Annals of Pure and Applied Mathematics

Vol. 11, No. 1, 2016, 1-8

ISSN: 2279-087X (P), 2279-0888(online)

Published on 1 January 2016 www.researchmathsci.org

Annals of
Pure and Applied
Mathematics

On Neighbourly Edge Irregular Bipolar Fuzzy Graphs

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Received 25 November 2015; accepted 10 December 2015

Abstract. In this paper, neighbourly edge irregular bipolar fuzzy graphs and neighbourly edge totally irregular bipolar fuzzy graphs are introduced. A relation between neighbourly edge irregular bipolar fuzzy graph and neighbourly edge totally irregular bipolar fuzzy graph is studied. A necessary and sufficient condition under which they are equivalent is provided. Some properties of neighbourly edge irregular bipolar fuzzy graphs are studied and they are examined for neighbourly edge totally irregular bipolar fuzzy graphs.

Keywords: Edge degree in bipolar fuzzy graph, total edge degree in bipolar fuzzy graph, edge regular bipolar fuzzy graph, totally edge regular bipolar fuzzy graph.

AMS Mathematics Subject Classification (2010): 05C12, 03E72, 05C72

1. Introduction

Euler first introduced the concept of graph theory in 1736. Fuzzy set theory was first introduced by Zadeh in 1965[10]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971[17]. In 1975, Rosenfeld introduced the concept of fuzzy graphs [7]. Now, fuzzy graphs have many applications in branches of engineering and technology. Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [4]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [4]. MiniTom and Sunitha introduced sum distance in fuzzy graphs [3]. Sunitha and Mathew discussed about fuzzy graphs in fuzzy graph theory-A survey [15].

Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Bipolar fuzzy sets whose range of membership degree is [-1,1]. In bipolar fuzzy sets, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree within (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree within [-1,0) of an element indicates the element somewhat satisfies the implicit counter property. It is

noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible [2].

Akram and Dudek introduced regular and totally regular bipolar fuzzy graphs. Also, they introduced the notion of bipolar fuzzy line graphs and presents some of their properties [2]. Samanta and Pal introduced irregular bipolar fuzzy graphs [8]. Pal and Rashmanlou introduced irregular interval-valued fuzzy graphs [16]. Mathew, Sunitha and Anjali introduced some connectivity concepts in bipolar fuzzy graphs [14]. Radha and Kumaravel introduced the concept of an edge degree, total edge degree in bipolar fuzzy graphs and edge regular bipolar fuzzy graphs and discussed about the degree of an edge in some bipolar fuzzy graphs [6]. Maheswari and Sekar introduced neighbourly edge irregular fuzzy graphs and discussed its properties [9]. Maheswari and Sekar introduced strongly edge irregular fuzzy graphs and discussed its properties [10]. Maheswari and Sekar introduced edge irregular fuzzy graphs and discussed its properties[11]. Maheswari and Sekar introduced an m-Neighbourly Irregular fuzzy graphs [12]. These motivates us to introduce neighbourly edge irregular bipolar fuzzy graphs and neighbourly edge totally irregular bipolar fuzzy graphs and discussed some of its properties [9]. Throughout this paper, the vertices take the membership value $A = (m_1^+,$ m_1 and edges take the membership value $B = (m_2^+, m_2^-)$ where (m_1^+, m_2^+) in [0,1] and (m_1, m_2) in [-1,0].

2. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1. A Fuzzy graph denoted by $G:(\sigma,\mu)$ on the graph $G^*:(V,E)$: is a pair of functions (σ,μ) where $\sigma:V\to[0;1]$ is a fuzzy subset of a set V and $\mu:V\times V\to[0;1]$ is a symmetric fuzzy relation on σ such that for all u,v in V the relation $\mu(u,v)=\mu(uv)\leq \sigma(u)$ $\Lambda\sigma(v)$ is satisfied[8].

Definition 2.2. The degree of an edge uv in the underlying graph is defined as $d_G(uv) = d_G(u) + d_G(v)$ -2[1].

Definition 2.3. A bipolar fuzzy graph with an underlying set V is defined to be a pair (A, B), where $A = (m_1^+, m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+, m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+(x, y) \le \min\{(m_1^+(x), m_1^+(y)\} \text{ and } m_2^+(x, y) \ge \max\{(m_1^-(x), m_1^-(y)\} \text{ for all } (x,y) \text{ in E. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E [8].$

Definition 2.4. Let G: (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. The positive degree of a vertex u in G is defined as $d^+(u) = \sum m_2^+(u,v)$, for uv in E. The negative degree of a vertex u in G is defined as $d^-(u) = \sum m_2^+(u,v)$, for uv in E and $\sum m_2^+(u,v) = \sum m_2^-(u,v) = 0$ if uv not in E. The degree of a vertex u is defined as $d^-(u) = d^+(u) = d^-(u) = 0$.

