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Some Characterizations of Semi Prime Ideal in Lattices

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Abstract. By Y. Rav, an ideal J of a lattice L is called a semi prime ideal if for all $x, y, z \in L$, $x \wedge y \in J$ and $x \wedge z \in J$ imply $x \wedge (y \vee z) \in J$. In this paper, for a subset A of L, we define $A^J = \{x \in L : x \wedge a = J \text{ for some } a \in A\}$. Here we prove that for a meet sub semi lattice A of a lattice L, A^J is an ideal, in fact a semi prime ideal if and only if J is semi prime. Then we include several characterizations of semi prime ideals J by using A^J where A is a filter of L. At the end we include a prime separation Theorem.

Keywords: Maximal filter, 0-distributive lattice, Semi prime ideal, Minimal Prime down set.

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1. Introduction

Varlet [9] introduced the concept of 0-distributive lattices to generalize the notion of pseudo complemented lattices. Then many authors including [2, 3, 6, 7, 10] have studied them explicitly for lattices and meet semilattices. A lattice L with 0 is called a 0-distributive lattice if for all $a,b,c \in L$, $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Of course every distributive lattice with 0 is 0-distributive. Also every pseudo complemented lattice is 0-distributive. It is well known that the non-distributive pentagonal lattice $R_5 = \{0, a, b, c, 1;\ a \leq b,\ a \wedge c = b \wedge c = 0,\ a \vee c = b \vee c = 1\}$ is 0-distributive; while the diamond lattice $M_3 = \{0, a, b, c, 1;\ a \wedge b = b \wedge c = c \wedge a = 0,\ a \vee b = a \vee c = b \vee c = 1\}$ is not 0-distributive. Again [8] has extended the concept of 0-distributivity by introducing the notion of *semi prime ideals* in a lattice. In a lattice L, an ideal J is called a semi prime ideal if for all $x, y, z \in L$, $x \wedge y \in J$, $x \wedge z \in J$ imply $x \wedge (y \vee z) \in J$. Of course, a lattice itself is always a semi prime ideal. In distributive lattices, every ideal is semi prime. Moreover, every prime ideal is semi prime. Observe that in R_5, $(0]$, $(b]$, $(c]$ and R_5 itself are all semi prime but $(a]$ is not. Again in M_3, only semi prime ideal is M_3 itself. Recently [1,4] have given several characterizations of these ideals for lattices including some prime separation theorems. On the other hand [5] have studied them for meet semilattices directed above and extended most of the results of [4]. Let J be an ideal of a lattice *L*. For a subset A of *L*, we define $A^J = \{x \in L : x \wedge a = J \ \text{for some}\ a \in A\}$. In this paper we give several characterizations of semi prime ideals in term of A^J.

A non-empty subset *I* of a lattice *L* is called a down set if for $x \in I$ and $y \leq x$ $(y \in L)$ imply $y \in I$. Down set *I* is called an ideal if for $x, y \in I$, $x \vee y \in I$.

A non-empty subset *F* of *L* is called an upset if $x \in F$ and $y \geq x$ $(y \in L)$ imply $y \in F$. An upset *F* of *L* is called a filter if for all $x, y \in F$, $x \wedge y \in F$. An ideal (down set) *P* is called a prime ideal (down set) if $a \wedge b \in P$ implies either $a \in P$ or $b \in P$. A filter *Q* of *L* is called prime if $a \vee b \in Q$ implies either $a \in Q$ or $b \in Q$.

A filter *F* of *L* is called a maximal filter if $F \neq L$ and it is not contained by any other proper filter of *L*. A prime down set *P* is called a minimal prime down set if it does not contain any other prime down set of *L*.

We include the following Lemmas which are very trivial.

2. Main results

Lemma1. For a non-empty subset A of a lattice L, A is a filter if and only if L-A is a prime down set. ☐

Lemma 2. For a non-empty proper subset of a lattice L, A is a prime ideal if and only if L-A is a prime filter. ☐

Following Lemma is due to [3] which is proved by using Zorn's Lemma.

