Progress in Nonlinear Dynamics and Chaos
Vol. 5, No. 1, 2017, 43-58
ISSN: 2321 – 9238 *(online)*
Published on 6 June 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/pindac.v5n1a6

Progress in Nonlinear Dynamics and Chaos

Searching Minimum Spanning Tree in a Type-2 Fuzzy Graph

Anita Pal **and** ***Saibal Majumder***

Departments of Mathematics
National Institute of Technology Durgapur
Durgapur-712209, West Bengal, India
E-mail: anita.buie@gmail.com; saibaltufts@gmail.com

Received 15 April 2017; accepted 2 June 2017

Abstract. Minimum spanning tree of a connected graph has a numerous real world applications. In this paper, we deal with an undirected connected graph whose edge weights are imprecise. We find its corresponding minimum spanning tree by using Borůvka's algorithm. The imprecise edge weights of the graph are expressed as type-2 fuzzy values. A numerical example is given where we compare the minimum spanning trees and their effective weights of two graphs, one with crisp edge weights and other with imprecise edge weights.

Keywords**:** Type-2 fuzzy set · Satisfaction function · Possibility · Necessity · Credibility, · Critical Value· Defuzzification · Minimum Spanning Tree ·Borůvka's algorithm.

Mathematics Subject Classifications (2010): 05C72, 05C85

1. Introduction

Minimum spanning tree is a fundamental problem in the area of graph theory which has many applications in different engineering domain. Usually, crisp values are generally used to represent the edge weights of a connected graph. But, in many cases of real world problems we find it difficult to define their exact edge weights. In those cases we can use type-1 fuzzy values instead of crisp value to express the imprecision of the weights. However, if these inexact or uncertain edge weights do vary under certain condition such as time then it becomes unsuitable to express such situation using type-1 fuzzy sets. In those cases we go for type-2 fuzzy sets as they are capable enough to handle such situations.

This paper deals with an undirected connected graph whose edge weights are type-2 fuzzy values. The weights are compared with the help of satisfaction function as proposed by Lee et al. **[7].** By applying Borůvka's algorithm we then find the minimum spanning tree (MST) and its effective weight by adding all the edge weights of the resultant MST applying Zadeh's extension principle. Finally we defuzzify the type-2 fuzzy weight of the resultant MST using the critical value reduction method **[10]** and compare the result with the equivalent crisp weight of the same MST.

The paper is organized as follows:
Section 2 depicts the preliminary concepts of Type-2 fuzzy sets, Regular Fuzzy Variable (RFV), Fuzzy Possibility Space, and addition of type-2 fuzzy sets using extension principle. Section 3 discusses the Type-2 fuzzy variable, their ranking using satisfaction function and the measures of a Fuzzy variable, CV- based reduction method of type-2 fuzzy set and finally its defuzzification using centroid method. Section 4 describes the proposed fuzzy approach of Borůvka's algorithm with a numerical example to compare its result with its crisp counterpart. Section 5 discuss the results of the previous section. And finally Section 6 concludes the paper.

2. Preliminary concepts

We recall some basic concepts of Type-2 fuzzy sets in this section, the concept of which, was introduced by Lotfi Zadeh (1975) as an extension of ordinary fuzzy set(also known as "type-1 fuzzy set").

2.1. Definition type-2 fuzzy set

A fuzzy set of type-2 **[1, 2, 3, 4,5]**, denoted by $\tilde{\tilde{A}}$ is characterized by a type-2 membership grade $\tilde{\mu}_{\tilde{\tilde{A}}}(x,k), where x \in X and k \in J_x \subseteq [0,1]$, where $0 \le \tilde{\mu}_{\tilde{\tilde{A}}}(x,k) \le 1$. Every type-2 fuzzy set has two types of membership grade namely primary membership grade and secondary membership grade. The secondary membership grade is the grade of its primary membership grade. According to the above definition of type-2 fuzzy set, *k* is the primary membership grade and $\tilde{\mu}_{\tilde{\tilde{A}}}(x,k)$ is the secondary membership grade of $\tilde{\tilde{A}}$.

In continuous domain, $\tilde{\tilde{A}}$ can be defined as $\tilde{\tilde{A}} = \int_{x\in X}\int_{k\in Jx} \tilde{\mu}_{\tilde{\tilde{A}}}(x,k)/(x,k), J_X \subseteq [0,1]$.

If X and J_x are both discrete then we defined $\tilde{\tilde{A}}$ as

$$\tilde{\tilde{A}} = \sum_{x\in X}\sum_{k\in J_x} \tilde{\mu}_{\tilde{\tilde{A}}}(x,k)/(x,k), J_X \subseteq [0,1]$$

From now onwards we consider only discrete type-2 fuzzy set since our work is confined to only on discrete domain of type-2 fuzzy set.

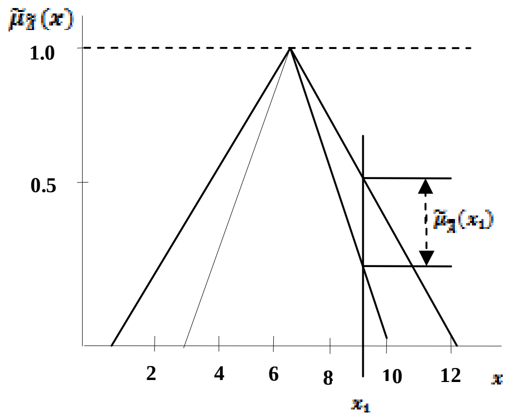

Figure 1: Membership grade of a type-2 fuzzy set

2.2. Definition convex type-2 fuzzy set [4]
Let
$J_x = \{k_1, k_2, k_3, \dots, k_n\}$ such that every $k_i \in$
$[0,1]$
and $k_1 < k_2 < \cdots < k_{i-1} < k_i < k_{i+1}$. A fuzzy grade $\sum_i \mu_{\tilde{A}}(k_i)/(k_i)$ in J_x is convex if for any integers i, l and q the following condition is satisfied:
$\mu_{\tilde{A}}(k_q) \geq \min\{\mu_{\tilde{A}}(k_i), \mu_{\tilde{A}}(k_l)\};\ \forall i \leq q \leq l$ Definition Ample Field: An ample field A[8]on U is a class of subsets on U that is closed under arbitrary union, intersection and complementation in U, where U is the universe of discourse.

2.3. Definition atom [8]: *Let A be an ample field on U, then an atom containing u is defined by $[u] \triangleq [u]_{\mathcal{A}} \triangleq \cap \{A|u \in A\}$. A is an atom in A if and only if $\emptyset \neq A \in$ A, and A is indivisible in A.*

2.4. Definition possibility space [9]: *If Pos: A→ $[0,1]$ be a set of functions on A then Pos is said to be the possibility measure if*

i. $Pos(\emptyset) = 0$ and
$Pos(U) = 1,\ U\ is\ the\ Universe\ of\ discourse$
ii. For any subclass of $\{A_i | i \in T\}$ of **A**, where T is the arbitrary index set $Pos(\cup_{i\in T} A_i) = \sup_{i\in T} Pos(A_i)$

The triplet (U, **A**, Pos) is the possibility space.

Now we know that a fuzzy variable is well defined as the function from Possibility space to a set of real numbers $\mathfrak{R}$ and the possibility measure of the fuzzy event $\{\zeta \in E\}, E \subset \mathfrak{R}$ is expressed as $Pos\{\zeta \in E\} = \sup_{e \in E} \mu_{\zeta}(e)$.

