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Progress in Nonlinear Dynamics and Chaos

A Study on Regularity in Vague Graphs

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Abstract. Theoretical concepts of graphs are highly utilized by computer scientists.Especially in research areas of computer science such as data mining, image segmentation, clustering image capturing and networking. The vague graphs are more flexible and compatible than fuzzy graphs due to the fact that they allowed the degree of membership of a vertex to an edge to be represented by interval valued in [0,1] rather than the crisp real values between 0 and 1. In this paper, some interesting properties of an edge regular vague graph are given. Also, new concepts such as strongly regular, edge regular, and biregular vague graphs are defined.

Keyword: Vague set, vague graph, biregular vague graph.

AMS Mathematics Subject Classification (2010): 05C76

1.Introduction

In 1965, Zadeh [24] first proposed the theory of fuzzy sets. Gau and Buehrer [5] proposed the concept of vague set 1993, by replacing the value of an element in a set with a subinterval of $[0,1]$. Namely, a true-membership function $t_v(x)$ and false membership function $f_v(x)$ are used to describe the bounderies of the memebership degree. The first definition of fuzzy graphs was proposed by Kafmann [6] in 1993, from Zadeh's fuzzy relations [24], [25], and [26]. But Rosenfeld [16] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. Ramakrishna [8] introduced the concept of vague graphs and studied some of their properties. Akram et al. [1] defined the vague hypergraphs. Borzooei and Rashmanlou [2, 3, 4] investigated domination in vague graphs, degree of vertices in vague graphs and ring sum in product intuitionistic fuzzy graphs. Rashmanlou et al. [9]-[15] introduced new concepts of bipolar fuzzy graphs, complete interval-valued fuzzy graphs, antipodal interval-valued fuzzy graphs, balanced interval-valued fuzzy graphs and some properties of highly irregular interval-valued fuzzy graphs. Samanta and Pal [17, 18, 19, 20] defined fuzzy tolerance graph, fuzzy threshold graph, fuzzy $k-$competition graph and $p-$competition fuzzy graph and new concepts of fuzzy planar graph. Karunambigai et al. [7] introduced edge regular intuitionistic fuzzy graph. In this paper, some properties of an edge regular vague graph are given. Particularly, strongly regular, edge regular and biregular vague graphs are defined and the necessary and sufficient condition for a vague graph to be strongly regular is studied. Also, we have introduced a partially edge regular vague graph and fully edge regular vague graph with suitable illustrations.

2. Preliminaries

A graph is an ordered pair $G=(V,E)$, where V is the set of vertices of G and E is the set of edges of G. A subgraph of a graph $G=(V,E)$ is a graph $H=(W,F)$, where $W\subseteq V$ and $F\subseteq E$. A fuzzy graph $G=(\sigma,\mu)$ is a pair of functions $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ with $\mu(u,v)\leq\sigma(u)\wedge\sigma(v)$, for all $u,v\in V$, where V is a finite non-empty set and $\wedge$ denote minimum.

A vague set A in an ordinary finite non-empty set X, is a pair (t_A, f_A), where $t_A: X\to[0,1]$, $f_A: X\to[0,1]$ are true and false membership functions, respectively such that $0\leq t_A(x)+f_A(x)\leq 1$, for all $x\in X$. Note that $t_A(x)$ is considered as the lower bound for degree of membership of x in A and $f_A(x)$ is the lower bound for negative of membership of x in A. So, the degree of membership of x in the vague set A, is characterized by the interval $[t_A(x), 1-f_A(x)]$.

Hence, a vague set is a special case of interval-valued sets studied by many mathematicians and applied in many branches of mathematics.

Let X and Y be two ordinary finite non-empty sets. We call a vague relation to be a vague subset of $X\times Y$, that is an expression R defined by

$$R=\{\langle(x,y), t_R(x,y), f_R(x,y)\rangle \mid x\in X, y\in Y\}$$

where $t_R: X\times Y\to[0,1]$, $f_R: X\times Y\to[0,1]$, which satisfies the condition $0\leq t_R(x,y)+f_R(x,y)\leq 1$, for all $(x,y)\in X\times Y$. (See [8]).

