Progress in Nonlinear Dynamics and Chaos
Vol. 6, No. 2, 2018, 71-77
ISSN: 2321 – 9238 *(online)*
Published on 26 November 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/pindac.v6n2a2

Progress in **Nonlinear Dynamics and Chaos**

On Fuzzy e^* (δs and δp)-continuous Multifunctions

***M.Poonguzhali*[1], *C.Loganathan*[2], *B.Vijayalakshmi*[3] and *A.Vadivel*[4]**

[1]Department of Mathematics, SSM College of Arts and Science, Komarapalayam
Namakkal, Tamilnadu-638183, India. email: poonguzhaliwin86@gmail.com
[2]Department of Mathematics, Maharaja Arts and Science College, Coimbatore
Tamilnadu- 641407, India. email: clogu@rediffmail.com
[3]Department of Mathematics, Govt. Arts College, Chidambaram
Tamilnadu-608002, India. email: mathvijaya2006au@gmail.com
[4]Department of Mathematics, Govt. Arts College (Autonomous), Karur
Tamilnadu-639005, India. email: avmaths@gmail.com
[2]Corresponding author

Received 30 October 2018; accepted 25 November 2018

Abstract. In this paper a new type of fuzzy multifunction termed as fuzzy e^* (respectively δs and δp) -continuous multifunction has been introduced and studied. Some characterizations and several properties of fuzzy lower and upper e^* (respectively δs and δp)-continuous multifunctions are obtained.

Keywords: Continuous multifunctions, Fuzzy *e**-open, fuzzy *e**-continuous multifunction

AMS Mathematics Subject Classification (2010): 54A40, 03E72

1. Introduction

The concept of fuzzy set was introduced by Zadeh [6]. Based on the concept of fuzzy sets, Chang [2] introduced and developed the concept of fuzzy topological spaces. Since then various important notion in the classical topology such as continuous functions [2] have been extended to fuzzy topological spaces. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduced various types of fuzzy continuity. One of them is fuzzy e-continuity. In 2014, Seenivasan [5] introduce the concept of fuzzy e-open and fuzzy e-continuity. Throughout this paper spaces (X,δ) and (Y,σ) (or simply X and Y) represent nonempty fuzzy topological spaces due to change [2] and the symbols I and I^X have been used for the unit closed interval $[0,1]$ and the set of all functions with domain X and codomain I, respectively. The support of a fuzzy set A is the set $\{x \in X: A(x) > 0\}$ and is denoted by $supp(A)$. A fuzzy set with only nonzero value $p \in (0,1]$ at only one element $x \in X$ is called a fuzzy point and is denoted by x_p and the set of all fuzzy points of a fuzzy topological space is denoted by $Pt(X)$. For any two fuzzy sets A and B of X, $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_p is said to be in a fuzzy set A (denoted by $x_p \in A$) if $x_p \leq A$, that is, if $p \leq A(x)$. The set of all fuzzy points having nonzero value ε, $0 < \varepsilon \leq 1$ and contained in the fuzzy set A is denoted by $Pr(A,\varepsilon)$. The constant fuzzy sets of X with values 0 and 1 are denoted by $\overline{0}$ and $\overline{1}$, respectively. A fuzzy set A is said to be quasi-coincident with B (written as $A\overline{q}B$) [3] if $A(x)+B(x)>1$ for some $x\in X$. A fuzzy set A is said to be not quasi-coincident with B (written as $A\overline{q}B$) [3] if $A(x)+B(x)\leq 1$ for all $x\in X$. A fuzzy open set A of X is called fuzzy quasi neighbourhood of a fuzzy point x_p if $x_p\overline{q}A$ and the collection of all fuzzy quasi neighbourhood of a fuzzy point x_p is denoted by $FQN(X,x_p)$. The fuzzy closure of A, fuzzy interior of A, fuzzy δ-closure of A and the fuzzy δ-interior of A are denoted by $Cl(A)$, $Int(A)$, $Cl_\delta(A)$ and $Int_\delta(A)$ respectively. A fuzzy subset A of space X is called fuzzy regular open [1] (respectively fuzzy regular closed) if $A=Int(Cl(A))$ (respectively $A=Cl(Int(A))$. The fuzzy δ-interior of fuzzy subset A of X is the union of all fuzzy regular open sets contained in A. A fuzzy subset A is called fuzzy δ-open [4] if $A=Int_\delta(A)$. The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e, $A=Cl_\delta(A)$). In this paper we use fuzzy e^*-open sets in order to obtain certain characterizations and properties of upper (lower) fuzzy e^*-continuous multifunctions.