2.5. Definition regular fuzzy variable (RFV) [9, 10]: A regular fuzzy variable ζ under a possibility space(U, **A**, Pos) is defined as the measurable map from U to the space $[0,1]$, such that $\forall k \in [0,1],\ \{\tau \in U | \zeta(\tau) \leq k\} \in$ **A.**
A discrete RFV is expressed as
$\zeta \sim \begin{pmatrix} \delta_1 & \delta_2 \dots & \delta_n \\ \mu_1 & \mu_2 \dots & \mu_n \end{pmatrix} \delta_i \in [0,1] and\ 0 < \mu_i \leq$
$1\ \forall i, max_{i=1}^{r} \mu_i = 1$

2.6. Definition fuzzy possibility space [9]: Let $\widetilde{Pos}$: **A**→ $\mathfrak{R}([0,1])$ be a set of function defined on **A** such that $\{\widetilde{Pos}(A) |$ **A** $\ni Aatom\}$ is a collection of mutually independent RFVs. $\widetilde{Pos}$ is said to be the fuzzy possibility measure **[6]** if it satisfies the conditions: (i.) $\widetilde{Pos}(\emptyset) = \tilde{0}$ (ii.) For any subclass $\{A_i | i \in T\}$ of **A** (finite, countable or uncountable) ,T is the arbitrary index set.
$\widetilde{Pos}(\cup_{i\in T} A_i) = \sup_{i \in T} \widetilde{Pos}(A_i)$. The triplet (U, **A**, $\widetilde{Pos}$) is called fuzzy possibility space.

2.7. Addition of type-2 fuzzy values: Addition of type-2 fuzzy numbers is performed by applying Zadeh's extension principle.

Extension principle: Let X be a Cartesian product of the universe of discourse $X = X_1 \times X_2 \times \ldots \times X_m$ and $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m$ be the m fuzzy sets in the universal set. Cartesian product of the fuzzy sets $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m$ forms a fuzzy universal set $\mathcal{F}(X) = \tilde{A}_1 \times \tilde{A}_2 \times \ldots \times \tilde{A}_m$ whose membership grade is defined as $\mu_{\tilde{A}_1 \times \tilde{A}_2 \times \ldots \times \tilde{A}_m}(x_1 \times x_2 \times \ldots \times x_m) = min[\mu_{\tilde{A}_1}(x_1), \mu_{\tilde{A}_1}(x_2), \ldots, \mu_{\tilde{A}_m}(x_m)]$.

Let g be a mapping function from universal set X to that of another universal set W,i.e. $g(x_1 \times x_2 \times \ldots \times x_m) : X \to W$. Then the fuzzy set B in W can be obtained by the function g and the fuzzy set $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_m$ as follows:

$$\mu_{\tilde{B}}(y) = \begin{cases} 0 & ; if\ g^{-1}(y) = \emptyset \\ max_{y=g(x_1 \times x_2 \times \ldots \times x_m)}[min\{\mu_{\tilde{A}_1}(x_1), \mu_{\tilde{A}_1}(x_2), \ldots, \mu_{\tilde{A}_m}(x_m)\}] ; & otherwise \end{cases}$$

where $g^{-1}(y)$ is the inverse image of y under g.

Union and intersection of type-2 fuzzy set uses max and min operators respectively, the detailed definition of which are given in **[4]**.

Addition of discrete type-2 fuzzy Costs: Let us consider two type-2 fuzzy costs associated with the adjacent edges of a graph which are expressed as $\tilde{\tilde{A}} = \sum_{x \in X} \tilde{\mu}_{\tilde{\tilde{A}}}(x)/x$ and $\tilde{\tilde{B}} = \sum_{y \in Y} \tilde{\mu}_{\tilde{\tilde{B}}}(y)/y$ such that $\tilde{\mu}_{\tilde{\tilde{A}}}(x) = \sum_{s \in J_x} f(s)/s$ and $\tilde{\mu}_{\tilde{\tilde{B}}}(y) = \sum_{t \in J_y} g(t)/t$ where X, Y are the universe of discourse and $J_x, J_y \subseteq [0,1]$.Then the membership grade of $\tilde{\tilde{A}} + \tilde{\tilde{B}}$ is defined as

$\vee_{w=x+y}(\tilde{\mu}_{\tilde{\tilde{A}}}(x) \wedge \tilde{\mu}_{\tilde{\tilde{B}}}(y)) = \vee_{w=x+y}[\sum_{s_i \in J_x, t_j \in J_y}\{f(s_i) \wedge g(t_j)\}/\{s_i \wedge t_j\}]$

where J_x and $J_y \subseteq [0,1]$.

Example: Let us consider two type-2 fuzzy values below:

$\tilde{\tilde{A}} = (\frac{0.3}{0.2} + \frac{0.9}{1.0})/2 + (\frac{0.6}{0.8} + \frac{0.7}{0.9})/3$ and $\tilde{\tilde{B}} = (\frac{0.6}{0.5} + \frac{0.8}{0.7})/4 + (\frac{0.3}{0.6} + \frac{0.8}{0.9})/6$

Now, $\tilde{\tilde{A}} + \tilde{\tilde{B}} = (\tilde{\mu}_{\tilde{\tilde{A}}}(2) \wedge \tilde{\mu}_{\tilde{\tilde{B}}}(4))/6 + (\tilde{\mu}_{\tilde{\tilde{A}}}(3) \wedge \tilde{\mu}_{\tilde{\tilde{B}}}(4))/7 + (\tilde{\mu}_{\tilde{\tilde{A}}}(2) \wedge \tilde{\mu}_{\tilde{\tilde{B}}}(6))/8 + (\tilde{\mu}_{\tilde{\tilde{A}}}(3) \wedge \tilde{\mu}_{\tilde{\tilde{B}}}(6))/9$

$= ({}^{0.3}/_{0.2} + {}^{0.6}/_{0.5} + {}^{0.8}/_{0.7})/6 + ({}^{0.6}/_{0.5} + {}^{0.7}/_{0.7})/7 + ({}^{0.3}/_{0.2} + {}^{0.3}/_{0.6} + {}^{0.8}/_{0.9})/8 + ({}^{0.3}/_{0.6} + {}^{0.6}/_{0.8} + {}^{0.7}/_{0.9})/9$

2.8. Definition centroid of a type-1 fuzzy set: Centroid is also known as *centre of gravity* or *centre of area* which is used to obtain the centre of area (x^*)defined by $x^* = \frac{\int x \times \mu(x)\, dx}{\int \mu(x) dx}$ for continuous case and $x^* = \frac{\sum_{i=1}^{n} \sum x \times \mu(x)}{\sum_{i=1}^{n} \sum \mu(x)}$ for discrete case.

