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An Analogue Circuit to Study the Forced and Quadratically Damped Mathieu-Duffing Oscillator

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Abstract. We report the results of study of the parametric excitation and nonlinear cubic characteristics of forced and quadratically damped Mathieu-Duffing oscillator. We designed the equivalent circuit of Matheiu-Duffing oscillator in PSpice and observed its dynamical behavior under various sets of control parameters. Verification of the results obtained from the PSpice circuit has been done by the numerical simulation.

Keywords: Nonlinear damping, Mathieu – Duffing oscillator, chaos

1. Introduction

The study of chaos in complex dynamical systems has been under the constant attention of researchers. There has been recent interest to study the nonlinear phenomenon and chaos in various physical systems through modelling them via electronic circuits [1, 2]. It serves the motives not only to understand the complex dynamics under different combination of system parameters, but also to capture the essential mechanism that generates chaos. The electronic circuits provide a very strong medium for both modelling and experimental studies of nonlinear systems. The real complex physical system can be simulated in real-time through an electronic circuit costing very less [3, 4, 5, 6]. Various nonlinear electronic circuits have been designed and analyzed to understand the dynamics of many complex physical systems of diverse nature under real-time conditions like: nonlinear physical pendulum, forced Duffing oscillator, forced van der pol oscillator, forced Duffing-van der pol oscillator, parametrically driven Duffing oscillator etc. [7].In [8], Sharma *et al.* have analyzed the dynamical behavior of forced Duffing oscillator under the presence of nonlinear damping term analytically as well as computationally. They have particularly observed the effect of nonlinear damping on the global dynamical behavior of forced Duffing oscillator through Melnikov analysis, characterization of parameter space through Lyapunov spectrum calculation, fractalness of basin boundaries and fractal dimensions etc.

In this communication we intend to study the dynamical system i.e., the Mathieu-Duffing oscillator system under the presence of nonlinear damping experimentally through an analog circuit. For this purpose we have designed a circuit and observed its dynamical behavior under various sets of control parameters and also compared the PSpice simulation results with the numerical simulation results. In the next paragraph we briefly describe the forced and quadratically damped Mathieu-Duffing oscillator system.

Here we consider the Mathieu-Duffing oscillator which can be described by the following equation of motion:

$$\ddot{x} + \alpha\dot{x}|\dot{x}|^{p-1} - \omega_o^2(\delta + \varepsilon\cos\omega t)\,x + \beta x^3 = 0 \qquad (1)$$

where, $\dot{x} = \frac{dx}{dt}$ represents the derivative with respect to time t, α is the damping coefficient, ε and ω are respectively, the amplitude and frequency of the parametric excitation, δ and β are, respectively, the Mathieu parameter and a stiffness constant. In our present work we consider a nonlinear damping term i.e. proportional to the pth power of velocity. Here modulus of velocity is taken to consider the sign of velocity. For all calculations we have considered $\omega_o^2 = 1, \beta = 1$ and $\omega = 1$. We have studied the dynamics of the system for different value of damping coefficient (α) and the amplitude of parametric excitation (ε).For our study, we have designed an electronic circuit for the model equation (1) and simulated it through the PSpice circuit simulator and numerical simulations. In the next section we give the description of the electronic circuit designed for our study. In Section 3, we discuss our PSpice circuit simulation results and compare them with the numerical simulation results and Section 4 concludes the paper.

2. Description of the circuit

We have constructed an analog circuit equivalent to eq. (1) using conventional operational amplifiers and five quadrant multipliers, which is shown in Figure 1.