Lemma 3. *Let F be a filter and I be an ideal of a lattice L, such that* $F \cap I = \varphi$. *Then there exists a maximal filter* $Q \supseteq F$ *such that* $Q \cap I = \varphi$. ☐

Theorem 4. Let J be an ideal of a lattice L. Then for any subset A of L, A^J is a down set containing J. Moreover, $A^J = L$ if $A \cap J \neq \varphi$.

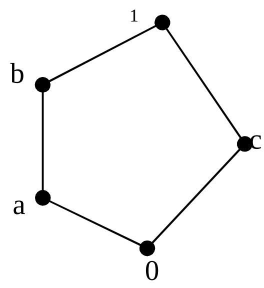
Proof: Let $x \in A^J$, $y \leq x$. Then $x \wedge a \in J$ for some $a \in A$. Now $y \wedge a \leq x \wedge a \in J$ implies $y \wedge a \in J$, so $y \in A^J$. Therefore A^J is a down set. Again let $j \in J$. Then $a \wedge j \in J$ for all $a \in A$, which implies $j \in A^J$, and so $J \subseteq A^J$. Hence A^J is a down set containing *J*. The proof of last part of the theorem is trivial. ☐

Now we include a characterization of semi prime ideals.

Theorem 5. *An ideal J of L is semi prime if and only if for every meet sub semi lattice A of L,* A^J *is a semi prime ideal of L containing J.*

Proof: Suppose J is *semi prime*. We already know that A^J is a down set containing *J*. Now let $x, y \in A^J$. Then $x \wedge a \in J$, $y \wedge b \in J$ for some $a, b \in A$. Then $x \wedge a \wedge b \in J$, $y \wedge a \wedge b \in J$. Since J is *semi prime*, so $a \wedge b \wedge (x \vee y) \in J$. Now $a \wedge b \in A$ implies $x \vee y \in A^J$, and so A^J is an ideal. Finally let $x \wedge y \in A^J$, and $x \wedge z \in A^J$. Then $x \wedge y \wedge a_1 \in J$, $x \wedge z \wedge b_1 \in J$ for some $a_1, b_1 \in A$. Thus $x \wedge a_1 \wedge b_1 \wedge y \in J$, $x \wedge a_1 \wedge b_1 \wedge z \in J$. Then by the *semi prime* property of *J*, $x \wedge a_1 \wedge b_1 \wedge (y \vee z) \in J$. Thus $x \wedge (y \vee z) \in A^J$ as $a_1 \wedge b_1 \in A$. Therefore A^J is semi prime. Conversely, if A^J is a semi prime ideal for every meet sub semilattice *A* of *S*, then in particular $(a)^J$ is an ideal for all $a \in L$.
Now, suppose $x \wedge a, x \wedge b \in J$. This implies $a, b \in (x)^J$. Since $(x)^J$ is an ideal, so $a \vee b \in (x)^J$ and so $x \wedge (a \vee b) \in J$. Therefore J is semi prime. ☐

Observe that in R_5



R_5

Figure 1:

J=(a] is not semiprime. Consider the filter A={b, 1}. It is easy to see that A^J={0, a, c} which is not an ideal at all.

Following result is a generalization of [3, Lemma 1.12]

Theorem 6. *Let A and B be filters of a lattice L, such that $A \cap B^J = \varphi$. Then there exists a minimal prime down set containing B^J and disjoint from A.*

Proof: Observe that $J \cap (A \vee B) = \varphi$. If not, let $j \in J \cap (A \vee B)$. Then $j \geq a \wedge b$ for some $a \in A$, $b \in B$. That is, $a \wedge b \in J$ as *J* is an ideal, which implies $a \in B^J$ gives a contradiction. Hence $J \cap (A \vee B) = \varphi$. Thus by Lemma-3, there exists a maximal filter *M* containing $A \vee B$ and disjoint to *J*. Now we prove that $M \cap B^J = \varphi$. If not, let $x \in M \cap B^J$. Then $x \in M$ and $x \wedge b_1 \in J$ for some $b_1 \in B \subseteq M$, so $x \wedge b_1 \in M$. This implies $M \cap J \neq \varphi$ which is a contradiction. Therefore, $M \cap B^J = \varphi$. Thus by

Lemma 1, $L-M$ is a minimal prime down set containing B^J. Moreover, $(L-M)\cap A=\varphi$. □

Now we extend [3, Lemma 1.13]

Theorem 7. *Let A be a filter of a lattice L. Then* A^J *is the intersection of all the minimal prime down sets* containing *J and disjoint from A.*

Proof: Let N be any minimal prime down set containing J and disjoint from *A*. If $x\in A^J$, then $x\wedge a\in J$ for some $a\in A$ and so $x\in N$ as *N* is prime.