2. Preliminaries

Definition 2.1. *[5] A fuzzy set λ of a fuzzy topological space X is said to be fuzzy e-open if $\lambda\leq Cl(Int_\delta\lambda)\vee Int(Cl_\delta\lambda)$, where $Cl(\lambda)=\wedge\{\mu:\mu\geq\lambda,\ \mu$ is fuzzy closed in $X\}$ and $Int(\lambda)=\vee\{\mu:\mu\leq\lambda,\ \mu$ is fuzzy open in $X\}$. If λ is fuzzy e-open, then $1-\lambda$ is fuzzy e-closed.*

Definition 2.2. *[5] Let X be a fuzzy topological space and λ be any fuzzy set in X. The fuzzy e-closure of λ in X is denoted by $eCl(\lambda)$ as follows: $eCl(\mu)=\wedge\{\lambda:\lambda\geq\mu,\lambda$ is a fuzzy e-closed set of $X\}$. Similarly we can define $eInt(\lambda)$.*

Definition 2.3. *[1] A fuzzy topological space X is product related to a fuzzy topological space Y if for fuzzy sets δ of X and η of Y whenever $1-\lambda\ngeq\delta$ and $1-\mu\ngeq\eta$ $\Rightarrow(1-\lambda\times 1)\wedge(1\times 1-\mu)\geq\delta\times\eta$, where λ is a fuzzy open set in X and μ is a fuzzy open set in Y, there exists λ_1 a fuzzy open set in X and μ_1 a fuzzy open set in Y such that $1-\lambda_1\geq\delta$ and $1-\mu_1\geq\eta$ and $(1-\lambda_1\times 1)\vee(1\times 1-\mu_1)=(1-\lambda\times 1)\vee(1\times 1-\mu)$.*

We know that a net $(x^{\alpha}_{\varepsilon_\alpha})$ in a fuzzy topological space (X,τ) is said to be eventually in the fuzzy set $\rho\leq X$ if there exists an index $\alpha_0\in J$ such that $(x^{\alpha}_{\varepsilon_\alpha})\in\rho$ for all $\alpha\geq\alpha_0$.

Definition 2.4. *Suppose that (X,τ), (Y,ν) and (Z,ω) are fuzzy topological spaces. It is known that if $F_1:X\to Y$ and $F_2:Y\to Z$ are fuzzy multifunctions, then the fuzzy multifunction $F_1\circ F_2:X\to Z$ is defined by $(F_1\circ F_2)(x_\varepsilon)=F_2(F_1(x_\varepsilon))$ for each $x_\varepsilon\in X$.*

Definition 2.5. *Suppose that $F:X\to Y$ is a fuzzy multifunction from a fuzzy topological space X to a fuzzy topological space Y. The fuzzy graph multifunction $G_F:X\to X\times Y$ of F is defined as $G_F(x_\varepsilon)=\{x_\varepsilon\}\times F(x_\varepsilon)$.*

3. Fuzzy e^* (respectively δs and δp)-continuous multifunctions

Definition 3.1. *Let $F:X\to Y$ be a fuzzy multifunction from a fuzzy topological space (X,τ) to a fuzzy topological space (Y,ν). Then it is said that F is:*

(1) Upper fuzzy e^* (respectively δs and δp)-continuous at $x_\varepsilon \in X$ if for each fuzzy open set μ of Y containing $F(x_\varepsilon)$, there exists a fuzzy e^* (respectively δs and δp)-open set ρ containing x_ε such that $\rho \leq F^+(\mu)$.

(2) Lower fuzzy e^* (respectively δs and δp)-continuous at $x_\varepsilon \in X$ if for each fuzzy open set μ of Y such that $x_\varepsilon \in F^-(\mu)$ there exists a fuzzy e^* (respectively δs and δp)-open set ρ containing x_ε such that $\rho \leq F^-(\mu)$.

