Ranking of Generalized Dodecagonal Fuzzy Numbers
Using Incentre of Centroids
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Abstract. This paper describes a ranking method for ordering generalized dodecagonal fuzzy numbers (DoFN) [16]. To find this ranking technique we first split the generalized DoFN into nine plane figures and then calculate the centroids of each plane figure followed by the centroid of these centroids and then find the incentre of this centroid which is a process of defuzzification proposed in this paper. This method is simple in evaluation and can rank various types of fuzzy numbers.

Keywords: Ranking function; centroid points; incentre; generalized dodecagonal fuzzy numbers.

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1. Introduction
2. Preliminaries

2.1. Fuzzy set
A fuzzy set \( \tilde{A} \) in \( X \) (set of real number) is a set of ordered pairs: \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\} \)
\( \mu_{\tilde{A}}(x) \) is called membership function of \( x \) in \( \tilde{A} \) which maps \( X \) to \([0,1]\).

2.2. Fuzzy number
A fuzzy set \( \tilde{A} \) defined on the set of real numbers \( \mathbb{R} \) is said to be a fuzzy number if its membership function \( \mu_{\tilde{A}}: \mathbb{R} \rightarrow [0,1] \) has the following characteristics
(i) \( \tilde{A} \) is normal. It means that there exists an \( x \in \mathbb{R} \) such that \( \mu_{\tilde{A}}(x) = 1 \)
(ii) \( \tilde{A} \) is convex. It means that for every \( x_1, x_2 \in \mathbb{R} \), \( \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0,1] \)
(iii) \( \mu_{\tilde{A}} \) is upper semi-continuous.
(iv) \( \text{supp} (\tilde{A}) \) is bounded in \( \mathbb{R} \).

2.3. \( \alpha \)-cut of fuzzy set
An \( \alpha \)-cut of fuzzy set \( \tilde{A} \) is a crisp set \( A_{\alpha} \) defined as \( A_{\alpha} = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\} \).

2.4. Convex fuzzy set
A fuzzy set \( \tilde{A} \) is a convex fuzzy set if and only if each of its \( \alpha \)-cut \( A_{\alpha} \) is a convex set.

2.6. Generalized dodecagonal fuzzy number
A fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}; u, v, w) \) is said to be generalized dodecagonal fuzzy number if its membership function is given by

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & x \leq a_1 \\
\left(\frac{x - a_1}{a_2 - a_1}\right) & a_1 \leq x \leq a_2 \\
u & a_2 \leq x \leq a_3 \\
u + (v - u)\left(\frac{x - a_3}{a_4 - a_3}\right) & a_3 \leq x \leq a_4 \\
v & a_4 \leq x \leq a_5 \\
v + (w - v)\left(\frac{x - a_5}{a_6 - a_5}\right) & a_5 \leq x \leq a_6 \\
w & a_6 \leq x \leq a_7 \\
v + (w - v)\left(\frac{x - a_7}{a_8 - a_7}\right) & a_7 \leq x \leq a_8 \\
k_2 & a_8 \leq x \leq a_9 \\
u + (v - u)\left(\frac{x - a_9}{a_{10} - a_9}\right) & a_9 \leq x \leq a_{10} \\
u & a_{10} \leq x \leq a_{11} \\
u + (v - u)\left(\frac{x - a_{11}}{a_{12} - a_{11}}\right) & a_{11} \leq x \leq a_{12} \\
0 & a_{12} \leq x 
\end{cases}
\]

where \( 0 < u < v < w \leq 1 \).
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3. Proposed ranking method of dodecagonal fuzzy number

The centroid of a DoFNN is considered to be the balancing point of the dodecagon (Fig. 1). Divide the dodecagon into eight triangles and one hexagon ABM, BCN, CDO, DEP, HIS, IJT, JKU, KWV and EFGRHQ respectively. Let the centroids of nine trapezoids be $G_1$, $G_2$, $G_3$, $G_4$, $G_5$, $G_6$, $G_7$, $G_8$, $G_9$ and $G_{10}$ respectively.