3. Type-2 Fuzzy variable

If (U, **A**, $\widetilde{Pos}$) is a fuzzy possibility space then a type-2 fuzzy variable $\tilde{\xi}$ is defined as a map from U to $\Re$ such that for any $r \in \Re$, $\{\tau \in U | \tilde{\xi}(\tau) \leq r\} \in$ **A.**

3.1. Ranking of type-2 fuzzy values

To compare the type-2 fuzzy values we use the proposed comparison model by Lee et al. **[7]**. This proposed model generally deals with the concept of *relative possibility of* *appearance.* To understand this concept let us consider two discrete ordinary fuzzy values $\tilde{A} = \frac{0.4}{2} + \frac{0.8}{6}$ and $\tilde{B} = \frac{0.3}{5} + \frac{0.7}{8}$. If we consider the *actual value* of a fuzzy variable $\tilde{X}$ then *actual value*, *av*($\tilde{X}$) can have a number of actual values which are implied by the possibility distribution of $\tilde{X}$. If we consider our example then, $av(\tilde{A})$ is either 2 or $6$with possibility 0.4 and 0.8 respectively. The detail discussion of *actual value of* a fuzzy variable is given in next sub section. Even though we never know whether the$av(\tilde{A})$is 2 or 6 but we can surely conclude that the possibility of$av(\tilde{A})$, $Pos(av(\tilde{A}) = 6) = 0.8$ is greater than the possibility of $Pos(av(\tilde{A}) = 2) = 0.4$. Similar assumption can be made for fuzzy variable$\tilde{B}$.If we consider $av(\tilde{A}) = 6$ and $av(\tilde{B}) = 5$ then $\tilde{A}$ will be greater than $\tilde{B}$ but if $av(\tilde{A}) = 2$ and $av(\tilde{B}) = 8$ then $\tilde{A}$ will be smaller than $\tilde{B}$ so we cannot conclude whether $\tilde{A} > \tilde{B}$ or$\tilde{A} < \tilde{B}$, but we can definitely find the possibility of $Pos(\tilde{A} > \tilde{B})$ or $Pos(\tilde{A} > \tilde{B})$ using the membership grade based on the above assumption. If we consider the relative possibility of a situation $\mathcal{R}$, $s(\mathcal{R})$ represents the *satisfaction degree*, i.e. to say if a proposition ($\tilde{A} > \tilde{B}$) exists then its satisfaction degree will be the degree to which the proposition is true. In this paper we consider the satisfaction degree only for those propositions where comparison operators$(<, >, =, \leq, \geq)$ are used. The value of satisfaction degree always lies between $[0,1]$. We estimate the measure of this satisfaction degree for a given proposition which is termed as the *satisfaction function,* the formal definition of which is stated in section 3.3.

3.2. Actual value of a fuzzy variable

The actual value, $av(\tilde{A})$of a fuzzy variable $\tilde{A}$is the exact value *v* which is implied by its possibility distribution of $\tilde{A}$ and it is expressed as $av(\tilde{A}) = v$.

Let, the fuzzy value of the height of a 6.2 feet tall person is expressed as 'about 6' feet because of observation imprecision. In this case, we can say 6.2 feet is the actual value so far the height of the person is concerned and 'about 6' feet is considered to be fuzzy value whose possibility distribution implies the possible location of its actual value i.e. $av(about\ 6) = 6.2$. We cannot isolate the actual value from a fuzzy value since the fuzzy value informs about the possible location of an actual value.

Possibility of actual value of type-2 fuzzy set

The possibility distribution of an actual value a_i of a type-2 fuzzy set $\tilde{\tilde{A}}$**[7]**is expressed as$(Poss\left(av\left(\tilde{\tilde{A}}\right) = a_i\right) = \sum_s s \times \tau_{\mu_{\tilde{\tilde{A}}}(a_i)}(s)$where s and $\tau_{\mu_{\tilde{\tilde{A}}}(a_i)}(s)$are the respective primary and secondary membership grade of a_i in $\tilde{\tilde{A}}$.

3.3. Satisfaction function

As it has already discussed that the possibility distribution of a fuzzy value does not give the possible location rather than the accurate location of actual value so the satisfaction degree can be expressed as the ratio of the combination of the actual values that satisfies the mathematical comparison relation to the exhaustive combination of the actual values. And this measure is said to be the satisfaction function.

Satisfaction function of type-1 fuzzy set [7]: The satisfaction function for a satisfaction degree $s(\tilde{A} * \tilde{B})$ is expressed as $S(\tilde{A} * \tilde{B})$ and is mathematically defined as

$$S(\tilde{A} * \tilde{B}) = \frac{\sum_{(x_k, y_j) \in \delta(x_k * y_j)} Poss(av(\tilde{A}) = x_k \wedge av(\tilde{B}) = y_j)}{\sum_{(x_k, y_j)} Poss(av(\tilde{A}) = x_k \wedge av(\tilde{B}) = y_j)};$$
$$x_k \in Ac_{\tilde{A}} \text{ and } y_j \in Ac_{\tilde{B}}$$

where $Ac_{\tilde{A}}$ and $Ac_{\tilde{B}}$ are the set of all actual values of $\tilde{A}$ and $\tilde{B}$ respectively. $*$ is the arithmetic comparison relation. And $\delta(x_k * y_j)$ is the set of all pair of actual values of $\tilde{A}$ and $\tilde{B}$ that satisfies the relation $(x_k * y_j)$. $\delta(x_k * y_j)$ is known as the *satisfaction set***.**

Satisfaction function of type-2 fuzzy set [7]: For two discrete type-2 fuzzy set $\tilde{\tilde{A}} = \sum_{x \in X} \tilde{\mu}_{\tilde{\tilde{A}}}(x)/x$ and $\tilde{\tilde{B}} = \sum_{y \in Y} \tilde{\mu}_{\tilde{\tilde{B}}}(y)/y$ such that $\tilde{\mu}_{\tilde{\tilde{A}}}(x)/x = \sum_{s \in J_x} f(s)/s$ and $\tilde{\mu}_{\tilde{\tilde{B}}}(y)/y = \sum_{t \in J_y} g(t)/t$ where X, Y is the universe of discourse and $J_x, J_y \subseteq [0,1]$. Now, the satisfaction degree for $av\left(\tilde{\tilde{A}}\right) = x$ and $av\left(\tilde{\tilde{B}}\right) = y$ is defined as

$$s\left(av\left(\tilde{\tilde{A}}\right) = x \wedge av\left(\tilde{\tilde{B}}\right) = y\right) = \sum_{s \in J_x} \sum_{t \in J_y} (s \times f(s) \times t \times g(t))$$

We now define the satisfaction function for $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ as

$$S\left(\tilde{\tilde{A}} > \tilde{\tilde{B}}\right) = \frac{\sum_{(x_k, y_j) \in \psi(\tilde{\tilde{A}} > \tilde{\tilde{B}})} s\left(av\left(\tilde{\tilde{A}}\right) = x_k \wedge av\left(\tilde{\tilde{B}}\right) = y_j\right)}{\sum_{(x_k, y_j) \in \psi(R)} s\left(av\left(\tilde{\tilde{A}}\right) = x_k \wedge av\left(\tilde{\tilde{B}}\right) = y_j\right)}$$

$$S\left(\tilde{\tilde{A}} = \tilde{\tilde{B}}\right) = \frac{\sum_{(x_k, y_j) \in \psi(\tilde{\tilde{A}} = \tilde{\tilde{B}})} s\left(av\left(\tilde{\tilde{A}}\right) = x_k \wedge av\left(\tilde{\tilde{B}}\right) = y_j\right)}{\sum_{(x_k, y_j) \in \psi(R)} s\left(av\left(\tilde{\tilde{A}}\right) = x_k \wedge av\left(\tilde{\tilde{B}}\right) = y_j\right)}$$

