Progress in Nonlinear Dynamics and Chaos
Vol. 8, No. 1 & 2, 2020, 25-33
ISSN: 2321 – 9238 *(online)*
Published on 16 December 2020
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/pindac.v8n1a08503

Progress in Nonlinear Dynamics and Chaos

Deflection of Light Rays in the Gravitational Field

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Received 12 September 2020; accepted 15 December 2020

Abstract. Discussion is about Deflection of Light Rays in the Gravitational Field. The first calculation of the deflection of light by mass was published by the German astronomer Johan George von Soldner in 1801. In this paper a comparative study will be done for gravitational deflection of light ray calculated by different authors using different methods. Einstein calculated that the deflection predicted general theory of relativity would be twice the Newtonian value. Soldner showed that rays from a distant star skimming the Sun's surface would be deflected through an angle of about 0.9 seconds of arc, or one quarter of thousands of degree. This angle corresponding to the apparent diameter of a compact disc (CD) viewed from a distance of about 30 kilometers (nearly 20 miles). In actual sense, Physicist do not know why light is affected by gravity if photons are mass less. Explanation given here is depended on the observation about gravitational deflection of light after reviewing the theories of relativity.

Keyword**:** Gravitation, deflection, space-time, mass, bending

1. Introduction

The first observation of light deflection was performed by noting the change in position of stars as they passed near the Sun on the celestial sphere. The observations were performed in 1919 by Arthur Eddington, Frank Watson Dyson, and their collaborators during the total solar eclipse on May 29. The exact solution of Einstein field equation for a static, spherical, uncharged body was found by Schwarzschild in 1916. Within a year of Schwarzschild solution, Reissner and Nordström independently, obtained the solution for Einstein field equation for a static, charged, spherically symmetric body, known as Reissner-Nordström solution [1].

Different authors have calculated the deflection angle under weak and strong field approximations using null geodesic method considering Schwarzschild space-time. Deflection of light rays in the gravitational field has been studied since long and well known Weinberg [2], Islam [3], Islam [4], Narlikar [5], Godel [6], Raychaudhuri [7], Wald [8], Roy and Sen [9]. Very recently, in 2019 Roy and Sen [9] also used the material medium to study of gravitational deflection of light ray.

2. Newton's law of gravitation

In this section we will show that Newton's law of gravitation may be deduced from Einstein law when the gravitation field is weak and the matter distribution is static.
The Einstein's law of gravitation is

$$R_{ij} - \frac{1}{2} g_{ij} R = -kT_{ij} \quad \ldots\ldots\ldots\ldots (1)$$

where $R = kT$ and k is the constant of Einstein's law of gravitation.
We first consider the Riemann- Christoffel curvature tensor,

$$R^i_{jkl} = \Gamma^i_{rk}\Gamma^r_{jl} - \Gamma^i_{lr}\Gamma^r_{jk} + \frac{\partial}{\partial x^k}\left(\Gamma^i_{jl}\right) - \frac{\partial}{\partial x^l}\left(\Gamma^i_{jk}\right) \quad \ldots\ldots\ldots\ldots (2)$$

From 1st kind of Christoffel symbol, we have

$$\Gamma^k_{ij} \approx \delta^{kr}[ij, r]$$
$$= [ij, r]$$
$$= \frac{1}{2}\left(\frac{\partial h_{jk}}{\partial x^i} + \frac{\partial h_{ik}}{\partial x^j} - \frac{\partial h_{ij}}{\partial x^k}\right) \quad \ldots\ldots\ldots\ldots (3)$$

If the product of h_{ij} are to be neglected the equation (3) can be written as

$$R^i_{jkl} = \frac{\partial}{\partial x^k}\left(\Gamma^i_{jl}\right) - \frac{\partial}{\partial x^l}\left(\Gamma^i_{jk}\right)$$

