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Multichannel System with Chances and Cost Considerations in The Queue Systems

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Abstract. Queue systems where more than one service points are involved are referred to as the multichannel system. Aspects of cost and chance considerations play vital roles whenever the concept of queue theory is under serious discussion. In this paper, chances associated with items in queue theory are analyzed. Also, since services to customers in queue systems are expected to be paid for, our main interests go to the total cost, stepping down the costs without necessarily reducing the services standard and acquiring reasonably, the interests and overall profits.

Keywords: probability, traffic intensity, cost, multi-channel systems, queue, companies, time.

1. Introduction

Chances and costs are associated with items in queue. The probabilities or chances of entities such as queuing on arrival, number of items in the system including the range of such as well as the chances of getting served immediately on arrival in which case, queuing at such a material time unnecessary. the other side includes the cost implications of rendering such services at every point in time. The total has to be considered and this, not at the expense or detriment of the required or the expected reasonable, commensurate and equitable profits acquired by the establishments involved.

2. Cost consideration

Just like all employees, there is utmost need to pay people who are rendering services to customers in the queue system for the services they render to the customers (see [13]). The following are some area of interest in this particular aspect. (i) the total cost of such services (ii) ways and means of reducing such costs without lowering the standard of the services and (iii) making profits in the ventures where queuing systems are in practice. All these are very imperative whenever any well-developed queue systems are in practice.

Take the following case as an example; Book suppliers draw books from bookstore in a large bookstore. The manager feels worried about the time spent by the cashier in issuing out receipts to the customers and wants to find out if by employing one assistant cashier, his worries could be eliminated. In his investigations, he discovers that; there exists simple queue, each book supplier is being paid \$5 per hr., the cashier receives \$7 per hr., also able to answer 24 customers every hr. and that if an assistant cashier is employed, would earn \$6 per hr. and the no of customers attended to would be increased to 30 pre hr. Also, an average number of 20 customers visit the bookstore per hr. from this phenomenon, we determine whether it's worthwhile to employ the service of an assistant cashier or not.

First, if we consider the existing system before deciding whether the service of an assistant cashier is needed at all. Here, the average arrival rate is given by : $\lambda = 20h^{-1}$ and the average service rate is given by: $\mu = 24h^{-1}$.

∴ The Trafic intensity = $\rho = {}^{\lambda}/_{\mu} \quad \frac{20}{24} = \frac{5}{6}$.

We have that the number in the system = $\frac{\rho}{1-\rho} = 5$.

∴ the cost of supplying book per hour = \$5 x 5 = \$25

Note also that for the new system, after the service of an assistant cashier has been employed , we have as follows:

Traffic intensity = ρ = 20/30 = 2/3 .

∴ Number in system = = $\frac{\rho}{1-\rho} = 2$.

We have that (i) Book supplying costs \$5 x 2 = \$$10h^{-1}$.

Assistant cashier's cost = \$$6h^{-1}$.

Therefore, Total cost = \$(10 + 6) = \$$16h^{-1}$, and the net savings per hour = \$(25-16) = \$$9h^{-1}$. From this analogy, it's clear that the employment of an assistant cashier would be worthwhile.

3. Chance consideration

Concerning chances, there are four of such which are considerable.(See [6,7, 8 and 14])

(i) The probability of queuing on arrival. This is equal to the traffic intensity ρ,

(ii) The probability of not queuing on arrival. This is equal to 1- ρ.

(iii) The probability that there are n items in the system. This is given by pr($x = n$) = (1- ρ)ρ^n

(iv) The probability that there are more than n items in the system. This is defined as pr($x > n$) = ρ^n

Take for example, in a simple queue, which has the traffic intensity of 2\3, it follows that the probability of queue on arrival is given by ρ =2\3.

The probability of not que on arrival is 1- ρ = 1- 2\3 = 1\3.

Also, the probability that there are n = 2 items in the system is given by

pr(n=2) = (1- ρ)ρ^n = (1- 2\3)$\left(\frac{2}{3}\right)^2$ = (1\3)(4\9) = 4\27, and the probability that there are more than two items in the system is given by

pr(n>2) = ρ^2 = $(2\backslash 3)^2$ = 4\9.

Again, consider a telephone operator who serves his customers @ the rate of 100 in 5hr. Customers arrive @ the telephone booth @ the rate of 60 in 4hr. It follows that the traffic intensity is given by 15\20 = 3\4.

The probability that there is no customer in the system is given by pr(n=0) = (1- $\rho)\rho^n$ = (1-3\4)$(3\backslash4)^0$ = 1- 3\4 = 1\4, and the probability that there are exactly three customers in the system is given by

pr(n=3) = (1- 3\4)$(3\backslash4)^3$ = (1\4)(27\64) = 27\256.

And finally, the probability that there are more than three customers in the system is given by

pr(n > 3) = ρ^n = $(3\backslash4)^3$ = 27\64. (also see [9-11])

4. Time consideration

Time is another important factor in queue system. This is in two parts:

(i) The average time in queue (before service is rendered)

This is defined as : $\bar{w} = \frac{\rho}{1-\rho} \times \frac{1}{\mu}\left\{or\ \frac{\bar{s}}{\mu}\right\}$, where $\bar{s} = \frac{\rho}{1-\rho}$

(ii) The average time in system.

This can be defined as : $\bar{t} = \frac{1}{1-\rho} \times \frac{1}{\mu}\left\{or\ \frac{\bar{\bar{n}}}{\mu}\right\}$ where $\bar{\bar{n}} = \frac{1}{1-\rho}$ where ρ is the traffic intensity.