Definition 2.5. Let G:(A,B) be a bipolar fuzzy graph on $G^*(V,E)$. The positive total degree of a vertex u in G is defined as $td^+(u) = \sum m_2^+ (u,v) + m_1^+ (u)$, for uv in E. The negative degree of a vertex u in E is defined as $td^-(u) = \sum m_2^- (u,v) + m_1^-(u)$, for uv in E[8].

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Definition 2.6. Let G: (A, B) be a bipolar fuzzy graph on G*(V, E). Then, G is said to be an irregular bipolar fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct degrees [8].

Definition 2.7. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be a highly irregular bipolar fuzzy graph if each vertex of a G is adjacent to the vertices with distinct degrees [8].

Definition 2.8. Let G: (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. The positive degree of an edge is defined as $d_G^+(uv) = d_G^+(u) + d_G^+(v) - 2m_2^+(uv)$. The negative degree of an edge is defined as $d_G^-(uv) = d_G^-(u) + d_G^-(v) - 2m_2^-(uv)$. The degree of an edge is defined as $d_G^-(uv) = (d_G^+(uv), d_G^-(uv))$. The minimum degree of an edge is $\Box_E^-(uv) = \min\{d_G^-(uv): uv \text{ in } E\}$. The maximum degree of an edge is $\Delta_E^-(uv) = \max\{d_G^-(uv): uv \text{ in } E\}$ [6].

Definition 2.9. Let G: (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. The total positive degree of an edge is defined as $td_G^+(uv) = td_G^+(u) + td_G^+(v) - m_2^+(uv)$. The total negative degree of an edge is defined as $td_G^-(uv) = td_G^-(u) + td_G^-(v) - m_2^-(uv)$. The degree of an edge is defined as $td_G^-(uv) = (td_G^+(uv), td_G^-(uv))$. The minimum degree of an edge is $\Box_E(uv) = \min\{td_G(uv): uv \text{ in } E\}$. The maximum degree of an edge is $\Delta_E(uv) = \max\{td_G(uv): uv \text{ in } E\}$ [6].

3. Neighbourly edge irregular bipolar fuzzy graphs and neighbourly edge totally irregular bipolar fuzzy graphs

Definition 3.1. Let G: (A, B) be a bipolar fuzzy graph on $G^*(V, E)$, where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a neighbourly edge irregular bipolar fuzzy graph if every pair of adjacent edges have distinct degrees.

Definition 3.2. Let G: (A, B) be a bipolar fuzzy graph on $G^*(V, E)$, where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V and $E \subseteq V \times V$ respectively. Then G is said to be a neighbourly edge totally irregular bipolar fuzzy graph if every pair of adjacent edges have distinct total degrees.

Example 3.3. Graph which is both neighbourly edge irregular bipolar fuzzy graph and neighbourly edge totally irregular bipolar fuzzy graph. Consider G^* : (V, E) where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.

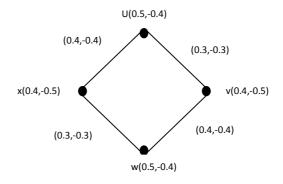


Figure 1:

From Figure 1,
$$d_G(u) = (0.7, -0.7) d_G(v) = (0.7, -0.7) d_G(w) = (0.7, -0.7) d_G(x) = (0.7, -0.7)$$

