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Approximate Controllability of Random Impulsive Integro Semilinear Differential Systems

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Abstract. In this paper, we study the existence of approximate controllability of random impulsive integro semilinear differential system under sufficient condition with non-densely defined system.

Keywords: Integro-semilinear differential system, random impulses, approximate controllability

AMS Mathematics Subject Classification (2010): 93B05, 35R60, 93E99

1. Introduction

Approximate controllability plays a vital role in engineering and science. Many authors have studied the problem of controllability for various kinds of differential and impulsive differential systems using different approaches, see [4, 5, 13, 16, 26] and the references therein. Most of the papers deal the problem with fixed time impulses, but in real time situation it need not be at fixed times may be at random time. When the impulses exist at random times, then the solutions of the differential equations are a stochastic process. It is very different from deterministic impulsive control systems and also it is different from impulsive stochastic control systems. Thus the random impulsive equations give more realistic than deterministic impulsive actions. There are few publications in this field, Wu and Meng first brought forward random impulsive ordinary differential equations and investigated boundedness of solutions to these models by Liapunov's direct function in [21]. Wu et al., studied some qualitative properties of random impulses in [22–25]. In [1], the author studied the existence and exponential stability for random impulsive semilinear functional differential equations through the fixed point technique under non-uniqueness. The existence, uniqueness and stability results were discussed in [2] through Banach fixed point method for the system of differential equations with random impulsive effect. In [3, 17–19] the author studied the existence results for the random impulsive neutral functional differential equations and differential inclusions with delays. In [27], the authors generalized the distribution of random impulses with the Erlang distribution.

Motivated by the above mentioned works, the main purpose of this paper is to study the approximate controllability of random impulsive integro semilinear differential systems. We relaxed the Lipschitz condition on the impulsive term and under our assumption it is enough to be bounded. We extend the results to densely define differential systems to fill the gap in the approximate controllability of abstract differential systems. To the best of our knowledge, there is no paper which studies the random impulsive integro differential systems. We utilize the technique developed in [6, 7, 8, 11, 13, 14, 15, 20, 23, 26].

The paper will be organized as follows: In section 2, we recall briefly the notations, definitions, preliminary facts which are used throughout this paper. In section 3, we study the approximate controllability of random impulsive integro semilinear differential systems.

2 Preliminaries

Let X be a real separable Hilbert space and Ω a nonempty set. Assume that τ_k is a random variable defined from Ω to D_k= $(0,\ d_k)$ for k = 1, 2, ..., where $0 < d_k < +\infty$. Furthermore, assume that τ_k follow the Erlang distribution, where k = 1,2,...., and let τ_i and τ_j are independent with each other as $i \neq j$ for i, j = 1,2,.., . For the sake of simplicity, we denote $R_\tau = [\tau, +\infty)$, $R^+ = [0, +\infty)$.

We consider a semilinear integro differential system with random impulses of the form

$$x'(t) = Ax(t) + (Bu)(t) + \int_0^t F\left(t, s, x\big(\sigma(s)\big)\right) ds,\ t \neq \varepsilon_k, \tag{2.1}$$

$$x(\varepsilon_k) = b_k(\varepsilon_k) x(\varepsilon_k^-), \quad k = 1,2, ..., \tag{2.2}$$

$$x_{t_0} = \phi, \tag{2.3}$$

where $A: D(A) \subset X \to X$ is a closed (not necessarily bounded) linear operator whose domain need not be dense in X, that is $\overline{D(A)} \neq X; F: \Delta X C \to X, \sigma: R^+ \to R^+, C = C([-r, 0], X)$ is the set of piecewise continuous functions mapping $[-r, 0]$ into X with some given $r > 0$; $u: [t_0, T] \to U$ is the control function spaces, $B: Y \to Z$ is bounded linear operator, X_t is a function when t is fixed, defined by $X_t(s) = X(t + s)$ for all $S \epsilon [-r, 0]$; $\varepsilon_{0=} t_0$ and $\varepsilon_k = \varepsilon_{k-1} + \tau_k$ for $k = 1,2, \ldots$ here $t_0 \in R_\tau$ is arbitrary given real number. The impulse moments $\{\varepsilon_k\}$ form a strictly increasing sequence, that is $t_0 = \varepsilon_0 < \varepsilon_1) < \varepsilon_2) < \ldots\ldots < lim_{k\to\infty} \varepsilon_k = \infty$; $b_k: D_k \to X$ for each $k = 1, 2, \ldots\ldots$, $x(\varepsilon_k^-) = lim_{t \uparrow \varepsilon_k} x(t)$ according to their paths with norm $\|x\|_t = sup\,|x(s)|$, $t - r \leq s \leq t$. For each t satisfying t ≥ t ≥ 0, ||.|| is any given norm in x; $\emptyset$ is a function defined from $[-r, 0]$ to X, here Δ denotes the set $\{(t, s): 0 \leq s \leq t < \infty\}$.