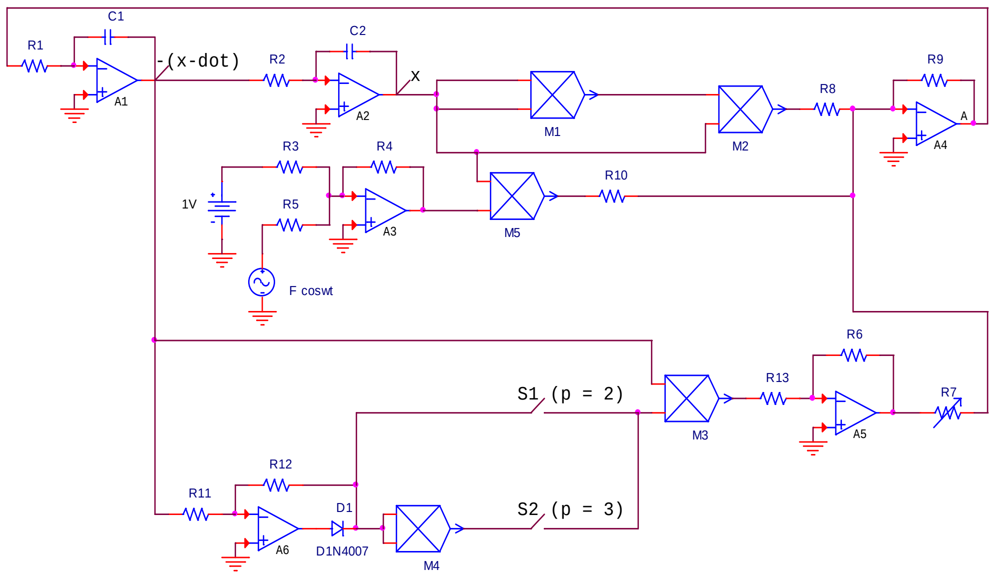


Figure 1: Analog circuit for the forced and quadratically damped Mathieu-Duffing oscillator, equivalent to eq. (1)

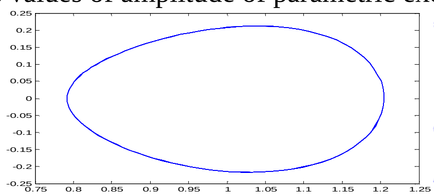
The circuit is constructed using conventional operational amplifiers and five quadrant multipliers. Here A1 and A2 represent two integrators; A3 and A4 are summing amplifiers, A5 is inverter; A6 is an absolute rectifier; and M1, M2, M3, M4 and M5 are four quadrant multipliers. The operational amplifiers used in A1, A2, A3, A4, A5 and A6 are op-amp µA741C and the five multipliers M1, M2, M3, M4 and M5 are Analog Devices multiplier AD633 with the inbuilt gain of 0.1. Analog Devices multiplier AD633 is the low cost chip with 2% error at Full Scale.
At junction A i.e. at the output of summing amplifier, we obtain the following equation

$$\ddot{x} = -\alpha\dot{x}|\dot{x}|^{p-1} + \omega_0^2(\delta + \varepsilon\cos\omega t)\, x - \beta x^3$$

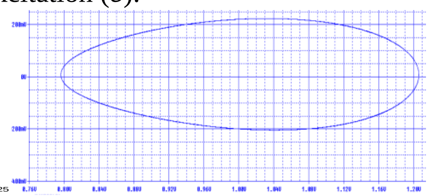
where, $p = 2$ (switch S1 closed) or 3 (switch S2 closed),
$|\omega_0{}^2| = \frac{(0.1)R9}{R10}, \alpha = \frac{R9}{R7}, \quad and \beta = \frac{(0.01)R9}{R8}$
The values of various resistors and capacitors used in the circuit are as given below: -
$R1 = 100k\Omega,\ R2 = 100k\Omega,\ R3 = 1k\Omega,\ R4 = 1k\Omega,\ R5 = 1k\Omega,$
$R6 = 10k\Omega(for p = 2) or\ 100k\Omega(for p = 3),\ R7 = Variable resistor of\ 100k\Omega,$
$R8 = 100\Omega,\ R9 = 10k\Omega,\ R10 = 1k\Omega,\ R11 = 1k\Omega,\ R12 = 1k\Omega,\ R13 = 1k\Omega$, and C1 = C2 = 0.01µF.
Here $\omega_0{}^2 = 1,\ \beta = 1, \delta = 1\ and \boldsymbol{\omega} = \mathbf{1}$.
$\omega = 1$ is obtained at external frequency f ≈160Hz since here $\omega = 2\pi * f * (R1 * C1) or\ 2\pi * f * (R2 * C2)$.