Degrees of the edges are calculated below.

```
\begin{array}{l} d_G^+(uv) = d_G^+(u) + d_G^+(v) - 2m_2^+(uv) = (0.7) + (0.7) - 2(0.3) = 0.8, \\ d_G^-(uv) = d_G^-(u) + d_G^-(v) - 2m_2^-(uv) = (-0.7) + (-0.7) - 2(-0.3) = -0.8 \\ d_G^-(uv) = (d_G^+(uv), d_G^-(uv)) = (0.8, -0.8), \quad d_G^-(vw) = (d_G^+(vw), d_G^-(vw)) = (0.6, -0.6) \\ d_G^-(wx) = (d_G^+(wx), d_G^-(wx)) = (0.8, -0.8), \quad d_G^-(xu) = (d_G^+(xu), d_G^-(xu)) = (0.6, -0.6). \\ \text{Here, } d_G^-(uv) = (0.6, -0.6), \quad d_G^-(vw) = (0.8, -0.8), \quad d_G^-(wx) = (0.6, -0.6), \quad d_G^-(xu) = (0.8, -0.8). \\ \text{It is noted that every pair of adjacent edges have distinct degrees. Hence $G$ is neighbourly edge irregular bipolar fuzzy graph. Total degrees of the edges are calculated below. \\ td_G^-(xu) = (d_G^+(uv), d_G^-(uv)) = (1.1, -1.1), \quad td_G^-(vw) = (d_G^+(vw), d_G^-(vw)) = (1, -1). \\ td_G^-(wx) = (d_G^+(wx), d_G^-(wx)) = (1.1, -1.1), \quad td_G^-(xu) = (d_G^+(xu), d_G^-(xu)) = (1, -1). \\ \text{It is observed that every pair of adjacent edges have distinct total degrees. So, $G$ is neighbourly edge totally irregular bipolar fuzzy graph. Hence $G$ is both neighbourly edge totally irregular bipolar fuzzy graph. Hence $G$ is both neighbourly edge irregular bipolar fuzzy graph. \\ \end{array}
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Definition 3.4. Let $G:(\sigma,\mu)$ be a bipolar fuzzy graph on $G^*:(V,E)$. If every pair of adjacent edges have the same edge degree and each edge e_i have edge degree (c_i,k_i) with $c_i=|k_i|$, then G is called an equally neighbourly edge irregular bipolar fuzzy graph. Otherwise it is unequally neighbourly edge irregular bipolar fuzzy graph.

Result 3.5. An equally neighbourly edge irregular bipolar fuzzy graph is neighbourly edge irregular bipolar fuzzy graph.

Result 3.6. A neighbourly edge irregular bipolar fuzzy graph need not be an equally neighbourly edge irregular bipolar fuzzy graph.

Theorem 3.7. Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. If G is neighbourly edge irregular bipolar fuzzy graph, then G is neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Assume that B is a constant function, let $B(uv) = (c_1,c_2)$ for all $uv \in E$, where c_1 and c_2 are constant. Let uv and xy be any pair of adjacent edges in E. Suppose that G is neighbourly edge irregular bipolar fuzzy graph. Then $d_G(uv) \neq d_G(xy)$, where uv and xy are pair of adjacent edges in E

```
\Rightarrow (d_G^+(uv), d_G^-(uv)) \neq (d_G^+(xy), d_G^-(xy)) \Rightarrow (d_G^+(uv), d_G^-(uv)) + (c_1, c_2) \neq (d_G^+(xy), d_G^-(xy)) + (c_1, c_2) \Rightarrow d_G(uv) + B(uv) \neq d_G(xy) + B(uv) \Rightarrow t d_G(uv) \neq t d_G(xy), where uv and xy are any pair of adjacent edges in E.
```

Hence G is neighbourly edge totally irregular bipolar fuzzy graph.

Theorem 3.8. Let $G : (\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. If G is neighbourly edge totally irregular bipolar fuzzy graph, then G is neighbourly edge irregular bipolar fuzzy graph.

Proof. Proof is similar to the above Theorem 3.7.

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Remark 3.9. Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. If G is both neighbourly edge irregular bipolar fuzzy graph and neighbourly edge totally irregular bipolar fuzzy graph. Then B need not be a constant function.

Example 3.10. The bipolar fuzzy graph given in example 3.3 is both neighbourly edge irregular bipolar fuzzy graph and neighbourly edge totally irregular bipolar fuzzy graph. But B is not constant.

Theorem 3.11. Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. If G is neighbourly edge irregular bipolar fuzzy graph, then G is an irregular bipolar fuzzy graph.

Proof. Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$. Assume that B is a constant function, let $B(uv)=(c_1,c_2)$ for all $uv\in E$, where c_1 and c_2 are constant. Let us suppose that G is neighbourly edge irregular bipolar fuzzy graph. Then every pair of adjacent edges have distinct degrees. Let uv and vw are adjacent edges in G with distinct degrees.