For $i = j$ the Ricci tensor is,

$$R_{jk} = \frac{\partial}{\partial x^k}\left(\Gamma^i_{ji}\right) - \frac{\partial}{\partial x^i}\left(\Gamma^i_{jk}\right)$$
$$= \frac{1}{2}\frac{\partial}{\partial x^k}\left(\frac{\partial h_{ij}}{\partial x^i} + \frac{\partial h_{ii}}{\partial x^j} - \frac{\partial h_{ji}}{\partial x^i}\right) - \frac{1}{2}\frac{\partial}{\partial x^i}\left(\frac{\partial h_{ik}}{\partial x^j} + \frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)$$
$$= \frac{1}{2}\left(\frac{\partial^2 h_{ij}}{\partial x^j \partial x^k} + \frac{\partial^2 h_{jk}}{\partial x^i \partial x^i} - \frac{\partial^2 h_{ji}}{\partial x^i \partial x^k} - \frac{\partial^2 h_{ik}}{\partial x^i \partial x^j}\right)$$

Putting $j = k = 4$

$$R_{44} = \frac{1}{2}\left(\frac{\partial^2 h_{ii}}{\partial x^4 \partial x^4} + \frac{\partial^2 h_{44}}{\partial x^i \partial x^i} - 2\frac{\partial^2 h_{i4}}{\partial x^i \partial x^4}\right) \quad \ldots\ldots\ldots\ldots (4)$$

For static distribution h_{ij} will be independent of t and equation (4) reduces to

$$R_{44} = \frac{1}{2}\frac{\partial^2 h_{44}}{\partial x^i \partial x^i}$$

Or,

$$R_{44} = \frac{1}{2}\nabla^2 h_{44} \quad \ldots\ldots\ldots\ldots (5)$$

where $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$. If V is the Newtonian potential we have

$$V = \frac{1}{2}C^2 h_{44}$$
$$h_{44} = \frac{2V}{C^2}$$

Substituting this in (5) we get,

$$R_{44} = \left(\frac{1}{C^2}\right)\nabla^2 V \quad \ldots\ldots\ldots\ldots (6)$$

Considering that no electromagnetic field is present, the energy momentum tensor for the mass distribution will be given by

$$T_{ij} = \theta_{ij} = \mu_{00} V_i V_j$$

where θ_{ij} is the kinetic energy-momentum tensor and $\mu_{00} = \left(1 - \frac{v^2}{c^2}\right)\mu$ is the proper density of proper mass, μ being ordinary mass density. For a static distribution, 4-velocity at every point becomes $\underline{V} = \left(\underline{0}, ic\right)$ and hence all the components of T_{ij}, with the exception of T_{44} are zero. So,

$$T_{44} = \mu_{00} V_4 V_4 = -\mu_{00} C^2 = -\mu C^2$$

Also,

$T = T_{11} + T_{22} + T_{33} + T_{44} = T_{ii}$

That is,

$T = T_{44} = -\mu C^2$

Then,

$R_{44} = k\left(\frac{1}{2} g_{44} T - T_{44}\right)$

Or, $\left(\frac{1}{C^2}\right) \nabla^2 V = k\left\{\frac{1}{2}\left(1 + \frac{2V}{c^2}\right) T - T_{44}\right\}$

Or, $\left(\frac{1}{C^2}\right) \nabla^2 V \approx k\left[\frac{1}{2} T + \mu c^2\right]$

Or, $\left(\frac{1}{C^2}\right) \nabla^2 V = \frac{1}{2} k\mu c^2$

Or,

$\nabla^2 V = \frac{1}{2} k\mu c^4$ .... (7)

According to the Newtonian theory, if μ is the density of matter, the gravitational field can be derived from a potential function V satisfying the equation,

$\nabla^2 V = 4\pi\gamma\mu$ .... (8)

Comparing equation (7) & (8) we get Einstein's gravitational constant in terms of Newton's gravitational constant as

$k = \frac{8\pi\gamma}{c^2}$

3. Gravitational deflection of light:

Let the events at the points having rectangular Cartesian co-ordinates (x, y, z) and $(x + dx, y + dy, z + dz)$ Occur at times $t, t + dt$ respectively. Then by definition of proper time we have,

$d\tau^2 = dt^2 - \left(\frac{1}{c^2}\right)(dx^2 + dy^2 + dz^2)$

The interval ds between the events will be defined by

$ds^2 = c^2 dt^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$

If the events (x, y, z, t) and $(x + dx, y + dy, z + dz, t + dt)$ connected by a line ray then,

$dx^2 + dy^2 + dz^2 - c^2 dt^2 = 0$

$\text{Or,} \quad ds^2 = 0$

$\text{Or,} \; ds = 0$

Equation of a geodesic in inertial frame is $\frac{d^2x^i}{ds^2} = 0$. Since the values of Γ^i_{jk} are all zero. On a null geodesic $ds = 0$ and so the equation $\frac{d^2x^i}{ds^2} = 0$ is not appropriate. The equation of a null geodesic is defined by $\frac{d^2x^i}{d\lambda^2} = 0$, where λ is a parameter on null geodesic, λ is an invariant satisfying $g_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} = 0$.