$$S\left(\tilde{\tilde{A}} < \tilde{\tilde{B}}\right) = \frac{\sum_{(x_k, y_j) \in \psi(\tilde{\tilde{A}} < \tilde{\tilde{B}})} s\left(av\left(\tilde{\tilde{A}}\right) = x_k \wedge av\left(\tilde{\tilde{B}}\right) = y_j\right)}{\sum_{(x_k, y_j) \in \psi(R)} s\left(av\left(\tilde{\tilde{A}}\right) = x_k \wedge av\left(\tilde{\tilde{B}}\right) = y_j\right)}$$

where, $\psi(\tilde{\tilde{A}} > \tilde{\tilde{B}})$ is the *satisfaction set* consisting of all possible pairs of (x_k, y_j) which satisfies $av(\tilde{\tilde{A}}) > av(\tilde{\tilde{B}})$; $\psi(\tilde{\tilde{A}} = \tilde{\tilde{B}})$ is the *satisfaction set* consisting of all possible pairs of (x_k, y_j) which satisfies $av\left(\tilde{\tilde{A}}\right) = av(\tilde{\tilde{B}})$; $\psi(\tilde{\tilde{A}} < \tilde{\tilde{B}})$ is the *satisfaction set* consisting of all possible pairs of (x_k, y_j) which satisfies $av\left(\tilde{\tilde{A}}\right) < av(\tilde{\tilde{B}})$ and

$$S\left(\tilde{\tilde{A}} > \tilde{\tilde{B}}\right) + S\left(\tilde{\tilde{A}} = \tilde{\tilde{B}}\right) + S\left(\tilde{\tilde{A}} < \tilde{\tilde{B}}\right) = 1.$$

3.4. Measures of a fuzzy event [10, 11]

For a fuzzy variable ζ having a membership grade $\mu_{\zeta}(x)$ and for any set $G \subseteq \Re$ the three measures of ζ namely Possibility, Necessity and Credibility measures**[7]** are expressed in below:

I. **Possibility measure:** For a fuzzy event$\{\zeta \in G\}$the possibility measure of which is defined as $Pos\{\zeta \in G\} = \max_{x \in G} \mu_{\zeta}(x)$

II. **Necessity measure:** For a fuzzy event$\{\zeta \in G\}$the necessity measure of which is defined as $Nec\{\zeta \in G\} = 1 - \max_{x \in G^c} \mu_{\zeta}(x)$

3.5. Credibility measure: For a fuzzy event$\{\zeta \in G\}$the credibility measure of which is defined as

$$Cr\{\zeta \in G\} = \frac{1}{2}\left[Pos\{\zeta \in G\} + Nec\{\zeta \in G\}\right]$$

Critical values of RFVs: In this section three different types of Critical Values (CV) as proposed by Qin et al. **[10]** are considered.

I. For a RFV ζ, the optimistic CV of ζ , $\overline{CV}[\zeta]$ is expressed as

$\overline{CV}[\zeta] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Pos}\{\zeta \geq \alpha\}]$.

II. The pessimistic CV of ζ , $\underline{CV}[\zeta]$ is expressed as , $\underline{CV}[\zeta] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Nec}\{\zeta \geq \alpha\}]$.

III. The CV of ζ, $CV[\zeta]$, is expressed as$CV[\zeta] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Cr}\{\zeta \geq \alpha\}]$.

Example: Let ζ be a discrete RFV, where $\zeta \sim \begin{pmatrix} 0.4 & 0.6 & 0.8 \\ 0.85 & 1.0 & 0.47 \end{pmatrix}$ then,

$$Pos(\zeta \geq \alpha) = \begin{cases} 1.0 & ;\alpha \leq 0.6 \\ 0.47 & ;0.6 < \alpha \leq 0.8 \\ 0.0 & ;0.8 < \alpha \leq 1.0 \end{cases}$$

$$Nec(\zeta \geq \alpha) = \begin{cases} 1.0 & ;\alpha \leq 0.4 \\ 0.15 & ;0.4 < \alpha \leq 0.6 \\ 0.0 & ;0.6 < \alpha \leq 1.0 \end{cases}$$

$$Cr(\zeta \geq \alpha) = \begin{cases} 1.0 & ;\alpha \leq 0.4 \\ 0.575 & ;0.4 < \alpha \leq 0.6 \\ 0.235 & ;0.6 < \alpha \leq\leq 0.8 \\ 0.0 & ;0.8 < \alpha \leq 1.0 \end{cases}$$

Now, $\overline{CV}[\zeta] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Pos}\{\zeta \geq \alpha\}]$

$$= \sup_{\alpha \in [0,0.6]} (\alpha \bigwedge \wedge 1.0) \vee \sup_{\alpha \in (0.6,0.8]} (\alpha \bigwedge \wedge 0.47) \vee \sup_{\alpha \in (0.8,1.0]} (\alpha \wedge 0.0) = (0.6 \vee 0.47 \vee 0.0) = 0.6$$

$$\underline{CV}[\zeta] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Nec}\{\zeta \geq \alpha\}]$$

$$= \sup_{\alpha \in [0,0.4]} (\alpha \wedge 1.0) \vee \sup_{\alpha \in (0.4,0.6]} (\alpha \wedge 0.15) \vee \sup_{\alpha \in (0.6,1.0]} (\alpha \wedge 0.0) = (0.4 \vee 0.15 \vee 0.0) = 0.4$$

$$CV[\xi] = \sup_{\alpha\in[0,1]} [\alpha\wedge Cr\{\xi \geq \alpha\}] = \sup_{\alpha\in[0,0.4]}(\alpha \bigwedge \quad \wedge 1.0)\vee \sup_{\alpha\in(0.4,0.6]}(\alpha \bigwedge \quad \wedge 0.575)$$
$$\vee \sup_{\alpha\in(0.6,0.8]}(\alpha\wedge 0.235)\vee \sup_{\alpha\in(0.8,1.0]}(\alpha\wedge 0.0) = (0.4\vee 0.575\vee 0.235\vee 0.0) = 0.575$$

2.5. CV-based reduction method

As proposed by Qin et al. **[10]** CV-based reduction method for a type -2 fuzzy variable, $\tilde{\xi}$ is defined as follows:

For a fuzzy possibility space (U, **A**, $\widetilde{Pos}$), the secondary membership grade $\tilde{\mu}_{\tilde{\xi}}(x)$ for a type -2 fuzzy variable, $\tilde{\xi}$ can be reduced to a representing value for RFV $\tilde{\mu}_{\tilde{\xi}}(x)$. These representing values are the CV's of $\widetilde{Pos}\{\tau \in U | \tilde{\xi}(\tau) = x\}$.

3.6. Defuzzification of type-2 fuzzy set

Once we apply CV-based reduction method on a type-2 fuzzy variable the resultant fuzzy variable is reduced to type-1 fuzzy variable which in turn is crispified by Appling the centroid method.

4. Fuzzy minimal spanning tree

The most fundamental concept in classical graph theory is to find the minimal spanning tree of an undirected graph since this concept has many application in engineering fields. A spanning tree of a graph $G$is an acyclic sub-graph of Gthat includes every vertex of $G$and is connected; every spanning tree has exactly $n-1$edges where n is the number of vertices of the graph. A minimum spanning tree (MST) is a spanning tree of minimum weight which is defined to be the sum of the weights of all its edges. Our problem is to find the MST of G. The deterministic case of this concept considers the exact weights or cost associated with the edges of the graph, but in practical case this may be a serious restriction as cost of the edges may well be imprecise or even the vertex set and/or the edge set of a graphical structure may also be imprecise. The simplest way to handle these imprecision is to express the graph as a fuzzy graph.