The simple counting process is denoted as $\{g_t, t \geq 0\}$ and it is generated by $\{\varepsilon_n\}$, that is, $\{g_t \geq n\} = \{\varepsilon_n \leq t\}$, and denote F_t the $\sigma - algebra$ generated by $\{g_t$, $t \geq 0\}$. Then $(\Omega, P, \{F_t\})$ is a probability space. Let $L_2 = L_2(\Omega, F_t, X)$ denote the Hilbert space of all F_t-measurable square integrable random variables with values in X.

Assume that $T > t_0$ is any fixed time to be determined later and let $\boldsymbol{B}$ denote the Banach space $B([t_0 - r,\ T], L_2)$, the family of all $F_t -$ measurable, C-valued random variable Ψ with norm

$$\|\Psi\|_B = (\sup_{[t_0 \le t \le T} E\|\Psi\|_t^2)^{1/2}$$

Let $L_2^0(\Omega, B)$ denote the family of all F_0 - measurable, B – valued random variable ϕ.

Definition 2.1. The control system (2.1) is said to be approximately controllable on $[t_0, T]$ if for any $\varepsilon > 0$, the intial function $\phi \epsilon C$ with $\phi(t_0) \epsilon \overline{D(A)}$ and $x_1 \epsilon \overline{D(A)}$, there exists a control u ϵY such that the integral solution (.) of (2.1) satisfies

$$E||x(T) - x_1||^2 \le \varepsilon.$$

Let $x_t(\phi(t_0), u)$ denotes the state value of the system (2.1) at time corresponding to the control $u \epsilon Y$ and the intial value $\phi(t_0)$. Now we introduce the following set, which is called the reachable set of the system (2.1) at terminal time T

$$K_T(F) = \{x_t(\phi(t_0), u); u \epsilon Y\}.$$

A control system is said to be approximately controllable on $[t_0, T]$, if $K_T(F)$ is dense in $\overline{D(A)}$. i.e., $\overline{K_T(F)} = \overline{D(A)}$.

The linear system (2.1*) is obtained by putting $F \equiv 0$ in (2.1) and is denoted by (2.1*). The linear system (2.1*) is approximately controllable if the reachable sets $\overline{K_T(F)} = \overline{D(A)}$.

Throughout, this work the operator A is assumed to satisfy the following Hille-Yosida (HY) condition.

Definition 2.2. We say that a linear operator A satisfies the (HY) condition. If there exist constants M ≥ 1 and $\widehat{\omega} \epsilon R$ such that $(\widehat{\omega}, +\infty) \subset \rho(A)$ and

$$SUP\{(\lambda - \widehat{\omega})^n \left||R(\lambda, A)^n|\right|^2 : n \epsilon N \text{ and } \lambda > \widehat{\omega}\} \le M,$$

where $R(\lambda, A) = (\lambda I - A)^{-1}$.

Theorem 2.1. The following assertions are equivalent:

(i) A is the generator of a non-degenerate, locally Lipschitz continuous integrated semigroup;
(ii) A satisfies condition (HY).

It is well known that above condition is equivalent to the fact that operator A is the generator of a locally Lipschitz integrated semigroup $((S(t))_{t \ge 0}$ on X see [9, 10, 14]. Let A_0 be a part of A defined by

$$D(A_0) = \{x \epsilon D(A) : A_x \epsilon \overline{D(A)}$$

$$A_0 x = Ax, for\ all\ x \epsilon D(A_0). \quad (2.3)$$

Then A_0 generates a C_0semigroup $(\mathrm{S}(\mathrm{t}-t_0))_{t\geq t_0}$ on $\overline{D(A)}$

Definition 2.3. For a given $T\in(t_0,+\infty)$, a stochastic process $\{x(t)\,\epsilon\, B, t_0-r\ \leq t\leq T\ \}$ is said to be an integral solution to the equation (2.1) in $(\Omega,P,\{F_t\})$, if

(i) $x(t)\epsilon\ B$ is F_t- adapted for $t\geq t_0$;

(ii) $x(t_0+s)=\phi(s)\ \epsilon\ L_2^0(\Omega,B)$, when $s\ \epsilon\ [-r,0]$ and

$$x(t)=\sum_{k=0}^{+\infty}\ \left[\ \prod_{i=1}^{k}b_i\,(\tau_i)S(t-t_0)\varphi(0)\ \right]\ I_{[\varepsilon_k,\varepsilon_{k+1})}(\mathrm{t})$$

$$+\lim_{\lambda\to\infty}\ \sum_{k=0}^{+\infty}\ \left[\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\varepsilon_{i-1}}^{\varepsilon_i}S(t-s)C(\lambda)[Bu(s)+f(s,x_s)]ds\ +\right.$$

$$\left.\int_{\varepsilon_k}^{t}S(t-s)C(\lambda)[Bu(s)+f(s,x_s)]ds\right]I_{[\varepsilon_k,\varepsilon_{k+1})},\qquad \mathrm{t}\epsilon[t_0,T] \tag{2.4}$$

where $C(\lambda)\ =\ \lambda R(\lambda,A)$;