(3) Upper (lower) fuzzy e^* (respectively δs and δp)-continuous if it has this property at each point of X.

Definition 3.2. *A sequence (x_{ε_n}) is said to e^* (respectively δs and δp)-converge to a point X if for every fuzzy e^* (respectively δs and δp)-open set μ containing x_ε there exists an index n_0 such that for $n \geq n_0$, $x_{\varepsilon_n} \in \mu$. This is denoted by $x_{\varepsilon_n} \to_{e^*} x_\varepsilon$ (respectively $x_{\varepsilon_n} \to_{\delta s} x_\varepsilon$ and $x_{\varepsilon_n} \to_{\delta p} x_\varepsilon$).*

Theorem 3.1. *Let $F: X \to Y$ be a fuzzy multifunction from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν). Then the following statements are equivalent:*

(1) F is upper fuzzy e^* (respectively δs and δp)-continuous.

(2) For each $x_\varepsilon \in X$ and for each fuzzy open set μ such that $x_\varepsilon \in F^+(\mu)$ there exists a fuzzy e^* (respectively δs and δp)-open set ρ containing x_ε such that $\rho \leq F^+(\mu)$.

(3) $F^+(\mu)$ is a fuzzy e^* (respectively δs and δp)-open set for any fuzzy open set $\mu \leq Y$.

(4) $F^-(\mu)$ is a fuzzy e^* (respectively δs and δp)-closed set for any fuzzy open set $\mu \leq Y$.

(5) for each $x_\varepsilon \in X$ and for each net $(x^\alpha_{\varepsilon_\alpha})$ which e^* (respectively δs and δp)-converges to x_ε in X and for each fuzzy open set $\mu \leq Y$ such that $x_\varepsilon \in F^+(\mu)$, the net $(x^\alpha_{\varepsilon_\alpha})$ is eventually in $F^+(\mu)$.

Proof. (1) $\Leftrightarrow$ (2): This statement is obvious.

(1) $\Leftrightarrow$ (3): Let $x_\varepsilon \in F^+(\mu)$ and let μ be a fuzzy open set. It follows from (1) that there exists a fuzzy e^* (respectively δs and δp)-open set ρ_{x_ε} containing x_ε such that $\rho x_\varepsilon \leq F^+(\mu)$. It follows that $F^+(\mu) = \vee_{x_\varepsilon \in F^+(\mu)} \rho_{x_\varepsilon}$ and hence $F^+(\mu)$ is fuzzy e^* (respectively δs and δp)-open. The converse can be shown easily.

(3) $\Rightarrow$ (4): Let $\mu \leq Y$ be a fuzzy open set. We have that $Y \backslash \mu$ is a fuzzy open set. From (3), $F^+(Y \backslash \mu) = X \backslash F^-(\mu)$ is a fuzzy e^* (respectively δs and δp)-open set. Then it is obtained that $F^-(\mu)$ is a fuzzy e^* (respectively δs and δp)-closed set.

(1) $\Rightarrow$ (5): Let $(x^\alpha_{\varepsilon_\alpha})$ be a net which e^* (respectively δs and δp)-converges to x_ε in X and let $\mu \leq Y$ be any fuzzy open set such that $x_\varepsilon \in F^+(\mu)$. Since F is an upper fuzzy e^* (respectively δs and δp)-continuous multifunction, it follows that there exists a fuzzy e^* (respectively δs and δp)-open set $\rho \leq X$ containing x_ε such that $\rho \leq F^+(\mu)$. Since $(x^\alpha_{\varepsilon_\alpha})$ e^* (respectively δs and δp)-converges to x_ε, it follows that there exists an index $\alpha_0 \in J$ such that $(x^\alpha_{\varepsilon_\alpha}) \in \rho$ for all $\alpha \geq \alpha_0$ from here, we obtain that $x^\alpha_{\varepsilon_\alpha} \in \rho \leq F^+(\mu)$ for all $\alpha \geq \alpha_0$. Thus the net $(x^\alpha_{\varepsilon_\alpha})$ is eventually in $F^+(\mu)$.

