a pseudo irregular bipolar fuzzy graph if there exits a vertex which is adjacent to the vertices with distinct pseudo degrees.

Definition 6.2. Let $G: (A, B)$ be a bipolar fuzzy graph on G*(V, E), where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a pseudo totally irregular bipolar fuzzy graph if there exits a vertex which is adjacent to the vertices with distinct total pseudo degrees.

Remark 6.3. A pseudo irregular bipolar fuzzy graph need not be a pseudo totally irregular bipolar fuzzy graph.

Remark 6.4. A pseudo totally irregular bipolar fuzzy graph need not be a pseudo irregular bipolar fuzzy graph.

Theorem 6.5. Let $G:(A,B)$ be a bipolar fuzzy graph on $G^*(V,E)$. If A is a constant function then the following conditions are equivalent.

(i) G is a pseudo irregular bipolar fuzzy graph.

(ii) G is a pseudo totally irregular bipolar fuzzy graph.

Proof. Assume that A is a constant function. Let $A(u) = (c1, c2)$, for all $u \in V$. Suppose G is a pseudo irregular bipolar fuzzy graph. Then atleast one vertex of G which is adjacent to the vertices with distinct pseudo degree. Let u_1 and u_2 be the adjacent vertices of u_3 with distinct pseudo degrees (k_1, k_2) and (f_1, f_2) respectively. Then $(k1, k2) \neq (f1, f2)$.

Suppose G is not a pseudo totally irregular bipolar fuzzy graph. Then every vertex of G which is adjacent to the vertices with same pseudo total degree $\Longrightarrow td_a(u_1) = td_a(u_2) \Longrightarrow d_a(u_1) + A(u_1) = d_a(u_2) + A(u_2) \Longrightarrow (k1, k2) + (c1, c2) = (f1, f2) + (c1, c2) \Longrightarrow (k_1 - f_1,\ k_2 - f_2) = 0 \Longrightarrow (k1, k2) = (f1, f2)$, which is a contradiction to $(k1, k2) \neq (f1, f2)$. Hence G is pseudo totally irregular bipolar fuzzy graph. Thus (i) $\Longrightarrow$ (ii) is proved.

Now, suppose G is a pseudo totally irregular bipolar fuzzy graph. Then at least one vertex of G which is adjacent to the vertices with distinct pseudo total degree. Let u_1 and u_2 be the adjacent vertices of u_3 with distinct pseudo total degrees $(g1, g2)$and $(h1, h2)$ respectively. Now, $td_a(u_1) \neq td_a(u_2) \Longrightarrow d_a(u_1) + A(u_1) \neq d_a(u_2) + A(u_2) \Longrightarrow (g1, g2) + (c1, c2) \neq (h1, h2) + (c1, c2) \Longrightarrow (g1, g2) \neq (h1, h2)$. Hence G is pseudo irregular bipolar fuzzy graph. Thus (ii) $\Longrightarrow$ (i) is proved. Hence (i) and (ii) are equivalent.

7. Highly pseudo irregular bipolar fuzzy graphs and highly pseudo totally irregular bipolar fuzzy graphs

Definition 7.1. Let $G:(A,B)$ be a bipolar fuzzy graph on G*(V, E), where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a highly pseudo irregular bipolar fuzzy graph if each vertex of G is adjacent to the vertices with distinct pseudo degrees .

Definition 7.2. Let $G:(A,B)$ be a bipolar fuzzy graph on G*(V, E), where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a highly pseudo totally irregular bipolar fuzzy graph if each vertex of G is adjacent to the vertices with distinct total pseudo degrees .

Remark 7.3. A highly pseudo irregular bipolar fuzzy graph need not be a highly pseudo totally Irregular fuzzy graph.
Remark 7.4. A highly pseudo totally irregular bipolar fuzzy need not be a highly pseudo irregular bipolar fuzzy graph.

Theorem 7.5. Let $G: (A, B)$ be a fuzzy graph on $G^*(V, E)$. If A is a constant function then the following conditions are equivalent.

(i) G is a highly pseudo irregular bipolar fuzzy graph.
(ii) G is a highly pseudo totally irregular bipolar fuzzy graph.

Proof. Proof is similar to the theorem 6.5.

Remark 7.6. The converse of the above theorem need not be true.

Theorem 7.7. Every highly pseudo irregular bipolar fuzzy graph is pseudo irregular bipolar fuzzy graph.
Proof. Let G be a pseudo highly irregular bipolar fuzzy graph. Then every vertex of G is adjacent to the vertices with distinct pseudo degrees $\Longrightarrow$ there is a vertex which is adjacent to the vertices with distinct pseudo degrees. Hence the graph G is pseudo irregular bipolar fuzzy graph.

Remark 7.8. The converse of the above theorem need not be true.

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