Definition 2.1. *[8] A vague graph is defined to be a pair $G=(A,B)$, where $A=(t_A, f_A)$ is a vague set on V and $B=(t_B, f_B)$ is a vague set on $E\subseteq V\times V$ such that $t_B(xy)\leq\min(t_A(x),t_A(y))$ and $f_B(xy)\geq\max(f_A(x),f_A(y))$, for each edge $xy\in E$.*

The underlying crisp graph of a vague graph $G=(A,B)$, is the graph $G=(V,E)$, where $V=\{v: t_A(v)>0$ and $f_A(v)>0\}$ and $E=\{\{u,v\}: t_B(\{u,v\})>0, f_B(\{u,v\})>0\}$. V is called the vertex set and E is called the edge set. A vague graph maybe also denoted as $G=(V,E)$.

A vague graph G is said to be strong if $t_B(v_iv_j)=\min\{t_A(v_i),t_A(v_j)\}$ and $f_B(v_iv_j)=\max\{f_A(v_i),f_A(v_j)\}$, for every edge $v_iv_j\in E$. A vague graph G is said to be complete if $t_B(v_iv_j)=\min\{t_A(v_i),t_A(v_j)\}$ and $f_B(v_iv_j)=\max\{f_A(v_i),f_A(v_j)\}$, for all $v_i,v_j\in V$. The complement of a vague graph $G=(A,B)$ is a vague graph

$\overline{G}=(\overline{A},\overline{B})$, where $\overline{A}=A=(t_A, f_A)$ and $\overline{B}=(\overline{t_B},\overline{f_B})$ is defined by:

$$\overline{t_B}(xy)=\min(t_A(x),t_A(y))-t_B(xy),\quad \overline{f_B}=f_B(xy)-\max(f_A(x),f_A(y)).$$

Definition 2.2. *[4] Let $G=(V,E)$ be a vague graph.*

(i) The neighborhood degree of a vertex v is defined as $d_N(v)=(d_{N_t}(v),d_{N_f}(v))$, where

$$d_{N_t}(v)=\sum_{w\in N(v)} t_A(w) \ \ and \ \ d_{N_f}(v)=\sum_{w\in N(v)} f_A(w).$$

(ii) The degree of a vertex v_i is defined by $d_G(v_i)=(d_t(v_i),d_f(v_i))=(k_1,k_2)$, where $k_1=d_t(v_i)=\sum_{v_i\neq v_j} t_B(v_iv_j)$ and $k_2=d_f(v_i)=\sum_{v_i\neq v_j} f_B(v_iv_j)$.

Definition 2.3. *[4] A vague graph $G=(V,E)$ is said to be*

(i) $(k_1,k_2)-$regular if $d_G(v_i)=(k_1,k_2)$, for all $v_i\in V$ and also G is said to be a regular vague graph of degree (k_1,k_2).

(ii) bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that

(a) $t_B(v_iv_j)=0$ and $f_B(v_iv_j)=0$, if $(v_i,v_j)\in V_1$ or $(v_i,v_j)\in V_2$

(b) $t_B(v_iv_j)=0$, $f_B(v_iv_j)>0$, if $v_i\in V_1$ or $v_j\in V_2$

(c) $t_B(v_iv_j)>0$, $f_B(v_iv_j)=0$, if $v_i\in V_1$ or $v_j\in V_2$, for some i and j.