Take the following case for instance (from [13], also see [5]):

An applicant got to a taxi park @ 8.00 am. The park which operates on simple queue system has an arrival rate of $4min^{-1}$ and service rate of $5min^{-1}$. The applicant boarded a taxi speeding @ $30kmh^{-1}$ to a company, 10km away to attend an interview. The interview which is also based on simple queue system has a service rate of $10h^{-1}$and an arrival rate of $8h^{-1}$. He joined the queue and was interviewed when it came to his turn. From calculations, observe that the total time spent from arrival @ the taxi park to the time his interview was conducted can be estimated to be 1+20+30 = 51mins. Now, since he got to the taxi park @ 8.00am, his interview was conducted around 8.51 am.

In the process of getting reasonable solutions, assumption of a more appropriate distribution such as the symmetrical normal distribution (see [1]) may be necessary in cases involving the analysis of complex and multidimensional situations. (also see [12 and 17]].

5. The multi-channel systems

A queue system in which there are more than one service point is referred to as the multi-channel system. The channels may be in series or parallel or it may be of the mixture of both.

Definition: Suppose that c is the number of channels. Then, we have as follows:

(a) The Traffic intensity is given by: $\rho = \frac{\lambda}{c\mu}$

(b) Given that n is the number of customers in the system. Then,

pr(n = 0) = $\rho_o = \frac{c!(1-\rho)}{(\rho c)^c + c!(1-\rho)\left(\sum_{n=0}^{c-1}\frac{1}{n!}(\rho c)^n\right)}$

(c) The probability of not queuing on arrival is given by: $\rho_n = \frac{(\rho c)^c}{c!(1-\rho)}\rho_o$

(d) The average number of customers in the system is given by: $\bar{s} = \frac{\rho(\rho c)^c}{c!(1-\rho)^2}\rho_o + \rho c$

(e) The average number of customers in the system is : $\bar{n} = \frac{\rho(\rho c)^c}{c!(1-\rho)^2}\rho_o$

(f) The average time spent in the system is : $\bar{t} = \frac{(\rho c)^c \rho_o}{c!(1-\rho)^2 c\mu} + \frac{1}{\mu}$

(g) Average time spent in the queue is : $\bar{w} = \frac{(\rho c)^c \rho_o}{c!(1-\rho)^2 c\mu}$

Consider fueling station which employs four petrol attendant who on the average each serves five customers an hour. The average number of customers requiring service is fifteen per hour.(see [15 and 16])

The following can be determined accordingly using the formulae. (i) The tragic intensity ρ, (ii) The probability that there is no customer in the system (iii) the probability of not requiring queue on arrival(iv) the average no of customer in the system (v) the average no of customer in the queue (vi) the average time spent in system (vii) the average time spent in the queue.

(i) $\rho = \frac{\lambda}{c\mu} = \frac{15}{4x5} = \frac{3}{4} = 0.75$

(ii) $\rho_o = \frac{c!(1-\rho)}{(\rho c)^c + c!(1-\rho)\left(\sum_{n=0}^{c-1}\frac{1}{n!}(\rho c)^n\right)}$

$= \frac{4!\left(\frac{1}{4}\right)}{\left(4X\frac{3}{4}\right)^4 + 24\left(\frac{1}{4}\right)\left(\frac{1}{0!}(3)^0 + \frac{1}{1!}(3)^1 + \frac{1}{2!}(3)^2 + \frac{1}{3!}(3)^3\right)} = \frac{6}{(3)^4 + 6\left(1 + 3 + \frac{9}{2} + \frac{9}{2}\right)}$

$= \frac{6}{81 + 6(13)} = \frac{2}{17 + 26} = \frac{2}{53} \cong 0.04$

(iii) $\rho_n = \frac{(\rho c)^c}{c!(1-\rho)}\rho_o = \frac{(3)^4 X\frac{2}{53}}{24\left(\frac{1}{4}\right)} = \frac{27}{53} \cong 0.51$

(iv) $\bar{s} = \frac{\rho(\rho c)^c}{c!(1-\rho)^2}\rho_o + \rho c = \frac{\frac{3}{4}(3)^4 \cdot \frac{2}{53}}{24\left(\frac{1}{16}\right)^2} + 3 = \frac{81}{53} + 3 = 4.53 \cong 5$

(v) $\bar{n} = \bar{s} - \rho c = 4.53 - 3 = 1.53 \ (\cong 2)$

(vi) $\bar{t} = \frac{(\rho c)^c \rho_o}{c!(1-\rho)^2 c\mu} + \frac{1}{\mu} = \frac{(3)^4 \cdot \frac{2}{53}\rho_o}{24\left(\frac{1}{4}\right)^2 4X5} + \frac{1}{5} \cong$ 18.1 min

(vii) $\bar{w} = \frac{(\rho c)^c \rho_o}{c!(1-\rho)^2 c\mu} = \frac{(3)^4 \cdot \frac{2}{53}\rho_o}{24 \cdot \frac{1}{16} \cdot 5x4} = \frac{27}{53x5} =$ 6.11min

6. Conclusion

The manufacturing process involves various operational steps in converting raw materials into finished products. In order to make the process efficient and cost effective, analytical tools such as queueing theory have to be used extensively with the advancement of technology. The variety of problems in manufacturing where queueing theory has been applied and identified in the comprehensive surveys Of these flow line and trans flow line systems, job shops can be easily identified as obvious application areas for queueing theory.

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