Therefore for a null geodesic we have,

$\frac{d^2x^1}{d\lambda^2} = \frac{d^2x^2}{d\lambda^2} = \frac{d^2x^3}{d\lambda^2} = \frac{d^2x^4}{d\lambda^2} = 0$, and also $g_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} = 0$

That is

$\frac{d^2x}{d\lambda^2} = \frac{d^2y}{d\lambda^2} = \frac{d^2z}{d\lambda^2} = \frac{d^2t}{d\lambda^2} = 0$

Hence,

$\frac{d^2x}{d\lambda^2} = \frac{d^2t}{d\lambda^2}, \frac{d^2y}{d\lambda^2} = \frac{d^2t}{d\lambda^2}, \frac{d^2z}{d\lambda^2} = \frac{d^2t}{d\lambda^2}$ ……..…………… (9)

Equation (9) implies that along a null geodesic x, y, z are linearly independent. This is also true for the co-ordinates of a light signal being propagated in an inertial frame. Hence, the world lines of light signals are null geodesic in inertial frames. It follows that the world lines of light signals are null geodesic in the presence of gravitational fields. By applying the principle we shall calculate the path of a light ray in the gravitational field of a spherical body for which the space-time metric is given by the Schwarzschild metric. Now in the presence of gravitational field the equation of a null geodesic is,

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} = 0$$

For $i = 1$

$$\frac{d^2x^1}{d\lambda^2} + \Gamma^1_{jk}\frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} = 0$$

$$\text{Or, } \frac{d^2x^1}{d\lambda^2} + \Gamma^1_{11}\left(\frac{dx^1}{d\lambda}\right)^2 + \Gamma^1_{22}\left(\frac{dx^2}{d\lambda}\right)^2 + \Gamma^1_{33}\left(\frac{dx^3}{d\lambda}\right)^2 + \Gamma^1_{44}\left(\frac{dx^4}{d\lambda}\right)^2 = 0$$

$$\text{Or, } \frac{d^2r}{d\lambda^2} - \frac{m}{r(r-2m)}\left(\frac{dr}{d\lambda}\right)^2 - (r-2m)\left\{\left(\frac{d\theta}{d\lambda}\right)^2 + \sin^2\theta\left(\frac{d\varphi}{d\lambda}\right)^2 - \frac{mc^2}{r^3}\left(\frac{dt}{d\lambda}\right)^2\right\} = 0$$

Similarly, for $i = 2,3,4$ respectively, we have the following equations

$$\frac{d^2\theta}{d\lambda^2} + \frac{2}{r}\frac{dr}{d\lambda}\frac{d\theta}{d\lambda} - \sin\theta\cos\theta\left(\frac{d\varphi}{d\lambda}\right)^2 = 0 \quad \text{……………………. (10)}$$

$$\frac{d^2\varphi}{d\lambda^2} + \frac{2}{r}\frac{dr}{d\lambda}\frac{d\varphi}{d\lambda} + 2\text{Cot}\theta\frac{d\theta}{d\lambda}\frac{d\varphi}{d\lambda} = 0 \quad \text{………………….... (11)}$$

$$\frac{d^2t}{d\lambda^2} + \frac{2m}{r(r-2m)}\frac{dr}{d\lambda}\frac{dt}{d\lambda} = 0 \quad \text{…………………… (12)}$$

Also for a null geodesic we have,

$$g_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} = 0$$

$$\text{Or, } g_{11}\left(\frac{dx^1}{d\lambda}\right)^2 + g_{22}\left(\frac{dx^2}{d\lambda}\right)^2 + g_{33}\left(\frac{dx^3}{d\lambda}\right)^2 + g_{44}\left(\frac{dx^4}{d\lambda}\right)^2 = 0$$

$$\text{Or, } \frac{m}{r(r-2m)}\left(\frac{dr}{d\lambda}\right)^2 + r^2\left\{\left(\frac{d\theta}{d\lambda}\right)^2 + \sin^2\theta\left(\frac{d\varphi}{d\lambda}\right)^2\right\} - \frac{c^2(r-2m)}{r}\left(\frac{dt}{d\lambda}\right)^2 = 0 \quad \text{……….. (13)}$$

Let us choose the spherical polar co-ordinates such that the ray is moving initially in the plane.