(5) $\Rightarrow$ (1): Suppose that is not true. There exists a point x_ε and a fuzzy open set μ with $x_\varepsilon \in F^+(\mu)$ such that $\rho \nleq F^+(\mu)$ for each fuzzy e^* (respectively δs and δp)-open set $\rho \leq X$ containing x_ε. Let $x_{\varepsilon_\rho} \in \rho$ and $x_\varepsilon \notin F^+(\mu)$ for each fuzzy e^* (respectively δs and δp)-open set $\rho \leq X$ containing x_ε. Then for the e^* (respectively δs and δp)-neighbourhood net (x_{ε_α}), $x_{\varepsilon_\alpha} \rightarrow_{e^*} x_\varepsilon$ (respectively $x_{\varepsilon_\alpha} \rightarrow_{\delta s} x_\varepsilon$ and $x_{\varepsilon_\alpha} \rightarrow_{\delta p} x_\varepsilon$), but (x_{ε_ρ}) is not eventually in $F^+(\mu)$. This is a contradiction. Thus, F is an upper fuzzy e^* (respectively δs and δp)-continuous multifunction.

Theorem 3.2. *Let $F: X \rightarrow Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, ν). Then the following statements are equivalent.*

(1) F is lower fuzzy e^* (respectively δs and δp)-continuous.
(2) For each $x_\varepsilon \in X$ and for each fuzzy open set μ such that $x_\varepsilon \in F^-(\mu)$ there exists a fuzzy e^* (respectively δs and δp)-open set ρ-containing x_ε such that $\rho \leq F^-(\mu)$.
(3) $F^-(\mu)$ is a fuzzy e^* (respectively δs and δp)-open set for any fuzzy open set $\mu \leq Y$.
(4) $F^+(\mu)$ is a fuzzy e^* (respectively δs and δp)-closed set for any fuzzy open set $\mu \leq Y$.
(5) For each $x_\varepsilon \in X$ and for each net $(x^\alpha_{\varepsilon_\alpha})$ which e^* (respectively δs and δp)-converges to x_ε in X and for each fuzzy open set $\mu \leq Y$ such that $x_\varepsilon \in F^-(\mu)$, the net $(x^\alpha_{\varepsilon_\alpha})$ is eventually in $F^-(\mu)$.

Proof: It can be obtained similarly as Theorem 3.1.

Theorem 3.3. *Let $F: X \rightarrow Y$ be a fuzzy multifunction from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν) and let $F(X)$ be endowed with subspace fuzzy topology. If F is an upper fuzzy e^* (respectively δs and δp)-continuous multifunction, then $F: X \rightarrow F(X)$ is an upper fuzzy e^* (respectively δs and δp)-continuous multifunction.*

Proof: Since F is an upper fuzzy e^* (respectively δs and δp)-continuous, $F(X \wedge F(X)) = F^+(\mu) \wedge F^+(F(X)) = F^+(\mu)$ is fuzzy e^* (respectively δs and δp)-open for each fuzzy open subset μ of Y. Hence $F: X \rightarrow F(X)$ is an upper fuzzy e^* (respectively δs and δp)-continuous multifunction.

Theorem 3.4. *Let (X, τ), (Y, ν) and (Z, ω) be fuzzy topological spaces and let $F: X \rightarrow Y$ and $G: Y \rightarrow Z$ be fuzzy multifunction. If $F: X \rightarrow Y$ is an upper (lower) fuzzy continuous multifunction and $G: Y \rightarrow Z$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction. Then $G \circ F: X \rightarrow Z$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction.*

Proof: Let $\lambda \leq Z$ be any fuzzy open set. From the definition of $G \circ F$, we have $(G \circ F)^+(\lambda) = F^+(G^+(\lambda))$ $((G \circ F)^-(\lambda) = F^-(G^-(\lambda)))$, since G is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous, it follows that $G^+(\lambda)(G^-(\lambda))$ is a fuzzy open set. Since F is an upper (lower) fuzzy continuous, it follows that $F^+(G^+(\lambda))(F^-(G^-(\lambda)))$ is a fuzzy e^* (respectively δs and δp)-open set, this shows that $G \circ F$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous.