```
Then (d_G^+(uv), d_G^-(uv)) \neq (d_G^+(vw), d_G^-(vw))

\Rightarrow ((d_G^+(u) + d_G^+(v) - 2c_1), (d_G^-(u) + d_G^-(v) - 2c_2)) \neq ((d_G^+(v) + d_G^+(w) - 2c_1), (d_G^-(v) + d_G^-(w) - 2c_2)) \Rightarrow (d_G^+(u) + d_G^+(v) - 2c_1) \neq (d_G^+(v) + d_G^+(v) - 2c_1)

(\text{or})(d_G^-(u) + d_G^-(v) - 2c_2) \neq (d_G^-(v) + d_G^-(w) - 2c_2)

\Rightarrow (d_G^+(u) + d_G^+(v)) \neq (d_G^+(v) + d_G^+(w)) (\text{or})(d_G^-(u) + d_G^-(v)) \neq (d_G^-(v) + d_G^-(w))

\Rightarrow d_G^+(u) \neq d_G^+(w) (\text{or})d_G^-(u) \neq d_G^-(w)

\Rightarrow (d_G^+(u), d_G^-(u)) \neq (d_G^+(w), d_G^-(w))

\Rightarrow d_G^-(u) \neq d_G^-(w)
```

⇒there exists a vertex v which is adjacent to a vertices u and w have distinct degrees. Hence G is an irregular bipolar fuzzy graph.

Theorem 3.12. Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. If G is neighbourly edge totally irregular bipolar fuzzy graph, then G is an irregular bipolar fuzzy graph.

Proof. Proof is similar to the above theorem 3.11.

Remark 3.13. Converse of the above theorems 3.11 and 3.12 need not be true.

Theorem 3.14. Let $G: (\sigma, \mu)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. Then, G is neighbourly edge irregular bipolar fuzzy graph if and only if G is highly irregular bipolar fuzzy graph.

Proof. Let G: (σ, μ) be a connected bipolar fuzzy graph on $G^*(V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2)$ for all $uv \in E$, where c_1 and c_2 are constants. Let v be any vertex adjacent with u, w and x. Then uv, vw and vx are adjacent edges in G. Let us suppose that G is neighbourly edge irregular bipolar fuzzy graph.

- ⇒ every pair of adjacent edges in G have distinct degrees.
- \implies $d_G(uv) \neq d_G(vw) \neq d_G(vx)$
- \Rightarrow $(d_G^+(uv), d_G^-(uv)) \neq (d_G^+(vw), d_G^-(vw)) \neq (d_G^+(vx), d_G^-(vx)).$

```
\begin{split} & \text{Consider } (d^+_G(uv), d^-_G(uv)) \neq (d^+_G(vw), d^-_G(vw)) \\ & \Longrightarrow d^+_G(uv) \neq d^+_G(vw) \text{ or } d^-_G(uv) \neq d^-_G(vw) \\ & \Longrightarrow d^+_G(u) + d^+_G(v) - 2m^+_2(uv) \neq d^+_G(v) + d^+_G(w) - 2m^+_2(vw) \text{ (or) } d^-_G(u) + d^-_G(v) - 2m^-_2(uv) \neq d^-_G(v) + d^-_G(v) - 2m^-_2(vw) \\ & \Longrightarrow d^+_G(u) + d^+_G(v) - 2c_1 \neq d^+_G(v) + d^+_G(w) - 2c_1 \text{ (or) } d^-_G(u) + d^-_G(v) - 2c_2 \neq d^-_G(v) + d^-_G(w) - 2c_2 \\ & \Longrightarrow d^+_G(u) + d^+_G(v) \neq d^+_G(v) + d^+_G(w) \text{ (or) } d^-_G(u) + d^-_G(v) \neq d^-_G(v) + d^-_G(w) \\ & \Longrightarrow d^+_G(u) \neq d^+_G(w) \text{ (or) } d^-_G(u) \neq d^-_G(w) \\ & \Longrightarrow d^+_G(u) \neq d^-_G(w) \neq d^-_G(w) \\ & \Longrightarrow d^+_G(u) \neq d^-_G(w) \\ & \Longrightarrow d^+_G(u) \neq d^-_G(w) \\ & \Longrightarrow d^-_G(u) \neq d^-_G(u) \\ & \Longrightarrow d^-_G(u) \Rightarrow d^-_G(u) \\ & \Longrightarrow d^-_G(u) \Rightarrow d^-_G(u) \Rightarrow d^-_G(u) \\ & \Longrightarrow d^-_G(u) \Rightarrow d^-_G(u) \\ & \Longrightarrow d^-_G(u) \Rightarrow d^-_G(u) \Rightarrow d^-_G(u) \\ & \Longrightarrow d^-_G(u) \Rightarrow d^-_G(u) \Rightarrow d^-_G(u) \\ & \Longrightarrow d^-_G(u) \Rightarrow d^-_G(u) \Rightarrow d^-_G(u)
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the vertex v is adjacent to the vertices u, w and x with distinct degrees. Hence G is highly irregular bipolar fuzzy graph.