Conversely, let $y\in L-A^J$. Then $y\wedge a\notin J$ for all $a\in A$. Hence $(A\vee[y))\cap J=\varphi$. If not, let $x\in(A\vee[y))\cap J$, implies $x\in J$ and $x\geq a\wedge y$ for some $a\in A$. That is $a\wedge y\in J$, which implies $y\in A^J$ gives a contradiction. Hence $(A\vee[y))\cap J=\varphi$. Then by Lemma 3, $A\vee[y)\subseteq M$ for some maximal filter *M* and disjoint to *J*. Thus by Lemma 1, $L-M$ is a minimal prime down set containing *J*. Clearly $(L-M)\cap A=\varphi$ and $y\notin L-M$. □

Now we generalize Theorem 3.3 of [3] to give some characterizations of semi prime ideals.

Theorem 8. Let L be lattice with J. Then the following statements are equivalent;

i) J is semi prime.

ii) If A and B are filters of L such that $A\cap B^J=\varphi$, then there is a minimal prime ideal containing B^J and disjoint from A.

iii) If A and B are filters of L such that $A\cap B^J=\varphi$, there is a prime filter containing A and disjoint from B^J.

iv) If A is a filter of L and B is a prime down set containing A^J, there is a prime filter containing L-B and disjoint from A^J.

v) If A is a filter of L and B is a prime down set containing A^J, there is a minimal prime ideal containing A^J and contained in B.

vi) For each $x\in L$ such that $x\notin J$ and each prime down set B containing $(x)^J$, there is a prime ideal containing $(x)^J$ and contained in B.

vii) For each $x\in L$ with $x\notin J$ and each prime down set B containing $(x)^J$, there is a prime filter containing L-B and disjoint from $(x)^J$.

Proof: *(i)* ⇒*(ii)* Suppose *(i)* holds. Let A and B be filters of L such that $A\cap B^J=\varphi$. By Theorem 6, there is a minimal prime down set M such that $M\supseteq B^J$ and $M\cap A=\varphi$. Then L-M is a maximal filter disjoint to J. Since J is semi prime so by [1], L-M is a prime filter, and so by Lemma-1, M is a prime ideal.

(ii) ⇒*(iii)* is trivial as L-M is a prime filter containing A and disjoint from B^J.

(iii) ⇒(iv) By Lemma 1, F=L-B is a maximal filter such that $F \cap A^J = \varphi$. So by (iii), there exists a prime filter R containing F such that $R \cap A^J = \varphi$.

(iv) ⇒(v). By iv), R is a prime filter containing F=L-B and disjoint with A^J. Thus L-R is a minimal prime ideal containing A^J and contained in B.

(v) ⇒(vi). Let $x \in L$. Replace A by [x) in (v). Now B is a prime down set containing $A^J = (x)^J = [x)^J$. Thus by (v), there exists a minimal prime ideal containing $A^J = (x)^J$ and contained in B.

(vi) ⇒(vii). By (vi), there exists a minimal prime ideal P containing $(x)^J$ and is contained in B. Thus L-P is a prime filter disjoint to $(x)^J$. Moreover $L - P \supset L - B$.

(vii) ⇒(i). Suppose (vii) holds and let $x \in L$ such that $x \notin J$. By Lemma 1, $L-[x)$ is prime down set not containing x. Let $t \in (x] \cap (x)^J$. Then $t \le x$ and $t \wedge x \in J$. This implies $t \ne x$. For otherwise $x \in J$ gives a contradiction. Thus, it follows that $t < x$. Hence $(x] \cap (x)^J \subset L-[x)$. L-[x) contains $(x)^J$, as $L-[x)$ is a prime down set. By (vii), there is a prime filter B containing $[x) = L-(L-[x))$ and disjoint from $(x)^J$. Clearly $x \in B$ and $B \cap J = \varphi$ as $J \subseteq (x)^J$.