![Generalized Dodecagonal Fuzzy Number](image)

Figure 1: Generalized Dodecagonal Fuzzy Number

The centroid of the nine plane figure is

$$\begin{align*}
G_1 &= \left( \frac{8a_1 + 2a_2 + \frac{a_3}{3}}{3}, \frac{a_3}{3} \right) \\
G_2 &= \left( \frac{8a_2 + 2a_3 + \frac{a_1}{3}}{3}, \frac{a_1}{3} \right) \\
G_3 &= \left( \frac{8a_3 + 2a_1 + \frac{a_2}{3}}{3}, \frac{a_2}{3} \right) \\
G_4 &= \left( \frac{8a_4 + 2a_5 + \frac{a_6}{3}}{3}, \frac{a_6}{3} \right) \\
G_5 &= \left( \frac{8a_5 + 2a_6 + \frac{a_4}{3}}{3}, \frac{a_4}{3} \right) \\
G_6 &= \left( \frac{8a_6 + 2a_4 + \frac{a_5}{3}}{3}, \frac{a_5}{3} \right) \\
G_7 &= \left( \frac{8a_7 + 2a_8 + \frac{a_9}{3}}{3}, \frac{a_9}{3} \right) \\
G_8 &= \left( \frac{8a_8 + 2a_9 + \frac{a_7}{3}}{3}, \frac{a_7}{3} \right) \\
G_9 &= \left( \frac{8a_9 + 2a_7 + \frac{a_8}{3}}{3}, \frac{a_8}{3} \right) \\
\end{align*}$$

(a) $G_1$, $G_2$ and $G_3$ are non-collinear and they form triangle. We define the centroid $G_{10}$ of the triangle with vertices $G_1$, $G_2$ and $G_3$ as

$$G_{10} = \left( \frac{8a_1 + 2a_2 + \frac{a_3}{3} + 2a_4 + \frac{a_5}{3} + \frac{a_6}{3}}{3}, \frac{a_1 + a_2 + a_3}{3} \right)$$

(b) $G_4$, $G_5$ and $G_6$ are non-collinear and they form triangle. We define the centroid $G_{11}$ of the triangle with vertices $G_4$, $G_5$ and $G_6$ as

$$G_{11} = \left( \frac{2(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8) + 3(a_9)}{9}, \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9}{9} \right)$$

(c) $G_7$, $G_8$ and $G_9$ are non-collinear and they form triangle. We define the centroid $G_{12}$ of the triangle with vertices $G_7$, $G_8$ and $G_9$ as

$$G_{12} = \left( \frac{2(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9)}{9} \right)$$

Also, $G_{10}$, $G_{11}$ and $G_{12}$ are non-collinear and they form triangle. We define the incentre $I_A$ of the triangle with vertices $G_{10}$, $G_{11}$ and $G_{12}$ as

$$I_A(x,y) = \left[ \alpha \left( \frac{a_1 + 3(a_2 + a_3) + 2a_4}{9} \right) + b \left( \frac{2(a_4 + a_5 + a_6 + a_7 + a_9)}{18} \right) + c \left( \frac{2a_9 + 3(a_12 + a_{13}) + a_{14}}{9} \right), \alpha \left( \frac{4u + v}{9} \right) + b \left( \frac{5v + w}{9} \right) + c \left( \frac{4u + v}{9} \right) \right]$$
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where $a = \frac{[2(a_1+a_2-a_3-a_4)+2(a_3+a_4-a_1-a_2)+6(a_1+a_2+a_3+a_4)-2(a_1+a_2-a_3-a_4)]^2 + [2(a_1+a_2-a_3-a_4)+2(a_3+a_4-a_1-a_2)-2(a_1+a_2+a_3+a_4)]^2}{16} ;$

$b = \frac{2(a_1+a_2-a_3-a_4)+2(a_3+a_4-a_1-a_2)+6(a_1+a_2+a_3+a_4)-2(a_1+a_2-a_3-a_4)}{9} ;$

$c = \frac{[2(a_1+a_2-a_3-a_4)+2(a_3+a_4-a_1-a_2)-2(a_1+a_2+a_3+a_4)]^2 + [2(a_1+a_2-a_3-a_4)+2(a_3+a_4-a_1-a_2)-2(a_1+a_2+a_3+a_4)]^2}{16}.$

Hence the ranking function of the dodecagonal fuzzy number is defined as $R(A) = \sqrt{a^2 + b^2}.$

4. Numerical example

Example 4.1. $A = (0.2, 0.5, 0.6, 0.9, 1, 1.2, 1.4, 1.6, 1.8, 2, 2.2, 2.4, 2.6, 2.8, 3, 3.2) &$

$B = (0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.9, 2, 2.2, 2.4, 2.6, 2.8, 3, 3.2)\) \)

Solution: By using the existing method [1], we get $A \approx B.$

By using the existing method [17], we get $A \approx B.$

Now by using the proposed method,

$I_A(x,y) = (10.96, 2.24)$

$R(A) = 11.18$

& $I_B(x,y) = (11.29, 2.29)$

$R(B) = 11.52$

Therefore we get, $A < B.$

5. Conclusion

This paper proposes a method that ranks fuzzy numbers which is simple and concrete. This method which is simple and easier for calculation not only gives satisfactory results, but also gives a correct ranking order to problems. Comparative example is used to illustrate the advantages of the proposed method.

REFERENCES

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