Definition 2.4. *[16] Let $G^*=(V,E)$ be a crisp graph and let $e=v_iv_j$ be an edge in G^*. Then, the degree of an edge $e=v_iv_j\in E$ is defined as*

$d_{G^*}(v_iv_j)=d_{G^*}(v_i)+d_{G^*}(v_j)-2.$

3. Some properties of regularity in vague graphs

Definition 3.1. *Let $G=(V,E)$ be a vague graph.*

(i) The degree of an edge $e_{ij}\in E$ is defined as

$d_t(e_{ij})=d_t(v_i)+d_t(v_j)-2t_B(v_iv_j)$

or $d_t(e_{ij})=\sum_{\substack{v_iv_k\in E\\ k\neq j}} t_B(v_iv_k)+\sum_{\substack{v_kv_j\in E\\ k\neq i}} t_B(v_kv_j)$

$d_f(e_{ij})=d_f(v_i)+d_f(v_j)-2f_B(v_iv_j)$

or $d_f(e_{ij})=\sum_{\substack{v_iv_k\in E\\ k\neq j}} f_B(v_iv_k)+\sum_{\substack{v_kv_j\in E\\ k\neq i}} f_B(v_kv_j)$

(ii) The minimum edge degree of G is $\delta_E(G)=(\delta_t(G),\delta_f(G))$, where $\delta_t(G) = \wedge\{d_t(e_{ij}) \mid e_{ij} \in E\}$ and $\delta_f(G) = \wedge\{d_f(e_{ij}) \mid e_{ij} \in E\}$.

(iii) The maximum edge degree of G is $\Delta_E(G) = (\Delta_t(G), \Delta_f(G))$,

where $\Delta_t(G) = \vee\{d_t(e_{ij}) \mid e_{ij} \in E\}$ and $\Delta_f(G) = \vee\{d_f(e_{ij}) \mid e_{ij} \in E\}$.

(iv) The total edge degree of an edge $e_{ij} \in E$ is defined as

$$td_t(e_{ij}) = \sum_{\substack{v_i v_k \in E \\ k \neq j}} t_B(v_i v_k) + \sum_{\substack{v_k v_j \in E \\ k \neq i}} t_B(v_k v_j) + t_B(e_{ij}),$$

$$td_f(e_{ij}) = \sum_{\substack{v_i v_k \in E \\ k \neq j}} f_B(v_i v_k) + \sum_{\substack{v_k v_j \in E \\ k \neq i}} f_B(v_k v_j) + f_B(e_{ij}).$$

(v) The edge degree of G is defined by $d_G(e_{ij}) = (d_t(e_{ij}), d_f(e_{ij}))$ and the total edge degree of G is defined by $td_G(e_{ij}) = (td_t(e_{ij}), td_f(e_{ij}))$.

Examples 3.2. *Consider a vague graph* $G = (V, E)$ *such that* $V = \{u_1, u_2, u_3, u_4\}$ *and* $E = \{u_1u_2, u_1u_4, u_2u_3, u_2u_4, u_3u_4\}$.

Here, $d_t(e_{12}) = 0.5$, $d_f(e_{12}) = 1.9$, $d_G(e_{12}) = (0.5, 1.9)$, $td_t(e_{12}) = 0.5 + 0.2 = 0.7$, $td_f(e_{12}) = 1.9 + 0.5 = 2.4$. *Hence,* $td_G(e_{12}) = (0.7, 2.4)$.

Definition 3.3. *Let* $G = (V, E)$ *be a vague graph.*

(i) If each edge in G has the same degree (l_1, l_2), then G is said to be an edge regular vague graph.

(ii) If each edge in G has the same total degree (t_1, t_2), then G is said to be a totally edge regular vague graph.

Theorem 3.4. *Let* $G = (V, E)$ *be a vague graph on a cycle* $G^* = (V, E)$. *Then* $\sum_{v_i \in V} d_G(v_i) = \sum_{v_i v_j \in E} d_G(v_i v_j)$.