Theorem 3.5. *Let $F: X \rightarrow Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, ν). If F is a lower (upper) fuzzy e^* (respectively*

δs and δp)-continuous multifunction and $\mu \leq X$ is a fuzzy set, then the restriction multifunction $F|_{\mu}: \mu \to Y$ is an lower (upper) fuzzy e^* (respectively δs and δp)-continuous multifunction.

Proof: Suppose that $\beta \leq Y$ is a fuzzy open set. Let $x_\varepsilon \in \mu$ and let $x_\varepsilon \in F^-|_{\mu}(\beta)$. Since F is a lower fuzzy e^* (respectively δs and δp)-continuous multifunction, it follows that there exists a fuzzy open set $x_\varepsilon \in \rho$ such that $\rho \leq F^-(\beta)$. From here we obtain that $x_\varepsilon \in \rho \wedge \mu$ and $\rho \wedge \mu \leq F|_{\mu}(\beta)$. Thus, we show that the restriction multifunction $F|_{\mu}$ is lower fuzzy e^* (respectively δs and δp)-continuous multifunction .

The proof for the case of the upper fuzzy e^* (respectively δs and δp)-continuity of the multifunction $F|_{\mu}$ is similar to above.

Theorem 3.6. *Let $F: X \to Y$ be a fuzzy multifunction from a fuzzy topological (X, τ) to a fuzzy topological space (Y, ν), let $\{\lambda_\gamma : \gamma \in \Phi\}$ be a fuzzy open cover of X. If the restriction multifunction $F_\gamma = F_{\lambda_\gamma}$ is lower (upper) fuzzy e^* (respectively δs and δp)-continuous multifunction for each $\gamma \in \Phi$, then F is lower (upper) fuzzy e^* (respectively δs and δp)-continuous multifunction.*

Proof: Let $\mu \leq Y$ be any fuzzy open set. Since F_γ is lower fuzzy e^* (respectively δs and δp)-continuous for each γ, we know that $F_\gamma^-(\mu) = Int_{\lambda_\gamma}(F_\gamma^-(\mu))$ and from here $F^-(\mu) \wedge \lambda_\gamma \leq Int_{\lambda_\gamma}(F^-(\mu) \wedge \lambda_\gamma)$ and $F^-(\mu) \wedge \lambda_\gamma \leq Int(F^-(\mu)) \wedge \lambda_\gamma$. Since $\{\lambda_\gamma : \gamma \in \Phi\}$ is a fuzzy open cover of X. It follows that $F^-(\mu) \leq Int(F^-(\mu))$. Thus we obtain that F is lower (upper) fuzzy e^* (respectively δs and δp)-continuous multifunction. The proof of the upper fuzzy e^* (respectively δs and δp)-continuity of F is similar to the above.

Theorem 3.7. *Let $F: X \to Y$ be a fuzzy multifunction from a fuzzy topological space (X, τ) to a fuzzy topological space (y, ν). If the graph function of F is lower (upper) fuzzy e^* (respectively δs and δp)-continuous multifunction, then F is lower (upper) fuzzy e^* (respectively δs and δp)-continuous multifunction.*

Proof: For the fuzzy sets $\beta \leq X$, $\eta \leq Y$, we take

$$(\beta \times \eta)(z, y) = \begin{cases} 0 & \text{if } z \notin \beta \\ \eta(y) & \text{if } z \in \beta \end{cases}$$

Let $x_\varepsilon \in X$ and let $\mu \in Y$ be a fuzzy open set such that $x_\varepsilon \in F^-(\mu)$. We obtain that $x_\varepsilon \in G_F^-(X \times \mu)$ and $X \times \mu$ is a fuzzy open set. Since fuzzy graph multifunction G_F is lower fuzzy e^* (respectively δs and δp)-continuous, it follows that there exists a fuzzy e^* (respectively δs and δp)-open set $\rho \leq X$ containing x_ε such that $\rho \leq G_F^-(X \times \mu)$. From here, we obtain that $\rho \leq F^-(\mu)$. Thus, F is lower fuzzy e^* (respectively δs and δp)-continuous multifunction.

The proof of the upper fuzzy e^* (respectively δs and δp)-continuity of F is similar to the above.