$\prod_{j=m}^{n}(.)=1\ as\ m>n,\ \prod_{j=i}^{k}b_{j(\tau_j)}\ =b_k(\tau_k)b_{k-1}(\tau_{k-1})\ ...\ b_i(\tau_i), and\ I_A(.)$ is the index function, i.e.,

$$I_A(\mathrm{t})=\begin{cases}1, & \text{if } t\in \mathrm{A},\\ 0, & \text{if } t\notin \mathrm{A}.\end{cases}$$

Define the following functions

$$F\colon L_2[t_0,T,C]\ \to Z$$

$$(Fx)(t)\ =\ \int_0^t F(t,s,x(\sigma(s))ds;x\in L_2[t_0,T;C]$$

$$L\colon Z\to\overline{D(A)},$$

$$L_x=\lim_{\lambda\to\infty}\int_0^T S(t-s)C(\lambda)x(s)ds.$$

It is clear that L is a well defined bounded linear operator and for the well posedness of operator F see [12].

3. Main results

Now we introduce following hypotheses used in our discussion:

(H_1) The linear system (2.1*) is approximately controllable up to $\overline{D(A)}$

(H_2) $R(F)\ \subseteq\ \overline{R(B)}$

(H_3) The function $F\colon[t_0,\ T]\ X[t_0\ ,\ T]X\ C\ \to X$ is continuous and it satisfies the Lipschitz condition with respect to x.

$$\|\mathrm{F}(\mathrm{t},\mathrm{s},x_1)-F(t,s,x_2)\|^2\leq \mathrm{L}(\mathrm{t},\mathrm{s},\|x_1\|^2,\|x_2\|^2)\|x_1-x_2\|_s^2\ ,\ (t,s)\in\Delta,\ x_1,x_2\in X,$$

where $L\colon[t_0,T]\ X\ [t_0,T]\ XR^+XR^+\to R^+$ and is monotonically nondecreasing with respect to the second and third arguments, and $||F(t,s,0)||^2\leq k_1,\ k_1\geq 0.$

(H_4) $E\ \left\{\max\limits_{i,k}\{\prod_{j=i}^{k}\|b_j(\tau_j)\|\}\ \right\}$ is uniformly bounded, that is , there is $C>0$ such that $E\ \left\{\max\limits_{i,k}\{\prod_{j=i}^{k}\|b_j(\tau_j)\|\}\ \right\}\leq C$ for all $\tau_j\in D_j,\ \ j=1,2,\dots..$

Theorem 3.1. Let the hypotheses $(H_1)\ -\ (H_4)$ be hold. Then the system (2.1) is approximately controllable.

Proof: Let $x(t)$ be the integral solution of (2.1*) corresponding to control u in $t \in [t_0, T]$, which can be written as

$$x(t) = \sum_{k=0}^{+\infty} \left\{\prod_{i=1}^{k} b_i(\tau_i) S(t-t_0)\varphi(0)\right\} I_{[\varepsilon_k,\varepsilon_{k+1})}(t) \quad (3.1)$$
$$+ \lim_{\lambda\to\infty} \sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\varepsilon_{i-1}}^{\varepsilon_i} S(t-s)C(\lambda)Bu(s)ds + \int_{\varepsilon_k}^{t} S(t-s)C(\lambda)Bu(s)ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t),$$

Since $F_x \in R(B)$ for a given $\varepsilon > 0$ there exists a $w \in \gamma$ such that

$$||F_x - B_w|| \le \varepsilon. \quad (3.2)$$

Now, let $y(t)$ be an integral solution of (2.1) corresponding to the control u-w, then

$$x(t) - y(t)$$
$$= \lim_{\lambda\to\infty} \sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\varepsilon_{i-1}}^{\varepsilon_i} S(t-s)C(\lambda)Bw(s)ds + \int_{\varepsilon_k}^{t} S(t-s)C(\lambda)Bw(s)ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t)$$
$$- \lim_{\lambda\to\infty} \sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\varepsilon_{i-1}}^{\varepsilon_i} S(t-s)C(\lambda)[F_y(s)]ds + \int_{\varepsilon_k}^{t} S(t-s)C(\lambda)F_y(s)]ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t)$$
$$= \lim_{\lambda\to\infty} \sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\varepsilon_{i-1}}^{\varepsilon_i} S(t-s)C(\lambda)[Bw-Fx](s)ds + \int_{\varepsilon_k}^{t} S(t-s)C(\lambda)[Bw-Fx](s)ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t)$$
$$+ \lim_{\lambda\to\infty} \sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\varepsilon_{i-1}}^{\varepsilon_i} S(t-s)C(\lambda)[Fx-Fy](s)ds + \int_{\varepsilon_k}^{t} S(t-s)C(\lambda)[Fx-Fy](s)ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t)$$