Proof. Let $G = (V, E)$ be a vague graph and G^* be a cycle $v_1 v_2 v_3 \cdots v_n v_1$. Then $\sum_{i=1}^{n} d_G(v_i v_{i+1}) = (\sum_{i=1}^{n} d_t(v_i v_{i+1}), \sum_{i=1}^{n} d_f(v_i v_{i+1}))$. Now we have

$$\sum_{i=1}^{n} d_t(v_i v_{i+1}) = d_t(v_1 v_2) + d_t(v_2 v_3) + \cdots + d_t(v_n v_1), \textit{where } v_{n+1} = v_1$$

$$= d_t(v_1) + d_t(v_2) - 2t_B(v_1 v_2) + d_t(v_2) + d_t(v_3)$$

$$- 2t_B(v_2 v_3) + \cdots + d_t(v_n) + d_t(v_1) - 2t_B(v_n v_1)$$

$$= 2d_t(v_1) + 2d_t(v_2) + \cdots + 2d_t(v_n) - 2(t_B(v_1 v_2) + t_B(v_2 v_3) + \cdots + t_B(v_n v_1))$$

$$= 2\sum_{v_i \in V} d_t(v_i) - 2\sum_{i=1}^{n} t_B(v_i v_{i+1}) = \sum_{v_i \in V} d_t(v_i) + 2\sum_{i=1}^{n} t_B(v_i v_{i+1}) - 2\sum_{i=1}^{n} t_B(v_i v_{i+1})$$

$= \sum_{v_i \in V} d_t(v_i).$

Similarly, $\sum_{i=1}^{n} d_f(v_i v_{i+1}) = \sum_{v_i \in V} d_f(v_i)$.

Hence, $\sum_{i=1}^{n} d_G(v_i v_{i+1}) = (\sum_{v_i \in V} d_t(v_i), \sum_{v_i \in V} d_f(v_i)) = \sum_{v_i \in V} d_G(v_i)$.

Remark 3.5. *Let $G = (V, E)$ be a vague graph on a crisp graph G^*. Then,*

$$\sum_{v_i v_j \in E} d_G(v_i v_j) = (\sum_{v_i v_j \in E} d_{G^*}(v_i v_j) t_B(v_i v_j), \sum_{v_i v_j \in E} d_{G^*}(v_i v_j) f_B(v_i v_j)),$$

where $d_{G^*}(v_i v_j) = d_{G^*}(v_i) + d_{G^*}(v_j) - 2$, for all $v_i v_j \in E$.

Theorem 3.6. *Let $G = (V, E)$ be a vague graph on a $k-$regular crisp graph G^*. Then,*

$$\sum_{v_i v_j \in E} d_G(v_i v_j) = ((k-1)\sum_{v_i \in V} d_t(v_i), (k-1)\sum_{v_i \in V} d_f(v_i)).$$

Proof: By Remark 3.5 we have

$$\sum_{v_i v_j \in E} d_G(v_i v_j) = (\sum_{v_i v_j \in E} d_{G^*}(v_i v_j) t_B(v_i v_j), \sum_{v_i v_j \in E} d_{G^*}(v_i v_j) f_B(v_i v_j))$$

$$= (\sum_{v_i v_j \in E} (d_{G^*}(v_i) + d_{G^*}(v_j) - 2) t_B(v_i v_j), \sum_{v_i v_j \in E} (d_{G^*}(v_i) + d_{G^*}(v_j) - 2) f_B(v_i v_j)).$$

Since G^* is a regular crisp graph, $d_{G^*}(v_i) = k$, for all $v_i \in V$ and we have

$$\sum_{v_i v_j \in E} d_G(v_i v_j) = ((k+k-2)\sum_{v_i v_j \in E} t_B(v_i v_j), (k+k-2)\sum_{v_i v_j \in E} f_B(v_i v_j)),$$

$$\sum_{v_i v_j \in E} d_G(v_i v_j) = (2(k-1)\sum_{v_i v_j \in E} t_B(v_i v_j), 2(k-1)\sum_{v_i v_j \in E} f_B(v_i v_j)),$$

$$\sum_{v_i v_j \in E} d_G(v_i v_j) = ((k-1)\sum_{v_i \in V} d_t(v_i), (k-1)\sum_{v_i \in V} d_f(v_i)).$$