Theorem 3.8. *Suppose that (X, τ) and (X_α, τ_α) are fuzzy topological space where $\alpha \in J$. Let $F: X \to \prod_{\alpha \in J} X_\alpha$ be a fuzzy multifunction from X to the product space $\prod_{\alpha \in J} X_\alpha$ and let $P_\alpha: \prod_{\alpha \in J} X_\alpha \to X_\alpha$ be the projection multifunction for each $\alpha \in J$ which is defined be $P_\alpha((x_\alpha)) = \{x_\alpha\}$. If F is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous* *multifunction, then $P_\alpha \circ F$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction for each $\alpha \in J$.*

Proof: Take any $\alpha_0 \in J$. Let μ_{α_0} be a fuzzy open set in (X_α, τ_α). Then $(P_{\alpha_0} \circ F)^+(\mu_{\alpha_0}) = F^+(P^+_{\alpha_0}(\mu_{\alpha_0})) = F^+(\mu_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$ (respectively $(P_{\alpha_0} \circ F)^-(\mu_{\alpha_0}) = F^-(P^-_{\alpha_0}(\mu_{\alpha_0})) = F^-(\mu_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$. Since F is upper (lower) fuzzy e^*-continuous multifunction and since $\mu_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha$ is a fuzzy open set, it follows that $F^+(\mu_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$ (respectively $F^-(\mu_{\alpha_0} \times \prod_{\alpha \neq \alpha_0} X_\alpha)$) is fuzzy e^* (respectively δs and δp)-open in (X, τ). It shows that $P_{\alpha_0} \circ F$ is upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction. Hence we obtain that $P_\alpha \circ F$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction for each $\alpha \in J$.

Theorem 3.9. *Suppose that for each $\alpha \in J$, (X_α, τ_α) and (Y_α, ν_α) are fuzzy topological spaces. Let $F_\alpha: X_\alpha \to Y_\alpha$ be a fuzzy multifunction for each $\alpha \in J$ and let $F: \prod_{\alpha \in J} X_\alpha \to \prod_{\alpha \in J} Y_\alpha$ be defined by $F((x_\alpha)) = \prod_{\alpha \in J} F_\alpha(x_\alpha)$ from the product space $\prod_{\alpha \in J} X_\alpha$ to product space $\prod_{\alpha \in J} Y_\alpha$. If F is an upper (lower) fuzzy α-continuous multifunction, then each F_α is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction for each $\alpha \in J$.*

Proof: Let $\mu_\alpha \leq Y_\alpha$ be a fuzzy open set. Then $\mu_\alpha \times \prod_{\alpha \neq \beta} Y_\beta$ is a fuzzy open set. Since F is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction, it follows that $F^+(\mu_\alpha \times \prod_{\alpha \neq \beta} Y_\beta) = F^+(\mu_\alpha) \times \prod_{\alpha \neq \beta} X_\beta$, ($F^-(\mu_\alpha \times \prod_{\alpha \neq \beta} Y_\beta) = F^-(\mu_\alpha) \times \prod_{\alpha \neq \beta} X_\beta$) is a fuzzy e^* (respectively δs and δp)-open set. Consequently, we obtain that $F^+(\mu_\alpha)(F^-(\mu_\alpha))$ is a fuzzy e^* (respectively δs and δp)-open set. Thus, we show that F_α is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction.

Theorem 3.10. *Suppose that (X_1, τ_1), (X_2, τ_2), (Y_1, ν_1) and (Y_2, ν_2) are fuzzy topological spaces and $F_1: X_1 \to Y_1$, $F_2: X_2 \to Y_2$ are fuzzy multifunctions and suppose that if $\eta \times \beta$ is fuzzy e^* (respectively δs and δp)-open set then η and β are fuzzy e^* (respectively δs and δp)-open sets for any fuzzy sets $\eta \leq Y_1$, $\beta \leq Y_2$. Let $F_1 \times F_2: X_1 \times X_2 \to Y_1 \times Y_2$ be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\varepsilon, y_\nu) = F_1(x_\varepsilon) \times F_2(y_\nu)$. If $F_1 \times F_2$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunctions, then F_1 and F_2 are upper (lower) fuzzy e^* (respectively δs and δp)-continuous mutifunctions.*