There are several ways of classifying a fuzzy graph which are:

- Type I— Fuzzy vertex and fuzzy edge sets.
- Type II — Crisp vertex set and fuzzy edge set.
- Type III— Fuzzy vertex set and crisp edge set.
- Type IV — Crisp graph with fuzzy weights

In our case, we consider fuzzy graph of Type IV only, the cost of which are expressed as the type-2 fuzzy value and we use Borůvka's algorithm to find the minimum spanning tree of a crisp graph and its fuzzy version and finally compare the length of the spanning tree for both the fuzzy graph and its crisp version.

Type-2 Fuzzy weighted graph

We can express the inexact costs of the graph by fuzzy weights. But there are certain cases where type-1 fuzzy weights become inefficient. If the weights of an edge of a graph may vary with time, or if we consider the cost of the edges of a graph according to the opinion of different experts, it may happen that two experts opinion about the weight of an edge differ. A type-2 fuzzy set outperforms type-1 fuzzy sets as far as modeling of these scenarios is concerned. In this paper, we have used two operators namely comparison and addition, that are both necessary to compare the costs associated with the edges of the graph and to calculate the effective of the minimum spanning tree. We will then crispify the resultant weight of the MST of the fuzzy graph and compare the same withits crisp counterpart. For simplicity, in our proposed problem only the discrete normal or sub-normal convex type-2 fuzzy values can be used.

4.1. Borůvka's algorithm [13]

One of the greedy strategies for finding the minimum spanning tree of a graph is the Borůvka's algorithm which runs in $O(mlogn)$ time; m is the number of edges and n is the number of vertices of the graph. The basic idea in Borůvka's algorithm is to contract simultaneously the minimum weight edges incident on each of the vertices in a graph. This algorithm is suitable for parallel computations since the algorithm builds the MST uniformly throughout the graph. We have used the following notations in the rest of the paper:

- A graph G, $G = (V_G, E_G)$ where V_G is the set of the vertices and E_G is the set of the edges.
- v_k is the k^{th} vertex of V_G
- e_{kj} is the edge connecting the vertices v_k and v_j
- M is the minimum spanning tree of the graph G.
- M' is the minimum spanning tree of G' where $G' \subseteq G$.
- $c(e_{kj})$ is the deterministic cost associated with the edge of G connecting vertices v_k and v_j
- $\tilde{\tilde{c}}(e_{kj})$ is the type-2 fuzzy cost associated with the edge e_{kj} of G
- $c(M)$ is the deterministic cost of M
- $\tilde{\tilde{c}}(M)$ is the type-2 fuzzy cost of M

An implementation of Borůvka's algorithm is as follows:

a. The edges to be contracted are marked first.
b. Determine the connected components formed by the marked edges.
c. Each of the connected components is replaced by a single vertex.
d. Eliminate the self loops and the parallel edges created by these contractions.
e. If G' is the resultant graph formed from the original G after a Borůvka's algorithm. The MST of G will be the union of the edges marked for contraction during the same step with the edges of MST of G'.

In our numerical example we discuss the crisp and our proposed fuzzy approach of Borůvka's algorithm based on which we will compare the effective cost of the MST's of a fuzzy graph and its crisp version in the next section.

4.2. Proposed Fuzzy approach of Borůvka's algorithm

Input: *For any graph* $G=(V_G,E_G)$, $G'=(V_{G'},E_{G'})$ *where* $V_{G'}\subseteq V_G$ *and* $E_{G'}\subseteq E_G$

Output: *M, resultant MST*

Step 1: $M \leftarrow EmptyGraph$

Step 2: $\mathbf{foreach}\, v_k\, in\, V_G\, do$

Step 3: *Calculate the satisfaction function between a possible pair of edges incident on vertex* v_k *i.e.* $(\tilde{\tilde{c}}(e_{kj}) * \tilde{\tilde{c}}(e_{kl}))$, *where* '$*$' *is the arithmetic comparison relation i.e.* $('<','>'$ *and* $'=')$.

The cost $\tilde{\tilde{c}}(e_{kj})$, *is selected as minimum between the two,* $\tilde{\tilde{c}}(e_{kj})$ *and* $\tilde{\tilde{c}}(e_{kl})$ *if* $S(\tilde{\tilde{c}}(e_{kj}) < \tilde{\tilde{c}}(e_{kl}))$ *has the largest value among the three satisfaction functions:*

$\{S(\tilde{\tilde{c}}(e_{kj}) < \tilde{\tilde{c}}(e_{kl})),\ S(\tilde{\tilde{c}}(e_{kj}) = \tilde{\tilde{c}}(e_{kl}))\ and\ S(\tilde{\tilde{c}}(e_{kj}) > \tilde{\tilde{c}}(e_{kl}))\}$

Repeat ***Step-3*** *among every pair of edges* (e_{kj}, e_{kl}) *incident on vertex* v_k *and finally select the edge say,* e_{kj} *having the minimum cost.*

Step 4: $M \leftarrow M + \{e_{kj}\}$

Step 5: $\mathbf{endfor}$

Step 6: $G' \leftarrow G\, with\, all\, edges\, in\, M\, contracted$

Step 7: M' ←*Recursively compute the* **MST** of G'

Step 8: *return* $M \leftarrow M + M'$

5. Numerical example

In this section we find the MSTs of a crisp and a fuzzy graph and compare the deterministic cost, $c(M)$ of M with that of defuzzified type-2 fuzzy cost of MST, $\tilde{\tilde{c}}(M)$ of the fuzzy graph and finally calculate the percentage difference between them.

Let us now consider the crisp graph and we apply the crisp version of the Borůvka's algorithm to get its corresponding MST. We describe the same in detail as follows:

We consider a graph G shown in Fig.2:

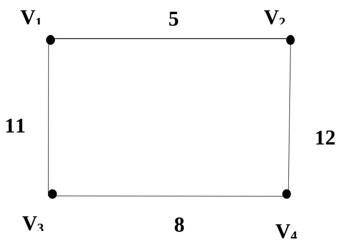

Figure 2: A graph G

After 1^{st} iteration the highlighted edges are added in M which is depicted in Fig. 3

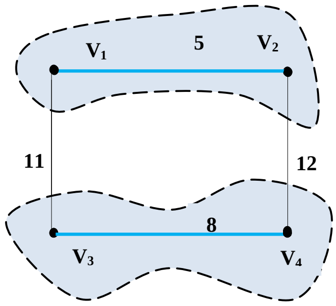

Figure 3: Edges e_{12} and e_{34} are selected after 1^{st} iteration of Borůvka's algorithm

After 2^{nd} iteration we get the required MST which is shown in Fig. 4:

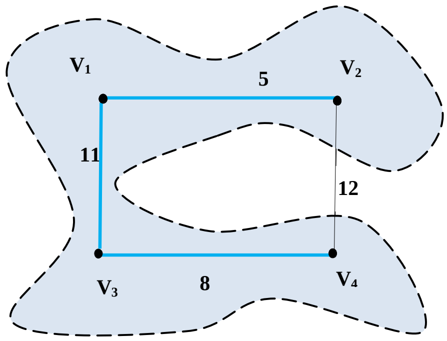

Figure 4: Edges e_{12}, e_{13} and e_{34} are selected after 2^{nd} iteration of Borůvka's algorithm

We observe that we get the MST after 2^{nd} iteration which is described as $M = \{e_{12}, e_{13} \text{ and } e_{34}\}$ and $c(M) = c(e_{12}) + c(e_{13}) + c(e_{34}) = 24$