Theorem 3.7. *Let $G = (V, E)$ be a vague graph on a crisp graph G^*. Then,*

$$\sum_{v_i v_i \in E} td_G(v_i v_j) = (\sum_{v_i v_j \in E} d_{G^*}(v_i v_j) t_B(v_i v_j) + \sum_{v_i v_j \in E} t_B(v_i v_j), \sum_{v_i v_j \in E} d_{G^*}(v_i v_j) f_B(v_i v_j) + \sum_{v_i v_j \in E} f_B(v_i v_j)).$$

Proof: By definition of total edge degree of G, we have

$$\sum_{v_i v_j \in E} td_G(v_i v_j) = (\sum_{v_i v_j \in E} td_t(v_i v_j), \sum_{v_i v_j \in E} td_f(v_i v_j))$$

$$= (\sum_{v_i v_j \in E} (d_t(v_i v_j) + t_B(v_i v_j)), \sum_{v_i v_j \in E} (d_f(v_i v_j) + f_B(v_i v_j)))$$

$$= (\sum_{v_iv_j\in E} d_t(v_iv_j) + \sum_{v_iv_j\in E} t_B(v_iv_j), \sum_{v_iv_j\in E} d_f(v_iv_j) + \sum_{v_iv_j\in E} f_B(v_iv_j)).$$

By Remark 3.5, we get

$$\sum_{v_iv_j\in E} td_G(v_iv_j) = (\sum_{v_iv_j\in E} d_{G^*}(v_iv_j)t_B(v_iv_j) + \sum_{v_iv_j\in E} t_B(v_iv_j),$$
$$\sum_{v_iv_j\in E} d_{G^*}(v_iv_j) f_B(v_iv_j) + \sum_{v_iv_j\in E} f_B(v_iv_j)).$$

Theorem 3.8. *Let* $G = (V, E)$ *be a vague graph. Then* (t_B, f_B) *is a constant function if and only if the following are equivalent.*

(i) G is a edge regular vague graph.

(ii) G is totally edge regular vague graph.

Proof: Assume that (t_B, f_B) is a constant function. Then $t_B(v_iv_j) = c_1$ and $f_B(v_iv_j) = c_2$, for every $v_iv_j \in E$, where c_1 and c_2 are constants. Let G be a (l_1,l_2) -edge regular vague graph. Then, $d_G(v_iv_j) = (l_1,l_2)$, for all $v_iv_j \in E$.

$$td_G(v_iv_j) = (d_t(v_iv_j) + t_B(v_iv_j), d_f(v_iv_j) + f_B(v_iv_j)) = (l_1 + c_1, l_2 + c_2),$$

for all $v_iv_j \in E$ which implies G is totally edge regular.

Let G be $(t_1,t_2)-$totally edge regular vague graph. Then $td_G(v_iv_j) = (t_1,t_2)$, for all $v_iv_j \in E$. So, we have

$$td_G(v_iv_j) = (d_t(v_iv_j) + t_B(v_iv_j), d_f(v_iv_j) + f_B(v_iv_j)) = (t_1,t_2).$$

Now,

$$(d_t(v_iv_j), d_f(v_iv_j)) = (t_1 - t_B(v_iv_j), t_2 - f_B(v_iv_j))$$
$$= (t_1 - c_1, t_2 - c_2).$$

Hence, G is $(t_1 - c_1, t_2 - c_2)$ edge regular vague graph.

Conversely, assume that (i) and (ii) are equivalent. We have to prove that (t_B, f_B) is a constant function. Suppose that (t_B, f_B) is not a constant function. Then $t_B(v_iv_j) \neq t_B(v_rv_s)$ and $f_B(v_iv_j) \neq f_B(v_rv_s)$ for at least one pair of $v_iv_j, v_rv_s \in E$. Let G be an (l_1,l_2) edge regular vague graph. Then, $d_G(v_iv_j) = d_G(v_rv_s) = (l_1,l_2)$.

$$td_G(v_iv_j) = (d_t(v_iv_j) + t_B(v_iv_j), d_f(v_iv_j) + f_B(v_iv_j))$$
$$= (l_1 + t_B(v_iv_j), l_2 + f_B(v_iv_j)),$$

for all $v_iv_j \in E$ and

$$td_G(v_rv_s) = (d_t(v_rv_s) + t_B(v_rv_s), d_f(v_rv_s) + f_B(v_rv_s))$$
$$= (l_1 + t_B(v_rv_s), l_2 + f_B(v_rv_s)),$$

for all $v_rv_s \in E$.