3. Results and discussion

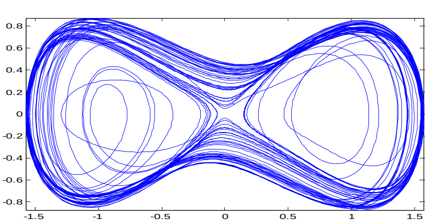
In this section we present the results of experimental study and their comparison with the numerical results. In Figure 2, we show the result for the fixed values of parameters $\omega_0{}^2 = 1$, $\beta = 1, \delta = 1, \omega = 1$, p= 2(drag like damping), damping coefficient (α)= 0.5 and various values of amplitude of parametric excitation (ε).



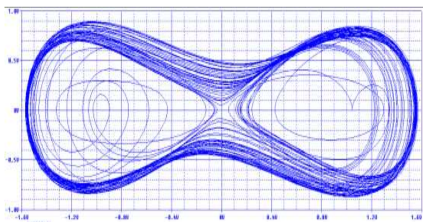
(a)



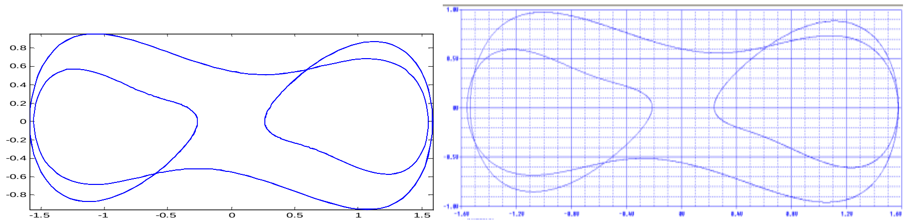
(b)



(c)



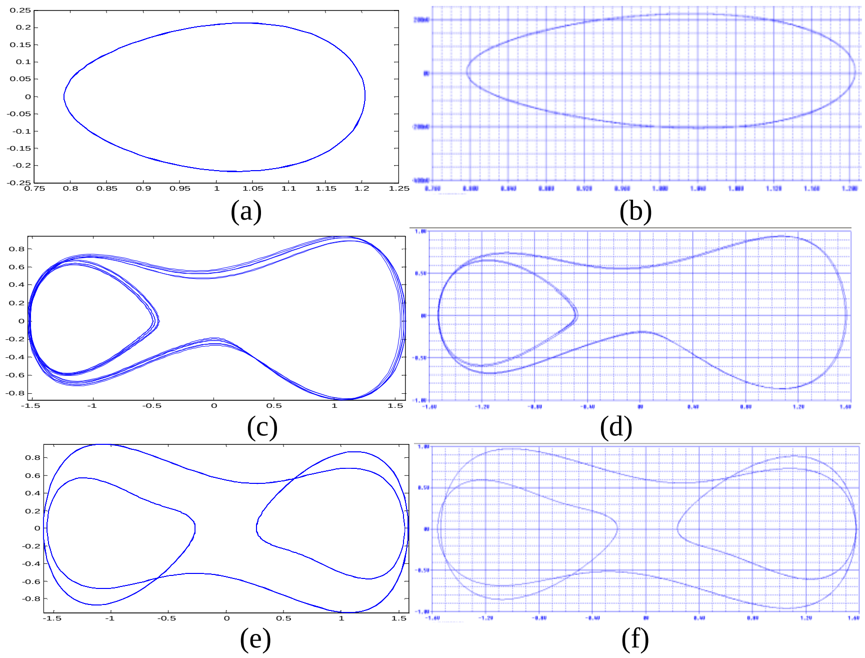
(d)



(e) (f)

Figure 2: First and second columns respectively show the computationally obtained phase plots ($\dot{x}$ vs x) and PSpice simulation results obtained from the circuit shown in Figure1. (a) and (b) are obtained at amplitude of parametric excitation (ε) = 0.2 V, (c) and (d) are obtained at amplitude of parametric excitation (ε) = 0.5 V and (e) and (f) are obtained at amplitude of parametric excitation (ε) = 0.65 V.