Now we consider the fuzzy approach of Borůvka's algorithm. In this case, we list out the cost associated with the edges of the graph depicted in Fig. 5.

$$\tilde{c}(e_{12}) = \{(\frac{0.25}{0.2} + \frac{0.63}{0.4} + \frac{0.49}{0.5})/4 + (\frac{0.28}{0.3} + \frac{0.69}{0.5} + \frac{0.57}{0.6})/8\}$$

$$\tilde{\tilde{c}}(e_{13}) = \{(\frac{0.42}{0.4} + \frac{0.65}{0.6} + \frac{0.62}{0.8})/8 + (\frac{0.29}{0.3} + \frac{0.47}{0.4} + \frac{0.44}{0.5})/14\}$$

$$\tilde{\tilde{c}}(e_{34}) = \{(\frac{0.72}{0.62} + \frac{1.0}{0.8} + \frac{0.8}{0.9})/6 + (\frac{0.60}{0.76} + \frac{0.61}{0.85} + \frac{0.59}{1.0})/9\}$$

$$\tilde{\tilde{c}}(e_{24}) = \{(\frac{0.6}{0.2} + \frac{0.9}{0.5} + \frac{0.7}{0.8})/9 + (\frac{0.23}{0.5} + \frac{0.46}{0.7} + \frac{0.32}{0.95})/15\}$$

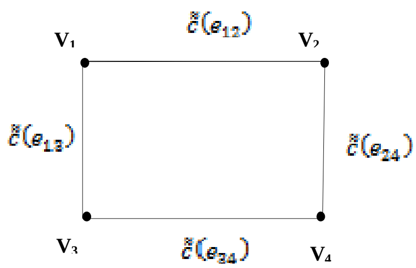

Figure 5: A graph G whose edge costs are type-2 fuzzy values

Now while applying the fuzzy approach of Borůvka's algorithm we calculate the satisfaction functions for every pair of the edges incident on each of the vertices of G. We give a sample calculation of the satisfaction function by considering the vertex v_1.

We calculate the satisfaction function of the remaining pair of the edges of G and tabulated them below:

Table 1: Satisfaction function for different pair of fuzzy costs

Vertex v_i	**Considering** $(\tilde{\tilde{c}}(e_{kj}),\tilde{\tilde{c}}(e_{kl}))$	**Satisfaction Set** $\psi\left(\tilde{\tilde{c}}(e_{kj}) * \tilde{\tilde{c}}(e_{kl})\right)$	**Satisfaction Function** $S\left(\tilde{\tilde{c}}(e_{kj}) * \tilde{\tilde{c}}(e_{kl})\right)$
v_1	$(\tilde{\tilde{c}}(e_{12}),\tilde{\tilde{c}}(e_{13}))$	$\psi\left(\tilde{\tilde{c}}(e_{12}) < \tilde{\tilde{c}}(e_{13})\right) = \{(4,8),(4,14),(6,8),(6,14)\}$	$S\left(\tilde{\tilde{c}}(e_{12}) < \tilde{\tilde{c}}(e_{13})\right) = 0.6018$
		$\psi\left(\tilde{\tilde{c}}(e_{12}) = \tilde{\tilde{c}}(e_{13})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{12}) = \tilde{\tilde{c}}(e_{13})\right) = 0.3981$
		$\psi\left(\tilde{\tilde{c}}(e_{12}) > \tilde{\tilde{c}}(e_{13})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{12}) > \tilde{\tilde{c}}(e_{13})\right) = 0.0$
v_2	$(\tilde{\tilde{c}}(e_{21}),\tilde{\tilde{c}}(e_{24}))$	$\psi\left(\tilde{\tilde{c}}(e_{21}) < \tilde{\tilde{c}}(e_{24})\right) = \{(4,9),(4,15),(8,9),$	$S\left(\tilde{\tilde{c}}(e_{21}) < \tilde{\tilde{c}}(e_{24})\right) = 1.0$
		$\psi\left(\tilde{\tilde{c}}(e_{21}) = \tilde{\tilde{c}}(e_{24})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{21}) = \tilde{\tilde{c}}(e_{24})\right) = 0.0$
		$\psi\left(\tilde{\tilde{c}}(e_{21}) > \tilde{\tilde{c}}(e_{24})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{21}) > \tilde{\tilde{c}}(e_{24})\right) = 0.0$
v_3	$(\tilde{\tilde{c}}(e_{34}),\tilde{\tilde{c}}(e_{31}))$	$\psi\left(\tilde{\tilde{c}}(e_{34}) < \tilde{\tilde{c}}(e_{31})\right) = \{(6,8),(6,14),(9,14)$	$S\left(\tilde{\tilde{c}}(e_{34}) < \tilde{\tilde{c}}(e_{31})\right) = 0.6985$
		$\psi\left(\tilde{\tilde{c}}(e_{34}) = \tilde{\tilde{c}}(e_{31})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{34}) = \tilde{\tilde{c}}(e_{31})\right) = 0.0$
		$\psi\left(\tilde{\tilde{c}}(e_{34}) > \tilde{\tilde{c}}(e_{31})\right) = \{(9,8)\}$	$S\left(\tilde{\tilde{c}}(e_{34}) > \tilde{\tilde{c}}(e_{31})\right) = 0.3014$
v_4	$(\tilde{\tilde{c}}(e_{43}),\tilde{\tilde{c}}(e_{42}))$	$\psi\left(\tilde{\tilde{c}}(e_{43}) < \tilde{\tilde{c}}(e_{42})\right) = \{(6,9),(6,15),(9,15)$	$S\left(\tilde{\tilde{c}}(e_{43}) < \tilde{\tilde{c}}(e_{42})\right) = 0.7323$
		$\psi\left(\tilde{\tilde{c}}(e_{43}) = \tilde{\tilde{c}}(e_{42})\right) = \{(9,9)\}$	$S\left(\tilde{\tilde{c}}(e_{43}) = \tilde{\tilde{c}}(e_{42})\right) = 0.2676$
		$\psi\left(\tilde{\tilde{c}}(e_{43}) > \tilde{\tilde{c}}(e_{42})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{43}) > \tilde{\tilde{c}}(e_{42})\right) = 0.0$
Iteration-2 In the second and final iteration, the minimum weight edge out of each of the two remaining components is added.			
	$(\tilde{\tilde{c}}(e_{13}),\tilde{\tilde{c}}(e_{24}))$	$\psi\left(\tilde{\tilde{c}}(e_{13}) < \tilde{\tilde{c}}(e_{24})\right) = \{(8,9),(8,15),(14,$	$S\left(\tilde{\tilde{c}}(e_{13}) < \tilde{\tilde{c}}(e_{24})\right) = 0.8069$
		$\psi\left(\tilde{\tilde{c}}(e_{13}) = \tilde{\tilde{c}}(e_{24})\right) = \{\emptyset\}$	$S\left(\tilde{\tilde{c}}(e_{13}) = \tilde{\tilde{c}}(e_{24})\right) = 0.0$
		$\psi\left(\tilde{\tilde{c}}(e_{13}) > \tilde{\tilde{c}}(e_{24})\right) = \{(14,9)\}$	$S\left(\tilde{\tilde{c}}(e_{13}) > \tilde{\tilde{c}}(e_{24})\right) = 0.1930$

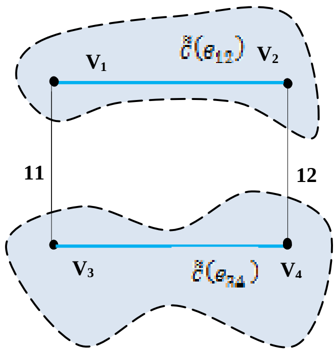

Figure 6: Selected **e**dges after 1^{st} iteration of Borůvka's algorithm (fuzzy approach)

After 1^{st} iteration of Borůvka's algorithm (fuzzy approach) edges, e_{12} and e_{34} having costs $\tilde{\tilde{c}}(e_{12})$ and $\tilde{\tilde{c}}(e_{34})$ respectively are added to M as shown in Fig-6.