Since

$$\|C(\lambda)\|^2 \le \frac{\lambda M}{\lambda - \hat{w}} \to M, \text{ as } \lambda \to \infty$$

Then, we have

$$\|x(t)-y(t)\|^2 \le 2\left[\sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} \|b_j(\tau_j)\| \int_{\varepsilon_{i-1}}^{\varepsilon_i} \|S(t-s)\|\|C(\lambda)\|\|[Bw-Fx](s)\|ds + \int_{\varepsilon_k}^{t} \|S(t-s)\|\|C(\lambda)\|\|[Bw-Fx](s)\|ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t)\right]^2$$
$$+2\left[\sum_{k=0}^{+\infty} \left[\sum_{i=1}^{k} \prod_{j=i}^{k} \|b_j(\tau_j)\| \int_{\varepsilon_{i-1}}^{\varepsilon_i} \|S(t-s)\|\|C(\lambda)\|\|[Fx-Fy](s)\|ds + \int_{\varepsilon_k}^{t} \|S(t-s)\|\|C(\lambda)\|\|[Fx-Fy](s)\|ds\right] I_{[\varepsilon_k,\varepsilon_{k+1})}(t)\right]^2$$

$$\leq 2\{\widehat{M}M\}^2 \left[\max_{i,k}\{1, \prod_{j=i}^{k} \|b_j(\tau_j)\|\} \right]^2 X$$

$$\left(\int_{t_0}^{t} \|[Bw - Fx]\,(s)\| ds I_{[\varepsilon_k,\varepsilon_{k+1})}(t)\right)^2$$

$$+ 2\{\widehat{M}M\}^2 \left[\max_{i,k}\{1, \prod_{j=i}^{k} \|b_j(\tau_j)\|\} \right]^2 X \left(\int_{t_0}^{t} \|[Fx - Fy]\,(s)\| ds I_{[\varepsilon_k,\varepsilon_{k+1})}(t)\right)^2$$

$$E\|x-y\|_t^2 \leq 2\{\widehat{M}M\}^2 \max\{1, c^2\}\,(t-t_0) \int_{t_0}^{t} E\|[Bw - Fx](s)\|^2 ds$$

$$+ 2\{\widehat{M}M\}^2 \max\{1, c^2\}\,(t-t_0) \int_{t_0}^{t} E\|[Fx - Fy](s)\|^2 ds$$

$$\leq 2\{\widehat{M}M\}^2 \max\{1, c^2\}\,(T-t_0)^2\, \varepsilon$$

$$+ 2\{\widehat{M}M\}^2 \max\{1, c^2\}\,(T-t_0)^2 \int_{t_0}^{t} L(s, s, E\|x\|^2, E\|y\|^2,)E\|x-y\|_s^2 ds$$

Taking supremum over t, and by Grownwall's inequality we get,

$$\|x-y\|_B^2 \leq\; 2\{\widehat{M}M\}^2 \max\{1, c^2\}\,(T-t_0)^2 \varepsilon \exp(2\{\widehat{M}\mathrm{M}\}^2 \max\{1. C^2\}(T-t_0)^2$$
$$\int_{t_0}^{t} L(s, s, E\|x\|^2, E\|y\|^2 ds$$

From the above inequality, it is clear that $\|x-y\|_B^2$ can be made arbitrarily small by choosing suitable w. It follows that reachable set of the system (2.1) is dense in the reachable set (2.1*) whichis dense in $\overline{\mathrm{D(A)}}$ due to condition (H_1). Hence the theorem is proved.

Theorem 3.2. Under assumption of the above theorem, system (2.1) – (2.3) is approximately controllable if its corresponding linear system is approximately controllable.

Proof: The proof is a particular case of Theorem 3.1 at $\propto = T$.

Remark 3.1. If $\overline{\mathrm{D(A)}} = X$, then the operator A generates a C_0 semigroup, then by taking mild solutions instead of integral solutions same proof leads us to the approximate controllability of the system (2.1) in the full space X. Hence the above theorem 3.1 is an extension of the results for densely defined control systems.

Remark 3.2. If the impulses are exist at fixed times in the system (2.1) – (2.3), then by the similar argument as in the Theorem 3.2, the system (2.1) – (2.3) is approximately controllable.

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