Now suppose $a,b,c \in L$ such that $a \wedge b \in J$ and $a \wedge c \in J$ but $a \wedge (b \vee c) \notin J$. By above proof there exists a prime filter B such that $a \wedge (b \vee c) \in B$ and disjoint from $(a \wedge (b \vee c))^J$, implies $a, b \vee c \in B$. Then either $b \in B$ or $c \in B$ as B is prime. This implies either $a \wedge b \in B$ or $a \wedge c \in B$. In any case $B \cap J \ne \varphi$, which gives a contradiction. Therefore $a \wedge (b \vee c) \in J$ a so J is semi prime. □

Hence by Theorem 5, we have the following characterization of semi prime ideals.

Corollary 9. *Let A be a filter and J be an ideal of a lattice L. Then J is semi prime if and only if* A^J *is the intersection of all the minimal prime ideals disjoint from A.* □

Now we include some characterizations of *a semi prime ideals of L* using the downs sets of the form A^J. This result is in fact a generalization of [3,Theorem 3.4]. In fact the results of [3] can be obtained by replacing J by (0].

Theorem 10. *Let L be a lattice. Then the following statements are equivalent;*

i) *J is semi prime.*
ii) *For each* $a \in S$, $(a)^J = [a)^J$ *is a semi prime ideal containing J.*
iii) *For any three filters A, B, C of L,*
$(A \vee (B \cap C))^J = (A \vee B)^J \cap (A \vee C)^J$
iv) *For all* $a,b,c \in L$, $([a) \vee ([b) \cap [c)))^J = ([a) \vee [b))^J \cap ([a) \vee [c))^J$
v) *For all* $a,b,c \in L$, $(a \wedge (b \vee c))^J = (a \wedge b)^J \cap (a \wedge c)^J$.

Proof: (i) ⇔ (ii). Follows by Theorem 3 and $(a)^J = [a)^J$ is trivial.

(i) $\Rightarrow$ (iii). Let $x \in (A \vee B)^J \cap (A \vee C)^J$. Then $x \in (A \vee B)^J$ and $x \in (A \vee C)^J$. Thus $x \wedge f \in J, x \wedge g \in J$ for some $f \in A \vee B$ and $g \in A \vee C$. Then $f \geq a_1 \wedge b$, and $g \geq a_2 \wedge c$ for some $a_1, a_2 \in A$, $b \in B$, $c \in C$. This implies $x \wedge a_1 \wedge b \in J$, $x \wedge a_2 \wedge c \in J$ and so $x \wedge a_1 \wedge a_2 \wedge b \in J$, $x \wedge a_1 \wedge a_2 \wedge c \in J$. Since J is *semi prime*, so $x \wedge a_1 \wedge a_2 \wedge (b \vee c) \in J$. Now $a_1 \wedge a_2 \in A$ and $b \vee c \in B \cap C$. Therefore, $(a_1 \wedge a_2) \wedge (b \vee c) \in A \vee (B \cap C)$ and so $x \in (A \vee (B \cap C))^J$. The reverse inclusion is trivial as $A \vee (B \cap C) \subseteq A \vee B, A \vee C$. Hence (iii) holds.

(iii) $\Rightarrow$ (iv) is trivial by considering $A = [a)$, $B = [b)$ and $C = [c)$ in (iii).

(iv) $\Rightarrow$ (v). Let (iv) holds. Suppose $x \in (a \wedge b)^J \cap (a \wedge c)^J$. Then $x \in ([a) \vee [b))^J \cap ([a) \vee [c))^J = ([a) \vee ([b) \cap [c)))^J$. This implies $x \wedge f \in J$ for some $f \in [a) \vee ([b) \cap [c))$. Then $f \geq a \wedge (b \vee c)$. It follows that $x \wedge a \wedge (b \vee c) \in J$ and so $x \in (a \wedge (b \vee c))^J$. On the other hand, $[a) \vee [b \vee c) \subseteq [a) \vee [b)$ and $[a) \vee [b \vee c) \subseteq [a) \vee [c)$ implies that $(a \wedge (b \vee c))^J \subseteq (a \wedge b)^J \cap (a \wedge c)^J$. Therefore (v) holds.