$\theta = \pi/2$, so that $\frac{d\theta}{d\lambda} = 0, \cos\theta = 0, \sin\pi/2 = 1$ initially. Hence by equation (10) we have $\frac{d^2\theta}{d\lambda^2} = 0$. Substituting $\theta = \pi/2$ in (11), (12) and (13) we get,

$$\frac{d^2\varphi}{d\lambda^2} + \frac{2}{r}\frac{dr}{d\lambda}\frac{d\varphi}{d\lambda} = 0 \quad \text{………………….. (14)}$$

$$\frac{d^2t}{d\lambda^2} + \frac{2m}{r(r-2m)}\frac{dr}{d\lambda}\frac{dt}{d\lambda} = 0 \quad \text{………………… (15)}$$

And,

$$\frac{m}{r(r-2m)}\left(\frac{dr}{d\lambda}\right)^2 + r^2\left(\frac{d\varphi}{d\lambda}\right)^2 - \frac{c^2(r-2m)}{r}\left(\frac{dt}{d\lambda}\right)^2 = 0 \quad \text{………………….. (16)}$$

Putting $\frac{d\varphi}{d\lambda} = \omega$ in (14) we get,

$$\frac{d\omega}{dr}\frac{dr}{d\lambda} + \frac{2}{r}\frac{dr}{d\lambda}\omega = 0$$

That is,

$\omega = \frac{d\varphi}{d\lambda} = \frac{\alpha}{r^2}$ …………………. (17)

Again putting $v = \frac{dt}{d\lambda}$ in (15) we get,

$v = \frac{dt}{d\lambda} = \frac{\beta r}{(r-2m)}$

Using these values in (16) we get,

$\left(\frac{\alpha}{r^2}\frac{dr}{d\varphi}\right)^2 + \frac{a^2}{r^2} = c^2\beta^2 + \frac{2m\alpha^2}{r^3}$

Putting $\frac{1}{r} = u$ the above equation become

$\left(\frac{du}{d\varphi}\right)^2 + u^2 = \frac{c^2\beta^2}{a^2} + 2mu^3$

$Or, 2\frac{du}{d\varphi}\frac{d^2u}{d\varphi^2} + 2u\frac{du}{d\varphi} = 6mu^2\frac{du}{d\varphi}$

$Or, \frac{d^2u}{d\varphi^2} + u = 3mu^2$ ………………………….. (18)

Neglecting the term $3mu^2$ we get,

$\frac{d^2u}{d\varphi^2} + u = 0$

The solution of this equation

$u = \frac{1}{R}\cos(\varphi + \alpha)$ ………….……………… (19)

where R and α are constant of integration.

If $\alpha = 0$, we have

$u = \frac{1}{R}\cos\varphi$

Putting the values of u in (18) we get,

$\frac{d^2u}{d\varphi^2} + u = \frac{3m}{R^2}\cos^2\varphi$

Or, $(D^2 + 1)u = \frac{3m}{R^2}\cos^2\varphi$

Here,

$P.I = \frac{m}{R^2}(2 - \cos^2\varphi)$

Hence the general solution is,

$u = \frac{1}{R}\cos\varphi + \frac{m}{R^2}(2 - \cos^2\varphi)$

Or, $x = R \pm \frac{2my}{r}$

That is,

$x = R + \frac{2my}{r}$ And $x = R - \frac{2my}{r}$

Also,

$y = \frac{R}{2m}x - \frac{R^2}{2m}$ And $y = -\frac{R}{2m}x + \frac{R^2}{2m}$

If α is the angle between these asymptotes, we have

$\tan\alpha = \frac{\left(\frac{R}{2m}\right)-\left(-\frac{R}{2m}\right)}{1+\left(\frac{R}{2m}\right)\left(-\frac{R}{2m}\right)}$

$\tan\alpha = \frac{4mR}{4m^2-R^2}$

Then,

$\sin \alpha = \frac{4mR}{4m^2+R^2} \approx \frac{4mR}{R^2} = \frac{4m}{R}$, Since $4m^2 \ll R^2$

Hence, the gravitational deflection is $\frac{4m}{R}$.