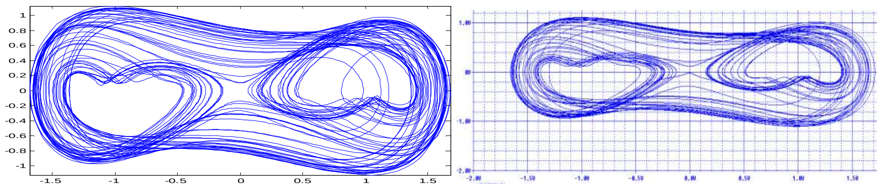
In Figure 3, we show the result for the fixed values of parameters $\omega_0{}^2 = 1$, $\beta = 1, \delta = 1, \omega = 1$, p= 3 (nonlinear third-power damping), damping coefficient (α)= 0.5 and various values of amplitude of parametric excitation (ε).



(a) (b) (c) (d) (e) (f)

Figure 3: First and second columns respectively show the computationally obtained phase plots ($\dot{x}$ vs x) and PSpice simulation results obtained from the circuit shown in Figure1. (a) and (b) are obtained at amplitude of parametric excitation (ε) = 0.2 V, (c) and (d) are obtained at amplitude of parametric excitation (ε) = 0.5 V and (e) and (f) are obtained at amplitude of parametric excitation (ε) = 0.65 V.

In Figure 4, we show the result for the fixed values of parameters $\omega_0{}^2 = 1$, $\beta = 1$, $\delta = 1$, $\omega = 1$, p= 2 (drag like damping), damping coefficient (α)= 0.3and amplitude of parametric excitation (ε) = 0.65 V.



(a) (b)

Figure 4: (a) and (b) respectively show the computationally obtained phase plots ($\dot{x}$ vs x) and PSpice simulation results obtained from the circuit shown in Figure1.

We have obtained various results for drag like damping (p=2) and nonlinear third-power damping (p=3). In figure 2 and figure 3, respectively for p=2 and p=3, the phase plot obtained for the amplitude of parametric excitation (ε) =0.2 V and 0.65 V are same i.e. periodic signal of period-1 and period-3 respectively. But for the amplitude of parametric excitation (ε) =0.5 V the result obtained shows the different nature for both the types of damping. For drag like damping (p=2) it is chaotic signal whereas for nonlinear third-power damping (p=3) it is periodic signal of period-2.

By changing the damping coefficient (α) we have found the changes in the nature of signals obtained by keeping same values of all other parameters. At damping coefficient (α) = 0.3, figure 4 shows the chaotic signals at the amplitude of parametric excitation (ε) =0.65 V for drag like damping (p=2).

4. Conclusions

We have designed and implemented an analogue circuit equivalent to a forced and quadratically damped Mathieu-Duffing oscillator to study the effect of nonlinear damping on the dynamical behavior of the forced oscillator. We have studied the designed circuit extensively for various combinations of system parameters and observed that the designed circuit is able to produce the correct sequence of the dynamical behavior as obtained with the numerical simulation. We have compared the result of PSpice simulation and computational result and found good degree of agreement. The designed circuit may also be used to study the real time behavior of complex systems modeled by forced and quadratically damped Mathieu-Duffing differential equation. One can also use this circuit in communication system for encryption and decryption of signal by using various combinations of system parameters like damping coefficient (α) and amplitude of parametric excitation (ε).

Acknowledgements

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