(v) $\Rightarrow$ (i). Suppose (v) holds. Let $a, b, c \in L$ with $a \wedge b \in J, a \wedge c \in J$. Then $a \wedge (a \wedge b) \in J, a \wedge (a \wedge c) \in J$ implies $a \in (a \wedge b)^J \cap (a \wedge c)^J = (a \wedge (b \vee c))^J$. Thus, $a \wedge (a \wedge (b \vee c)) \in J$. That is $a \wedge (b \vee c) \in J$. So J is *semi prime*. □

For any subset *A* of a lattice *L*, we define $A^{\perp_J} = \{x \in L : x \wedge a = J \ \text{for some}\ j \in J\}$. $A^{\perp_J}$ is always a down set. By [4], $A^{\perp_J}$ is a semi prime ideal containing *J* if and only if *J* is semi prime, clearly, for any $a \in L$, $(a)^J = [a)^J = (a)^{\perp_J} = (a]^{\perp_J}$.

Corollary 11. *Let J be an ideal of a lattice L, J is semi prime if and only if* $J = \bigcap_{a \in L} (a)^J$.

Proof: By Theorem-4, $J \subseteq (a)^J$ for every $a \in L$, *and so* $J \subseteq \bigcap_{a \in L} (a)^J$. For reverse inclusion, let $x \in \bigcap_{a \in L} (a)^J$. Then $x \in (a)^J$ for every $a \in L$. Thus, in particular, $x \in (x)^J$. This implies $x \wedge x = x \in J$ and so $\bigcap_{a \in L} (a)^J \subseteq J$. Therefore $J = \bigcap_{a \in L} (a)^J$. □

We conclude with few more characterizations of *semi prime ideal of L*. This is also a generalization of [3, Theorem3.5].

Theorem 12. *Let L be a lattice. Then the following are equivalent;*

i) *J is semi prime.*

ii) *For any three filters A, B, C of L.*
$((A \cap B) \vee (A \cap C))^J = A^J \cap (B \vee C)^J$

iii) *For any two filters A, B of L,* $(A \cap B)^J = A^J \cap B^J$

iv) *For all* $a,b\in L$, $(a)^J\cap(b)^J=(a\vee b)^J$.

v) *For all* $a,b\in L$, $(a]^{\perp_J}\cap(b]^{\perp_J}=(a\vee b]^{\perp_J}$.

Proof: (i) $\Rightarrow$ (ii). Suppose J is *semi prime,* Since $(A\cap B)\vee(A\cap C)\subseteq A$ and $B\vee C$, so $((A\cap B)\vee(A\cap C))^J\subseteq A^J\cap(B\vee C)^J$. Now suppose $x\in A^J\cap(B\vee C)^J$. Then $x\in A^J$ and $x\in(B\vee C)^J$. Thus $x\wedge a\in J$ for some $a\in A$ and $x\wedge b\wedge c\in J$ for some $b\in B$, $c\in C$. Hence $x\wedge a\in J$, $x\wedge b\wedge c\in J$ implies $x\wedge c\wedge a\in J$; $x\wedge c\wedge b\in J$. Since J is semi prime, so $x\wedge c\wedge(a\vee b)\in J$. Then $a\vee b\in A\cap B$. Now $x\wedge a\in J$ implies $x\wedge a=x\wedge(a\vee b)\wedge a\in J$. Also $x\wedge(a\vee b)\wedge c\in J$. Since J is semi prime, so $x\wedge(a\vee b)\wedge(c\vee a)\in J$. But $a\vee b\in A\cap B$ and $c\vee a\in C\cap A$. Hence $x\in((A\cap B)\vee(A\cap C))^J$ and so (ii) holds.