Conversely, let uv and vw are any two adjacent edges in G. Let us suppose that G is highly irregular bipolar fuzzy graph⇒every vertex adjacent to the vertices in G having distinct degrees.

```
\begin{split} &\Rightarrow \mathrm{d_G}(u) \neq \mathrm{d_G}(w) \Rightarrow (d_G^+(u), d_G^+(u)) \neq (d_G^-(w), d_G^-(w)) \\ &\Rightarrow (d_G^+(u) \neq d_G^+(w)) \text{ (or) } (d_G^-(u) \neq d_G^-(w)) \\ &\Rightarrow (d_G^+(u) + d_G^+(v)) \neq (d_G^+(v) + d_G^+(w)) \text{ (or) } (d_G^-(u) + d_G^-(v)) \neq (d_G^-(v) + d_G^-(w)) \\ &\Rightarrow (d_G^+(u) + d_G^+(v) - 2c_1) \neq (d_G^+(v) + d_G^+(w) - 2c_1) \text{ (or) } (d_G^-(u) + d_G^-(v) - 2c_2) \neq \\ (d_G^-(v) + d_G^-(w) - 2c_2) \Rightarrow (d_G^+(u) + d_G^+(v) - 2m_2^+(uv)) \neq (d_G^+(v) + d_G^+(w) - 2m_2^+(vw)) \text{ (or) } (d_G^-(u) + d_G^-(v) - 2m_2^-(uv)) \neq (d_G^-(v) + d_G^-(w) - 2m_2^-(vw)) \\ &\Rightarrow (d_G^+(uv), d_G^-(uv)) \neq (d_G^-(vw), d_G^-(vw)) \\ \Rightarrow d_G^-(uv) \neq d_G^-(vw) \end{split}
```

⇒every pair of adjacent edges have distinct degrees. Hence G is neighbourly edge irregular bipolar fuzzy graph.

Theorem 3.15. Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. Then G is neighbourly edge totally irregular bipolar fuzzy graph if and only if G is highly irregular bipolar fuzzy graph.

Proof. Proof is similar to the above theorem 3.14.

 \implies $d_G(u) \neq d_G(w) \neq d_G(x)$

Definition 3.16. Let G: (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. Then, G is said to be a strongly irregular bipolar fuzzy graph if every pair of vertices in G have distinct degrees.

Theorem 3.17 Let $G:(\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V,E)$ and B is a constant function. If G is strongly irregular bipolar fuzzy graph, then G is neighbourly edge irregular bipolar fuzzy graph.

Proof. Let $G: (\sigma, \mu)$ be a connected bipolar fuzzy graph on $G^*(V, E)$. Assume that B is a constant function, let $B(uv) = (c_1, c_2)$ for all $uv \in E$, where c_1 and c_2 are constants. Let uv and vw are any two adjacent edges in G. Let uv suppose that G is strongly irregular bipolar fuzzy graph

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- ⇒ every pair of vertices in G having distinct degrees.
- $\Longrightarrow d_G(u) \neq d_G(v) \neq d_G(w)$
- \implies $d_G(u)+d_G(v)\neq d_G(v)+d_G(w)$.
- \Rightarrow $(d_{G}^{+}(u), d_{G}^{-}(u)) + (d_{G}^{+}(v), d_{G}^{-}(v)) \neq (d_{G}^{+}(v), d_{G}^{-}(v)) + (d_{G}^{+}(w), d_{G}^{-}(w)).$
- \Rightarrow $(d_{G}^{+}(u)) + d_{G}^{+}(v), d_{G}^{-}(u)) + d_{G}^{-}(v)) \neq (d_{G}^{+}(v)) + d_{G}^{+}(w), d_{G}^{-}(v) + d_{G}^{-}(w)).$
- $\Rightarrow d^{+}_{G}(u) + d^{+}_{G}(v) \neq d^{+}_{G}(v) + d^{+}_{G}(w) \text{ (or) } d^{-}_{G}(u) + d^{-}_{G}(v) \neq d^{-}_{G}(v) + d^{-}_{G}(w)$
- \Rightarrow $d^+_G(u) + d^+_G(v) 2c_1 \neq d^+_G(v) + d^+_G(w) 2c_1$ (or) $d^-_G(u) + d^-_G(v) 2c_2 \neq d^-_G(v) + d^-_G(w) 2c_2$
- $\Longrightarrow d^{+}_{G}(u) + d^{+}_{G}(v) 2m_{2}^{+}(uv) \neq d^{+}_{G}(v) + d^{+}_{G}(w) 2m_{2}^{+}(vw) \text{ (or) } d^{-}_{G}(u) + d^{-}_{G}(v) 2m_{2}^{-}(uv) \neq d^{-}_{G}(v) + d^{-}_{G}(w) 2m_{2}^{-}(vw)$
- \Longrightarrow (d⁺_G(uv), d⁻_G(uv)) \neq (d⁺_G(vw), d⁻_G(vw))
- \implies d_G(uv) \neq d_G(vw)
- every pair of adjacent edges have distinct degrees. Hence G is neighbourly edge irregular bipolar fuzzy graph.