Proof: We know that $(\mu^* \times \beta^*)(x_\varepsilon, y_\nu) = min\{\mu^*(x), \beta^*(y)\}$ for any fuzzy sets μ^*, β^* and for any fuzzy point x_ε, y_ν. Let $\mu \times \beta \leq Y_1 \times Y_2$ be a fuzzy open set. It known that $(F_1 \times F_2)^+(\mu \times \beta) = F_1^+(\mu) \times F_2^+(\beta)$. Since $F_1 \times F_2$ is an upper fuzzy e^*-continuous multifunction, it follows that $F_1^+(\mu) \times F_2^+(\beta)$ is a fuzzy e^* (respectively δs and δp)-open set. From here, $F_1^+(\mu)$ and $F_2^+(\beta)$ are fuzzy e^* (respectively δs and δp)-open sets. Hence, it is obtain that F_1 and F_2 are upper fuzzy e^* (respectively δs and δp)-continuous multifunctions. The proof of the lower fuzzy e^* (respectively δs and δp)-continuity of the multifunctions F_1 and F_2 is similar to the above.

Theorem 3.11. *Suppose that (X, τ), (Y, ν) and (Z, ω) are fuzzy topological spaces and $F_1: X \to Y$, $F_2: X \to Z$ are fuzzy multifunction and suppose that if $\eta \times \beta$ is a fuzzy e^**

(respectively δs and δp)-opens set, then η and β are fuzzy e^* (respectively δs and δp)-open sets for any fuzzy set $\eta \leq Y$, $\beta \leq Z$. Let $F_1 \times F_2 : X \to Y \times Z$ be a fuzzy multifunction which is defined by $(F_1 \times F_2)(x_\varepsilon) = F_1(x_\varepsilon) \times F_2(x_\varepsilon)$. If $F_1 \times F_2$ is an upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunction, then F_1 and F_2 are upper (lower) fuzzy e^* (respectively δs and δp)-continuous multifunctions.

Proof: Let $x_\varepsilon \in X$ and let $\mu \leq \beta$, $\beta \leq Z$ be fuzzy e^* (respectively δs and δp)-open sets such that $x_\varepsilon \in F_1^+(\mu)$ and $x_\varepsilon \in F_2^+(\beta)$. Then we obtain that $F_1(x_\varepsilon) \leq \mu$ and $F_2(x_\varepsilon) \leq \beta$ and from here, $F_1(x_\varepsilon) \times F_2(x_\varepsilon) = (F_1 \times F_2)(x_\varepsilon) \leq \mu \times \beta$. We have $x_\varepsilon \in (F_1 \times F_2)^+(\mu \times \beta)$. Since $F_1 \times F_2$ is an upper fuzzy e^* (respectively δs and δp)-continuous multifunction, it follows that there exist a fuzzy e^* (respectively δs and δp)-open set ρ containing x_ε such that $\rho \leq (F_1 \times F_2)^+(\mu \times \beta)$. We obtain that $\rho \leq F_1^+(\mu)$ and $\rho \leq F_2^+(\beta)$. Thus we obtain that F_1 and F_2 are fuzzy e^* (respectively δs and δp)-continuous multifunctions.

The proof of the lower fuzzy e^* (respectively δs and δp)-continuity of the multifunctions F_1 and F_2 is similar to the above.

REFERENCES

1. K.K.Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, *J. Math. Anal. Appl*., 82 (1981) 297-305.
2. C.L.Chang, Fuzzy topological space, *J. Math. Anal. Appl*., 24 (1968) 182-190.
3. P.Pao-Ming and L.Ying-Ming, Fuzzy topology I. neighbourhood structure of a fuzzy point and Moore-simith convergence, *J. Math. Anal. Appl*., 76(1980) 571-599.
4. A.Mukherjee and S.Debnath, δ-semi-open sets in fuzzy setting, *J. Tri. Math. Soc*, 8 (2006) 51-54.
5. V.Seenivasan and K.Kamala, Fuzzy e-continuity and fuzzy e-open sets, *Annals of Fuzzy Mathematics and Informatics*, 8(1) (2014) 141-148.
6. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.