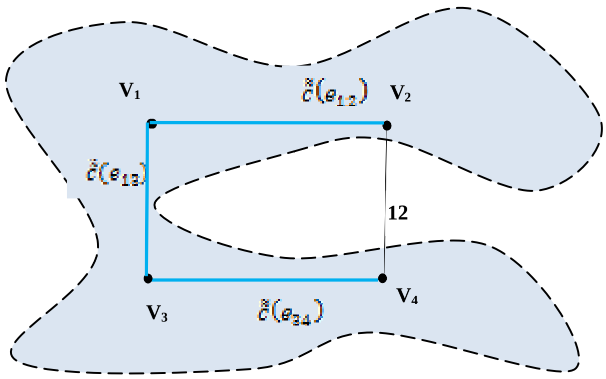

Figure 7: Edges e_{12}, e_{13} and e_{34} are selected after 2^{nd} iteration of Borůvka's algorithm (fuzzy approach)

After 2^{nd} iteration of Borůvka's algorithm (fuzzy approach) edges, e_{12}, e_{13} and e_{34} having costs $\tilde{\tilde{c}}(e_{12})$, $\tilde{\tilde{c}}(e_{13})$ and $\tilde{\tilde{c}}(e_{34})$ respectively are added to M and it becomes a MST for the fuzzy graph under consideration as depicted in Fig. 7.

We observe that by executing the fuzzy approach of Borůvka's algorithm recursively minimal spanning tree M exactly matches with that of its crisp version i.e. $M = \{e_{12}, e_{13} \text{ and } e_{34}\}$ and

$$\begin{aligned}\tilde{\tilde{c}}(M) = {} & \tilde{\tilde{c}}(e_{12}) + \tilde{\tilde{c}}(e_{13}) + \tilde{\tilde{c}}(e_{34}) \\ = {} & \{(0.25/0.2 + 0.63/0.4 + 0.49/0.5)/18 + (0.25/0.2 + 0.61/0.4 + 0.49/0.5)/21 \\ & + (0.28/0.3 + 0.42/0.4 + 0.65/0.5 + 0.57/0.6)/22 \\ & + (0.25/0.2 + 0.29/0.3 + 0.47/0.4 + 0.44/0.5)/24 \\ & + (0.20/0.3 + 0.42/0.4 + 0.61/0.5 + 0.57/0.6)/25 \\ & + (0.25/0.2 + 0.29/0.3 + 0.47/0.4 + 0.44/0.5)/27 + (0.29/0.3 + 0.47/0.4 + 0.44/0.5)/28 \\ & + (0.29/0.3 + 0.47/0.4 + 0.44/0.5)/31\}\end{aligned}$$

Now we defuzzify $\tilde{\tilde{\xi}}(M)$ using cv-based reduction method **[10]** for which we calculate the possibility, necessity and credibility measures of $\tilde{\tilde{\xi}}(M)$

$$Pos(\tilde{\mu}_{\tilde{\xi}(M)}(18) \geq \alpha) = \begin{cases} 0.63 & ;\alpha \leq 0.4 \\ 0.49\,; 0.4 < \alpha \leq 0.5 \\ 0.0\,; 0.5 < \alpha \leq 1.0 \end{cases}$$

$$Nec(\tilde{\mu}_{\tilde{\xi}(M)}(18) \geq \alpha) = \begin{cases} 1.0 & ;\alpha \leq 0.2 \\ 0.75\,; 0.2 < \alpha \leq 0.4 \\ 0.37\,; 0.4 < \alpha \leq 1.0 \end{cases}$$

$$Cr(\tilde{\mu}_{\tilde{\xi}(M)}(18) \geq \alpha) = \begin{cases} 0.815 & ;\alpha \leq 0.2 \\ 0.69\,; 0.2 < \alpha \leq 0.4 \\ 0.43\,; 0.4 < \alpha \leq 0.5 \\ 0.185\,; 0.5 < \alpha \leq 1.0 \end{cases}$$

$$\overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(18)] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Pos}\{\tilde{\mu}_{\tilde{\xi}(M)}(18) \geq \alpha\}]$$
$$= \sup_{\alpha \in [0,0.4]}(\alpha \wedge 0.63) \vee \sup_{\alpha \in (0.4,0.5]}(\alpha \wedge 0.49) \vee \sup_{\alpha \in (0.5,1.0]}(\alpha \wedge 0.0)$$
$$= (0.4 \vee 0.49 \vee 0.0) = 0.49$$
$$\underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(18)] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Nec}\{\tilde{\mu}_{\tilde{\xi}(M)}(18) \geq \alpha\}]$$
$$= \sup_{\alpha \in [0,0.2]}(\alpha \wedge 1.0) \vee \sup_{\alpha \in (0.2,0.4]}(\alpha \wedge 0.75) \vee \sup_{\alpha \in (0.4,1.0]}(\alpha \wedge 0.37)$$
$$= (0.2 \vee 0.4 \vee 0.37) = 0.4$$
$$CV[\tilde{\mu}_{\tilde{\xi}(M)}(18)] = \sup_{\alpha \in [0,1]} [\alpha \wedge \mathrm{Cr}\{\tilde{\mu}_{\tilde{\xi}(M)}(18) \geq \alpha\}]$$
$$= \sup_{\alpha \in [0,0.2]}(\alpha \wedge 0.815) \vee \sup_{\alpha \in (0.2,0.4]}(\alpha \wedge 0.69) \vee \sup_{\alpha \in (0.4,0.5]}(\alpha \wedge 0.43) \vee \sup_{\alpha \in [0.5,1.0]}(\alpha \wedge 0.185)$$
$$= (0.2 \vee 0.4 \vee 0.43 \vee 0.185) = 0.43$$

Similarly we get,

$$\overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(21)] = 0.49, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(21)] = 0.4, CV[\tilde{\mu}_{\tilde{\xi}(M)}(21)] = 0.44, \overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(22)] = 0.57, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(22)] = 0.5, CV[\tilde{\mu}_{\tilde{\xi}(M)}(22)] = 0.5, \overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(24)] = 0.44, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(24)] = 0.53, CV[\tilde{\mu}_{\tilde{\xi}(M)}(24)] = 0.485, \overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(25)] = 0.57, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(25)] = 0.5, CV[\tilde{\mu}_{\tilde{\xi}(M)}(25)] = 0.5, \overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(27)] = 0.44, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(27)] = 0.53, CV[\tilde{\mu}_{\tilde{\xi}(M)}(27)] = 0.485, \overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(28)] = 0.44, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(28)] = 0.53, CV[\tilde{\mu}_{\tilde{\xi}(M)}(28)] = 0.485, \overline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(31)] = 0.44, \underline{CV}[\tilde{\mu}_{\tilde{\xi}(M)}(31)] = 0.53, CV[\tilde{\mu}_{\tilde{\xi}(M)}(31)] = 0.485$$