For a light ray grazing the surface of the sun

$R(\text{sun}'\text{sradius}) = 6.95 \times 10^{10}\text{cm}$

And, $m = 1.5 \times 10^5\text{cm}$

So that

$\alpha = \frac{4\times1.5\times10^5}{6.95\times10^{10}}$ radius

$= 8.62 \times 10^{-6}$ radius

$= 1.77''$

This implies that a light ray grazing the sun's surface will be deflected by $1.77''$. This result is an accord with the experimental finding.

4. Gravitational deflection of light in Newtonian theory:

Let us consider the ray of light is moving in the parallel line with the y-axis and it passes through a particle of mass m at a distance $x = r$. Then acceleration on x-axis,

$$\frac{d^2x}{dt^2} = -\frac{m}{r^2}.\frac{x}{r} = \frac{mx}{(x^2+y^2)^{3/2}} \qquad \text{…………………..} (20)$$

For the light ray which moves parallel with the y-axis,

$\frac{dy}{dx} = 1$ And $\frac{d^2y}{dx^2} = 0$

Now,

$$\frac{dt}{dx} = \frac{dx}{dy}\frac{dy}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dx}\frac{dy}{dt}\right) = \frac{d^2x}{dy^2}\left(\frac{dy}{dt}\right)^2 + \frac{dx}{dy}\frac{d^2y}{dt^2} = \frac{d^2x}{dy^2}.1 + \frac{dx}{dy}.0 = \frac{d^2x}{dy^2}$$

$$\therefore \frac{d^2x}{dt^2} = \frac{d^2x}{dy^2}$$

Substituting these values in in (20) we get,

$\frac{d^2x}{dy^2} = -\frac{mR}{(R^2+y^2)^{3/2}}$ When $x = R$

Integrating with respect to y we get,

$$\frac{dx}{dy} = -\frac{m}{R}\sin\theta + C = -\frac{my}{R\sqrt{R^2+y^2}} + C \qquad \text{……………….} (21)$$

$$x = -\frac{m}{R}\sqrt{R^2 + y^2} + Cy + C_1 \qquad \text{……………….} (22)$$

Applying the conditions $\frac{dy}{dx} = 0, x = R, y = 0$ we get,

$C = 0$ and $C_1 = m + R$

Then from (5.14) we get,

$$x = R + \left[m - \frac{m}{R}\sqrt{R^2 + y^2}\right] \qquad \text{……………….} (23)$$

According to Newtonian theory (23) is the equation of deflection of light.

Here $m - \frac{m}{R}\sqrt{R^2 + y^2}$ is amount of deflection from the path $x = R$.

Considering $x << y$ we get,

$x = R + m - \frac{m}{R}(\pm y)$

Then,

$y = -\frac{Rx}{m} + \frac{R}{m}(R + m)$ ………………… (23)

Now, β be the angle then,

$\tan\beta = \frac{\frac{R}{m} - \left(-\frac{R}{m}\right)}{1 + \frac{R}{m} - \left(-\frac{R}{m}\right)} = \frac{2mR}{m^2 - R^2}$

$\sin\beta = \frac{2mR}{m^2 + R^2}$

Since β is very small and $m^2 \ll R^2$ then we have $\sin\beta = \beta = \frac{2mR}{R^2}$

That is, $\beta = \frac{2m}{R}$

But we know, $\alpha = \frac{4m}{R} = 2.\frac{2m}{R} = 2\beta$

Thus the deflection in the path of light due to the relativistic field of a heavy mass is twice that predicted by the Newtonian theory.

5. If light has no mass then how can it be affected by gravity?

This leads to two thoughts requiring the basic ideas about light, rest mass, zero mass, relativistic mass, gravitation and so on. These are:

May light has mass, or Gravitational deflection is not affected by mass.