Since, $t_B(v_iv_j) \neq t_B(v_rv_s)$ and $f_B(v_iv_j) \neq f_B(v_rv_s)$ we have $td_G(v_iv_j) \neq td_G(v_rv_s)$. Hence, G is not a totally edge regular that is contradiction to our assumption. Therefore, (t_B, f_B) is a constant function. Similarly we can show that (t_B, f_B) is a constant function when G is a totally edge regular vague graph.

Theorem 3.9. *Let $G = (V, E)$ be a vague graph on a $k-$regular crisp graph G^*. Then, (t_B, f_B) is a constant if and only if G is both regular and edge regular vague graph.*

Proof. Let $G = (V, E)$ be a vague graph on G^* and let G^* be a $k-$regular crisp graph. Assume that t_B and f_B are constant functions, i.e., $t_B(v_iv_j) = c$ and $f_B(v_iv_j) = t$, for all $v_iv_j \in E$, where c, t are constants. By definition of degree of a vertex we have

$$d_G(v_i) = (d_t(v_i), d_f(v_i)) = (\sum_{v_iv_j \in E} t_B(v_iv_j), \sum_{v_iv_j \in E} f_B(v_iv_j)) = (\sum_{v_iv_j \in E} c, \sum_{v_iv_j \in E} t),$$

for all $v_i \in V$. Hence, $d_G(v_i) = (kc, kt)$. Therefore, G is regular vague graph. Now, $td_G(v_iv_j) = (td_t(v_iv_j), td_f(v_iv_j))$, where

$$td_t(v_iv_j) = \sum t_B(v_iv_k) + \sum t_B(v_kv_j) + t_B(v_iv_j) = \sum_{\substack{v_iv_k \in E \\ k \neq j}} c + \sum_{\substack{v_kv_j \in E \\ k \neq i}} c + c$$

$$= c(k-1) + c(k-1) + c = c(2k-1).$$

Similarly, $td_f(v_iv_j) = t(2k-1)$, for all $v_iv_j \in E$. Hence, G is also totally edge regular vague graph.

Conversely, assume that G is both regular and edge regular vague graph. We prove that (t_B, f_B) is a constant function. Since G is regular, $d_G(v_i) = (c_1, c_2)$, for all $v_i \in V$. Also, G is totally edge regular so, $td_G(v_iv_j) = (t_1, t_2)$, for all $v_iv_j \in E$.

By definition of totally edge degree we have $td_G(v_iv_j) = (td_t(v_iv_j), td_f(v_iv_j))$, where $td_G(v_iv_j) = d_t(v_i) + d_t(v_j) - t_B(v_iv_j)$, for all $v_iv_j \in E$, $t_1 = c_1 + c_2 - t_B(v_iv_j)$. So, $t_B(v_iv_j) = 2c_1 - t_1$. Similarly we have $f_B(v_iv_j) = 2c_2 - t_2$, for all $v_iv_j \in E$. Hence, (t_B, f_B) is a constant function.

Definition 3.10. *A vague graph $G = (V, E)$, where $V = \{v_1, v_2, \cdots, v_n\}$ is said to be strongly regular, if it satisfies the following axioms:*

(i) G is $k = (k_1, k_2)-$regular vague graph

(ii) The sum of membership values and non-membership values of the common neighborhood vertices of any pair of adjacent vertices and non-adjacent vertices v_i, v_j of G has the same weight and is denoted by $\lambda = (\lambda_1, \lambda_2)$, $\delta = (\delta_1, \delta_2)$, respectively.