(ii) $\Rightarrow$ (iii) is trivial by considering $B=C$ in (iii).

(iii) $\Rightarrow$ (iv). Choose $A=[a)$ and $B=[b)$ in (iii). Then by (iii), $(a)^J\cap(b)^J=[a)^J\cap[b)^J=([a)\cap[b))^J=([a\vee b))^J=(a\vee b)^J$.

(iv) $\Leftrightarrow$ (v) is obvious.

(v) $\Rightarrow$ (i).Suppose (v) holds and for $a,b,c\in L$, $a\wedge b\in J$, $a\wedge c\in J$. Then $a\in(b]^{\perp_J}\cap(c]^{\perp_J}=(b\vee c]^{\perp_J}$. Therefore, $a\wedge(b\vee c)\in J$ and so J is *semi prime.* □

Observe that in Figure-1 of R_5, $J=(a]$ is not semi prime. Here we can easily check that

$(b\wedge(a\vee c))^J=\{b\}^J=\{0,a,c\}$, $(b\wedge a)^J\cap(b\wedge c)^J=(a)^J\cap(0)^J=L\cap L=L$.

Thus $(b\wedge(a\vee c))^J\neq(b\wedge a)^J\cap(b\wedge c)^J$.

Moreover, $(a)^J\cap(c)^J=L\cap\{0,a,b\}=\{0,a,b\}$, while $(a\vee c)^J=\{1\}^J=\{0,a\}$. Thus $(a)^J\cap(c)^J\neq(a\vee c)^J$.

We conclude the paper with a prime Separation Theorem by using A^J. For this we need the following results which are due to [4]

Lemma 13. *Let I be an ideal of a lattice L. A filter M disjoint from I is a maximal filter disjoint from I if and only if for all* $a\notin M$ *, there exists* $b\in M$ *such that* $a\wedge b\in I$. □

Theorem 14. *Let L be a lattice and J be an ideal of L. The following conditions are equivalent;*

i) *J is semi prime.*

ii) $\{a\}^{\perp_J}=\{x\in L:x\wedge a\in J\}$ *is a semi prime ideal containing J.*

iii) $A^{\perp_J}=\{x\in L:x\wedge a\in J \text{ for all } a\in A\}$ *is a semi prime ideal containing J.*

iv) $I_J(L)$ *is pseudo complemented.*

v) *$I_J(L)$ is a 0-distributive lattice.*

vi) *Every maximal filter disjoint from J is prime.* □

Thus we have the following Separation Theorem.

Theorem 15. *Let J be a semi prime ideal of a lattice L and A be a meet sub semi lattice of L. Then for a filter F disjoint from A^J, there exists a prime ideal containing A^J and disjoint from F.*

Proof: By lemma 3, there exists maximal filter *M* containing *F* and disjoint from A^J. We claim that $A \subseteq M$. If not then there exists $a \in A$ but $a \notin M$. Then $M \vee [a) \supseteq M$. By the maximality of *M*, $(M \vee [a)) \cap A^J \neq \varphi$. If $t \in (M \vee [a)) \cap A^J$, the $t \geq m \wedge a$ for some $m \in M$ and $t \wedge a_1 \in J$ for some $a_1 \in A$.This implies $m \wedge a \wedge a_1 \leq t \wedge a_1 \in J$, and $a \wedge a_1 \in A$. Thus $m \in A^J$ which is a contradiction. Hence $A \subseteq M$. Now let $z \notin M$. Then by maximality of *M*, $(M \vee [z)) \cap A^J \neq \varphi$. Suppose $y \in (M \vee [z)) \cap A^J$. Then $y \geq m_1 \wedge z$ and $y \wedge a_2 \in J$ for some $a_2 \in A$. Hence $m_1 \wedge a_2 \wedge z \in A^J$ and $m_1 \wedge a_2 \in M$. Therefore by lemma 13, *M* is a maximal filter disjoint to A^J. Since by Theorem 5, A^J is semi prime, so by [4, Theorem 2], *M* must be prime. Therefore, *L-M* is a prime ideal containing A^J, but disjoint from *F*. □

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