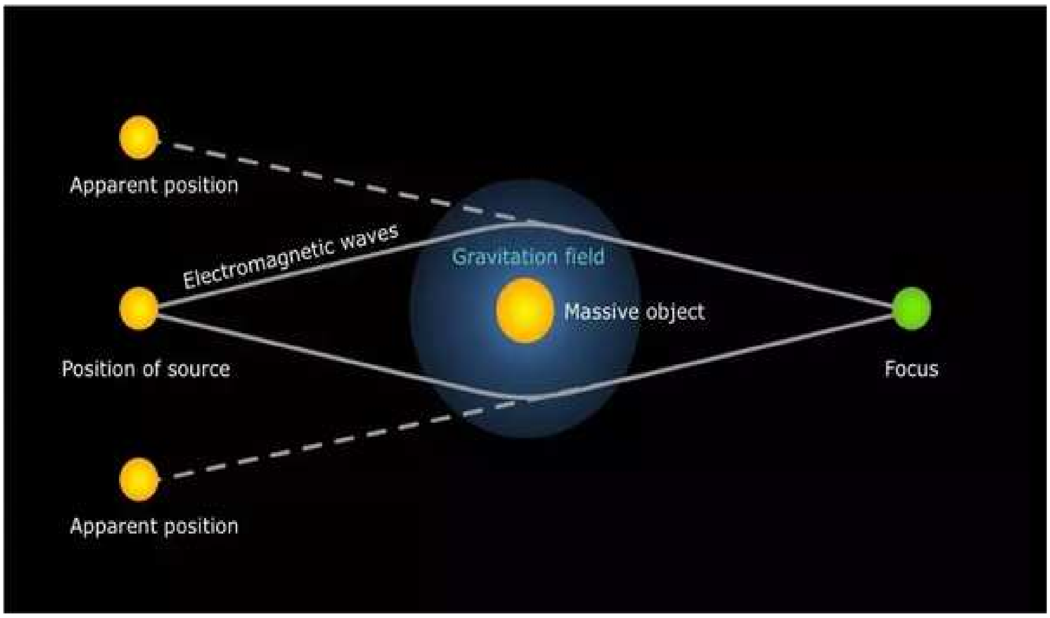

Figure 1:

Let's have a discussion about two topics mentioned above. Before that we should know about rest mass and zero mass.

Rest mass is the mass when the particle under consideration is at rest relative to the observer. Zero mass is that a mass of zero or which has no mass.

According to Newton's gravitational law gravitational force is proportional to the product of the masses. Actually Einstein's field equation would be more appropriate.

Anything with mass-energy, wraps space-time and so everything travels through space-time. So everything is affected by gravity. Also according to Einstein's theory of relativity light will be affected in the same way as matter is affected by gravity. Thus considering the above theories it can be said that if gravity affects light and light has mass. Lights do have mass and it is relativistic which is equivalent to energy $E = mc^2$. Thus light is a relativistic particle. It should be noted that mass can be destroyed but charge cannot be. Now it is time to explain about my second thought which says that gravitation is not affected by mass. Gravitation has a unique significance. Before Einstein, people assumed that because light is a wave, it is not attracted by gravity. After special relativity, Einstein assumed it would be attracted because it has mass. He developed general relativity, and that showed that light was deflected twice as much as he had previously calculated. A few years later, Edenton measured the light deflected by the sun, and that is what catapulted Einstein to fame. To explain this feature we should take help from Newton's 2^{nd} law of motion. Combining this law with Newton's gravitational equation the mass of the falling body removes. That's how we can relate that mass doesn't matter. According to principle of equivalence mass less particles are affected by gravitation just like massive particles. Thus gravitational deflection is not related to mass.

In actual sense, Physicist does not know why light is affected by gravity if photons are mass less.

Here we're going to explain our own observation about gravitational deflection of light after reviewing the theories of relativity. Gravity doesn't affect light, it affects space or time which

Light travels through. Let us consider the light ray is like a water wave in a lake where space or time is the shape of the lake. Some natural disaster or mankind related activities can change the shape of the lake and the water flows in new direction. In the same way light flows through space or time and deflect with the effect that gravity imposes on space or time.

6. **Conclusion**

From the above discussion, it can be concluded that the Gravitation has a unique significance. The question may arise "if light has no mass, how can it be affected by gravity?" According to Einstein's theory of relativity light will be affected by gravity in the same as a matter do. Some theories say light is not affected by gravity, as light is mass less. In actual sense Physicist have also confusion about this matter. Our work goes for a different explanation about gravitational deflection of light. Actually gravity doesn't affect light, it affect space or time and the deflected with the effects that gravity imposes on space or time.

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