Now when we have got $\overline{CV}[\zeta], \underline{CV}[\zeta]$ and $CV[\zeta]$ for $\tilde{\tilde{\xi}}(M)$ we have three different type-1 fuzzy sets derived from $\tilde{\tilde{\xi}}(M)$ which are as follows:

$$Optimistic_{\tilde{\xi}(M)} = (0.49/18 + 0.49/21 + 0.57/22 + 0.44/24 + 0.57/25 + 0.44/27 + 0.44/28 + 0.44/31)$$
$$Pessimistic_{\tilde{\xi}(M)} = (0.4/18 + 0.4/21 + 0.5/22 + 0.53/24 + 0.5/25 + 0.53/27 + 0.53/28 + 0.53/31)$$
$$Moderate_{\tilde{\xi}(M)} = (0.43/18 + 0.44/21 + 0.5/22 + 0.485/24 + 0.5/25 + 0.485/27 + 0.485/28 + 0.485/31)$$

Calculating centroid of each of the above type-1 fuzzy sets we get:

$cen_{Optimistic_{\tilde{\tilde{c}}(M)}}$

$$= \frac{\{(18 \times 0.49) + (21 \times 0.49) + (22 \times 0.57) + (24 \times 0.44) + (25 \times 0.57) + (27 \times 0.44) + (28 \times 0.44) + (31 \times 0.44)\}}{(0.49 + 0.49 + 0.57 + 0.44 + 0.57 + 0.44 + 0.44 + 0.44)}$$

$$= \frac{94.3}{3.88} = 24.30$$

$cen_{Pessimistic_{\tilde{\tilde{c}}(M)}} = 24.84$ and $cen_{Moderate_{\tilde{\tilde{c}}(M)}} = 24.62$

If we consider the value of $cen_{Moderate_{\tilde{\tilde{c}}(M)}}$, we can conclude that the cost of the MST of the fuzzy graph is more than its crisp version by $\frac{|cen_{Moderate_{\tilde{\tilde{c}}(M)}} - c(M)|}{c(M)} \times 100 = 2.58\%$

6. Results and discussions

In the above example we have find the MSTs of a crisp and a fuzzy graph respectively. We observe that in both the cases the corresponding MSTs constitute the same arcs i.e. $M = \{e_{12}, e_{13} \text{ and } e_{34}\}$. Henceforth, we compare the deterministic cost, $c(M)$ with that of defuzzified type-2 imprecise cost of MST, $\tilde{\tilde{c}}(M)$ of the fuzzy graph and observe that crispified $\tilde{\tilde{c}}(M)$ is more than $c(M)$ by *2.58%*. In the above example, while constructing $\tilde{\tilde{c}}(M)$ we have used the concept satisfaction function to compare the associated type-2 fuzzy costs of the edges. After the completion of MST we calculate $\tilde{\tilde{c}}(M)$ by finding its possibility, necessity and credibility measures and finally used the cv-based reduction method to crispify $\tilde{\tilde{c}}(M)$.

This work of implementing Borůvka's algorithm on a type-2 fuzzy weighted graph is unique in the literature hence numerical comparison of this work with other works could not be done.

6. Conclusion

In this paper, a modified Borůvka's algorithm on a type-2 fuzzy weighted graph has been explained. The proposed method is based on the possibility theory for type-2 fuzzy weighted graph. We have consider a simple graphical sequence to explain our proposed method, since at present the most serious concern dealing with possibility based approach of type-2 fuzzy sets is the computational complexity as we have to calculate the satisfaction function of every possible pair of edges incident on a particular vertex of a fuzzy graph. For graphs with larger size and orders computer programs can be written for the proposed method. But as we increase the size and order of the graph the computational complexity also increases rapidly which eventually makes this problem of NP class.

REFERENCES

1. J.M.Mendel and R.I.Bob John, Type-2 fuzzy sets made simple, *IEEE Transactions on Fuzzy Systems*, 10(2) (2002) 117-127.
2. N.N.Karnik and J.M.Mendel, Introduction to type-2 fuzzy logic systems, 1998. IEEE World Congresson Computational Intelligence. The 1998 IEEE International Conference on Fuzzy Systems Proceedings, 2 (1998) 915-920.

3. N.N.Karnik and J.M.Mendel, Operations on type-2 fuzzy sets, *Fuzzy Sets and Systems*, 122 (2001) 327–348.
4. M.Mizumoto and K.Tanaka, Some properties of fuzzy sets of type 2, *Information and Control*, 31 (1976) 312-340.
5. M.Mizumoto and K.Tanaka, Fuzzy sets of type 2 under algebraic product and algebraic sum, *Fuzzy Sets and Systems*, 5 (1981) 277-290.
6. K.M.Lee, C.Ho and K.H.Lee, Ranking fuzzy values with satisfaction function, *Fuzzy Sets and Systems*, 64 (1994) 295-309.
7. S. Lee, K.H.Lee and D.Lee, Comparison of type-2 fuzzy values with satisfaction function, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 12(5) (2004) 601-611.
8. P.Z.Wang, Fuzzy contactability and fuzzy variables, *Fuzzy Sets and Systems*, 8 (1982) 81-92.
9. Z.Q.Liu and Y.K.Liu, Fuzzy possibility space and type-2 fuzzy variable, Proceedings of the 2007 IEEE Symposium on Foundations of Computational Intelligence (FOCI 2007) (2007) 616-621.
10. R.Qin, Y.K.Liu and Z.Q.Liu, Methods of critical value reduction for type-2 fuzzy variables and their applications, *Journal of Computational and Applied Mathematics*, 235 (2011)1454–1481.
11. X.Li and D.A.Ralescu, Credibility measure of fuzzy set and applications, Proceedings of IPMU, (2008) 960-965.
12. F.Herrera and J.L.Verdegay, Fuzzy sets and operations research: Perspectives, *Fuzzy Sets and Systems*, 90 (1997) 207-218.
13. J.Nešetřil, E.Milkov, H.Nešetřilová and O.Borůvka, On minimum spanning tree problem translation of both the 1926 papers, comments, history, Discrete Mathematics, 233, (2001) 3–36.
14. M.S.Sunitha, Studies on Fuzzy Graphs PhD. Thesis, Department of Mathematics, Cochin University of Science and Technology, (2001).
15. T.Dinesh and T.V.Ramakrishnan, On generalised fuzzy graph structures, *Applied Mathematical Sciences*, 5(4) (2011) 173 – 180.
16. F.Bazlamacc and K.S.Hindi, Minimum-weight spanning tree algorithms: A survey and empirical study, *Computers & Operations Research*, 28 (2001) 767-785.
17. L.A.Zadah, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.
18. H.J.Zimmermann, *Fuzzy set theory*, Advanced Review, John Wiley & Sons, Inc. (2010).
19. SMA. Nayeem and M.Pal, Shortest path problem on a network with imprecise edge weight, *Fuzzy Optimization and Decision Making*, 4 (4) (2005) 293-312.
20. S.Samanta and M.Pal, Fuzzy planar graphs, *IEEE Transactions on Fuzzy Systems*, 23 (6) (2015) 1936-1942.
21. T.Pramanik, S.Samanta, B.Sarkar and M.Pal, Fuzzy \phi-tolerance competition graphs, Soft Computing, 21(13) (2017) 3723–3734.