Note 1. *Any strongly vague graph* G *is denoted by* $G=(n,k,\lambda,\delta)$.

Examples 3.11. *Consider a vague graph* $G=(V,E)$ *where* $V=\{u_1,u_2,u_3,u_4\}$ *and* $E=\{u_1u_2,u_2u_4,u_3u_4,u_1u_3,u_2u_3,u_1u_4\}$.
Here, $n=4$, $k=(k_1,k_2)=(0.3,1.5)$, $\lambda=(\lambda_1,\lambda_2)=(0.3,0.8)$, $\delta=(\delta_1,\delta_2)=(0,0)$.
So, G is strongly regular vague graph.

Theorem 3.12. *If* $G=(V,E)$ *is a complete vague graph with* (t_A, f_A) *and* (t_B, f_B) *as constant functions, then* G *is a strongly regular vague graph.*

Proof: Let $G=(V,E)$ be a complete vague graph where $V=\{v_1,v_2,\cdots,v_n\}$. Since t_A, f_A, t_B and f_B are constant functions. That is, $t_A(v_i)=r$, $f_A(v_i)=s$, for all $v_i\in V$ and $t_B(v_iv_j)=c$ and $f_B(v_iv_j)=t$, for all $v_iv_j\in E$ where r,s,c,t are constants. To prove that G is a strongly regular vague graph, we have to show that G is $k=(k_1,k_2)-$ regular vague graph and the adjacent vertices have the same common neighborhood $\lambda=(\lambda_1,\lambda_2)$ and non-adjacent vertices have the same common neighborhood $\delta=(\delta_1,\delta_2)$. Now,

$$d_G(v_i)=(d_t(v_i),d_f(v_i))=(\sum_{v_iv_j\in E} t_B(v_iv_j),\sum_{v_iv_j\in E} f_B(v_iv_j))$$

$$=((n-1)c,(n-1)t)\ \ (Since\ G\ is\ complete).$$

Hence, G is an $((n-1)c,(n-1)t)-$ regular vague graph. The sum of membership values and non-membership values of common neighborhood vertices of any pair of adjacent vertices $\lambda=((n-2)r,(n-2)s)$ are the same and the sum of membership values and non-membership values of common neighborhood vertices of any pair of non-adjacent vertices $\delta=0$, since G is complete vague graph. So we have the proof.

Remark 3.13. *If* G *is a strongly regular disconnected vague graph then,* $\delta=0$.

Definition 3.14. *A vague graph* $G=(V,E)$ *is said to be a biregular vague graph if it satisfies the following axioms:*
(i) G is $k=(k_1,k_2)-$ regular vague graph.
(ii) $V=V_1\cup V_2$ be the bipartition of V and every vertex in V_1 has the same neighborhood degree $M=(M_1,M_1)$ and every vertex in V_2 has the same neighborhood degree $N=(N_1,N_2)$, where M and N are constants.

Theorem 3.15. *If* $G=(V,E)$ *is a strongly regular vague graph which is strong then,* $\overline{G}$ *is* $(k_1,k_2)-$*regular.*

Proof: Let $G=(V,E)$ be a strongly regular vague graph. Then by definition, G is

$(k_1, k_2)-$regular. Since G is strong, we have

$$\overline{t_B}(v_i v_j) = \begin{cases} 0 & \text{for all } v_i v_j \in E \\ \min(t_A(v_i), t_A(v_j)) & \text{for all } v_i v_j \notin E \end{cases}$$

$$\overline{f_B}(v_i v_j) = \begin{cases} 0 & \text{for all } v_i v_j \in E \\ \max(f_A(v_i), f_A(v_j)) & \text{for all } v_i v_j \notin E. \end{cases}$$