Theorem 3.18. Let $G : (\sigma,\mu)$ be a connected bipolar fuzzy graph on $G^*(V, E)$ and B is a constant function. If G is strongly irregular bipolar fuzzy graph, then G is neighbourly edge totally irregular bipolar fuzzy graph.

Proof. Proof is similar to the above Theorem 3.17.

Remark 3.19. Converse of the above theorems need not be true.

Acknowledgement. This work is supported by F.No:4-4/2014-15, MRP-5648/15 of the University Grant Commission, SERO, Hyderabad.

REFERENCES

- 1. S.Arumugam and S.Velammal, Edge domination in graphs, *Taiwanese Journal of Mathematics*, 2(2) (1998) 173-179.
- 2. M.Akram and Wieslaw A.Dudek, Regular bipolar fuzzy graphs, *Neural Comput. & Applic.*, 21 (Suppl 1) (2012) S197- S205.
- 3. Mini Tom and M.S.Sunitha, Sum distance in fuzzy graphs, *Annals of Pure and Applied Mathematics*, 7(2) (2014) 73-89.
- 4. A.Nagoor Gani and S.R.Latha, On irregular fuzzy graphs, *Applied Mathematical Sciences*, 6 (2012) 517-523.
- 5. A.Nagoor Gani and R.Radha, The degree of a vertex in some fuzzy graphs, *Intern. Journal of Algorithms, Computing and Mathematics*, 2(3) (2009) 107-116.
- 6. S.P.Nandhini and E.Nandhini, Strongly irregular fuzzy graphs, *International Journal of Mathematical Archive*, 5 (5) (2014) 110-114.
- 7. K.Radha and N.Kumaravel, On edge regular bipolar fuzzy graphs, *Annals of Pure and Applied Mathematics*, 10(2) (2015) 129-139.
- 8. A.Rosenfeld, Fuzzy graphs, In: L.A.Zadeh, K.S.Fu, M.Shimura, EDs., Fuzzy sets and Their Applications, *Academic Press* (1975),77-95.
- 9. N.R.Santhi Maheswari and C.Sekar, On neighbourly edge irregular fuzzy graphs, *International Journal Mathematical Archive*, 6(10) (2015) 224-231.

- 10. N.R.Santhi Maheswari and C.Sekar, On strongly edge irregular fuzzy graphs, Accepted in *Kragujevac Journal of Mathematics*.
- 11. N.R.Santhi Maheswari and C.Sekar, On edge irregular fuzzy graphs, Accepted in *International Journal Mathematical and Soft Computing*
- 12. N.R.Santhi Maheswari and C.Sekar, *On m-neighbourly irregular fuzzy graphs, International Journal of Mathematics and Soft computing*, 5(2) (2015) 145-153.
- 13. S.Samantha and M.Pal, Irregular bipolar fuzzy graphs, *International Journal of Application of Fuzzy Sets*, 2 (2012) 91-102.
- 14. M.S.Sunitha and Sunil Mathew, Fuzzy graph theory: a survey, *Annals of Pure and Applied Mathematics*, 4(1) (2013) 92-110.
- 15. S.Mathew, M.S.Sunitha and Anjali, Some connectivity concepts in bipolar fuzzy graphs, *Annals of Pure and Applied Mathematics*, 7(2) (2014) 98-108.
- 16. M.Pal and H.Rashmanlou, Irregular interval-valued fuzzy graphs, *Annals of Pure and Applied Mathematics*, 3(1) (2013) 56-66.
- 17. L.A.Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.