Now the degree of a vertex v_i in $\overline{G}$ is $d_{\overline{G}}(v_i) = (d_{\bar{t}}(v_i), d_{\bar{f}}(v_i))$, where

$$d_{\bar{t}}(v_i) = \sum_{v_i \neq v_j} \bar{t}_B(v_i v_j) = \sum_{v_i \neq v_j} t_{\overline{A}}(v_i) \wedge t_{\overline{A}}(v_j) = k_1,$$

for all $v_i \in V$ and

$$d_{\bar{f}}(v_i) = \sum_{v_i \neq v_j} \bar{f}_B(v_i v_j) = \sum_{v_i \neq v_j} f_{\overline{A}}(v_i) \wedge f_{\overline{A}}(v_j) = k_2,$$

for all $v_i \in V$. (Since G is strong)

Hence, $d_{\overline{G}}(v_i) = (k_1, k_2)$, for all $v_i \in V$. So, $\overline{G}$ is a (k_1, k_2)-regular vague graph.

Theorem 3.16. *Let $G = (V, E)$ be a strong vague graph. Then G is a strongly regular if and only if $\overline{G}$ is a strongly regular.*

Proof: Assume that $G = (V, E)$ is a strongly regular vague graph. Then by definition, we have G is $(k_1, k_2)-$regular and the adjacent vertices and the non-adjacent vertices have the same common neighborhood $\lambda = (\lambda_1, \lambda_2)$ and $\delta = (\delta_1, \delta_2)$, respectively. We have to prove that $\overline{G}$ is a strongly regular vague graph. If G is strongly regular vague graph which is strong then $\overline{G}$ is a $(k_1, k_2)-$regular vague graph by Theorem 3.17. Next, let S_1 and S_2 be the sets of all adjacent vertices and non-adjacent vertices of G. That is, $S_1 = \{v_i v_j \mid v_i v_j \in E\}$, where v_i and v_j have same common neighborhood $\lambda = (\lambda_1, \lambda_2)$ and $S_2 = \{v_i v_j \mid v_i v_j \notin E\}$, where v_i and v_j have same common neighborhood $\delta = (\delta_1, \delta_2)$. Then, $\overline{S_1} = \{v_i v_j \mid v_i v_j \in \overline{E}\}$, where v_i and v_j have same common neighborhood $\delta = (\delta_1, \delta_2)$ and $\overline{S_2} = \{v_i v_j \mid v_i v_j \notin \overline{E}\}$, where v_i and v_j have same common neighborhood $\lambda = (\lambda_1, \lambda_2)$. Which implies $\overline{G}$ is a strongly regular. Similarly we can prove the converse.

Theorem 3.17. *A strongly regular vague graph G is a biregular vague graph if the adjacent vertices have the same common neighborhood $\lambda = (\lambda_1, \lambda_2) \neq 0$ and the non-adjacent vertices have the same common neighborhood $\delta = (\delta_1, \delta_2) \neq 0$.*

Proof: Let $G = (V, E)$ be a strongly regular vague graph. Then we have

$d(v_i) = (k_1, k_2)$, for all $v_i \in V$. Assume that the adjacent vertices have the same common neighborhood $\delta = (\delta_1, \delta_2) \neq 0$. Let S be the sets of all non-adjacent vertices. That is $S = \{v_i v_j \mid v_i$ is not adjacent to $v_j, i \neq j, v_i, v_j \in V\}$. Now the vertex partition of G is $V_1 = \{v_i \mid v_i \in S\}$ and $V_2 = \{v_j \mid v_j \in S\}$. Then V_1 and V_2 have the same neighborhood degree, since G is a strongly regular. Hence, G is a biregular vague graph.

4. Conclusion

Graph theory is an extremely useful tool in solving the combinatorial problemsin different areas including geometry, algebra, number theory, topology, operations research, optimization, and computer science. The concept of vague sets is due toGau and Buehrer who studied the concept with the aim of interpreting the real lifeproblems in better way than the existing mechanisms such as Fuzzy sets.In this paper,novel properties of an edge regular vague graph are given. Likewise, new concepts such as strongly regular, edge regular, and biregular vague graphs are defined.

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