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Reduction of Rough Set Based on Generalized Neighborhood System Operator

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Abstract. The theory of generalized neighborhood system-based approximation operators plays an important role in the theory of generalized rough sets since it includes both the neighborhood-based approximation operators and the covering-based approximation operators as its special circumstances. The theory of reduction is one of the most significant directions in rough sets. In this work, the reduction of rough set based on generalized neighborhood system operator is defined and discussed. In particular, the conditions for two generalized neighborhood system operator to generate the same lower or upper approximation are provided.

Keywords: rough set; neighborhood system; reduct; lower approximation operator; upper approximation operator

AMS Mathematics Subject Classification (2010): 03E02，68T30

1. Introduction

Rough set theory, proposed by Pawlak [9], is an effective mathematical approach to deal with uncertainty, granularity and incompleteness of knowledge. It has been successfully applied to intelligent control, economic, biology, data mining, medical diagnosis, and elsewhere [10,11,25,26].

The classical Pawlak's rough sets are based on partition or equivalent relation. This is too restrictive for many applications of Pawlak's rough sets. To address this problem, many extensions of a partition or equivalence relation have been proposed, such as tolerance relation [13], binary relations [17,22], similarity relations [14], coverings [1,7,12,19,21,24], neighborhood systems [2,18].

The rough sets based on generalized neighborhood system is introduced by Lin -Yao [3,4], and then researched by Yao [18], Lin-Michael [2,8], Syau- Lin [15] and Zhang et al. [27]. It is observed in [27] that the generalized neighborhood system-based rough sets is more general than the neighborhood-based (binary relation-based) rough sets and covering-based rough sets. It is well known that reduction theory is an important part of rough set theory [20,23]. However, as to our knowledge, there is no work in the reduction of generalized neighborhood system-based rough sets. The main objective of this paper is to serve such a purpose.

This paper is organized as follows. In Section 2, we recall some notions and results about generalized neighborhood system-based rough sets. In Section 3, we present the theory of reduction of rough sets based on generalized neighborhood system operator. In Section 4, we make a conclusion.

2. Preliminaries

In this section, we will introduce some basic concepts about generalized neighborhood system and rough sets based on generalized neighborhood system.

Definition 2.1. [2] Let U be the universe of discourse, and 2^U denote the power set of *U*. Then a function $N : U \to 2^{2^U}$ is called a generalized neighborhood system operator on *U*. For any $x \in U$, $N(x)$ is non-empty. Usually, $N(x)$ is called generalized neighborhood system of x and any $K \in N(x)$ is called neighborhood of *x*.

Definition 2.2. Let N be a generalized neighborhood system operator of U and $x \in U$. Then the set family

$$MD_N(x) = \{K \in N(x) | \forall V \in N(x) \wedge V \subseteq K \Rightarrow K = V\}$$

is called the minimal description of N at *x*.

Definition 2.3. [5,6] Let N be a generalized neighborhood system operator of U. For each subset X of U, the lower and upper approximations of X, $\underline{N}$ and $\overline{N}$, respectively, are defined as follows:

$$\underline{N}(X) = \{x \in U | \exists K \in N(x), K \subseteq X\}, \ \overline{N}(X) = \{x \in U | \forall K \in N(x), K \cap X \neq \emptyset\}.$$

3. Reduction of rough sets based on generalized neighborhood system

In this section, we shall present the theory of reduction of rough sets based on generalized neighborhood system.

Definition 3.1. Let N be a generalized neighborhood system operator of a universe U and $x \in U$.

(1) For any $K \in N(x)$, we say K is a reducible element of N at point x if there exists an $V \in N(x)$ such that $V \subset K$ (i.e., $V \subseteq K$ and $V \neq K$) otherwise K is an irreducible element of N at point *x*.

(2) If for any $K \in N(x)$, K is irreducible element of N at point x, then we say N is irreducible at point *x*, otherwise N is reducible at point *x*.

Let N be a generalized neighborhood system operator of a universe U. For any reducible element K of N at point x, we define a operator $N_K : U \to 2^{2^U}$ as

$$N_K(z) = \begin{cases} N(z) - K, & z = x; \\ N(z), & others. \end{cases}$$

It is easy to observe that the family $N_K(x) = N(x) - K$ is still non-empty since K is reducible element of N at point x. This shows that N_K is also a generalized neighborhood system operator of the universe U.

Proposition 3.1. Let N be a generalized neighborhood system operator of a universe U and K be a reducible element of N at point x. Then for any $K_1 \in N_K(x)$, K_1 is a reducible element of N at point x if and only if it is a reducible element of N_K at point x.

Proof. $(\Leftarrow)$ It is obviously since $N_K(x) \subseteq N(x)$.

$(\Rightarrow)$ Let K_1 be a reducible element of N at point x. Then there exists an $M \in N(x)$ such that $M \subset K_1$. If $M \neq K$ then $M \in N_K(x)$ and it follows that K_1 is a reducible element of N_K at point x. If M=K, from that K is a reducible element of N at point x, there exists an $H \in N(x)$ such that $H \subset K = M \subset K_1$. Obviously, $H \in N_K(x)$, it follows that K_1 is a reducible element of N_K at point x.

From Proposition 3.1 we observe easily that deleting a reducible element in a neighborhood system will not generate any new reducible elements or make other originally reducible element become irreducible elements of the new neighborhood system. Thus we can get the reduction of a neighborhood system of a universe U by deleting all reducible elements at each point in the same time or by deleting one reducible element at each point in a step. The remainder still consists of a neighborhood system of the universe U, and it is irreducible. Thus we give the definition of neighborhood system reduction as follows:

Definition 3.2. Let N be a generalized neighborhood system operator of a universe U. The generalized neighborhood system operator generated by deleting all reducible elements at each point, is called the reduct of N, and denoted by $reduct(N)$.

Lemma 3.1. Let N be a generalized neighborhood system operator of a universe U and $x \in U$. Then K is a reducible element of N at point x if and only if $K \notin MD_N(x)$.

Proof. $(\Rightarrow)$ Let K be a reducible element of N at point x, then there exists an $V \in N(x)$ such that $V \subset K$, by the definition 2.2, we have $K \notin MD_N(x)$.

$(\Leftarrow)$ Let $K \in N(x)$ but $K \notin MD_N(x)$, then by the definition 2.2, there exists an $S \in N(x)$ such that $S \subset K$, hence K is a reducible element of N at point x.

By Lemma 3.1 and Definition 3.2 we get the following theorem.

Theorem 3.1. Let N be a generalized neighborhood system operator of a universe U.
Then N and $reduct(N)$ have the same minimal description at all $x \in U$.

3.1. For lower approximation operator

Lemma 3.1.1. Let N be a generalized neighborhood system operator of a universe U and K be a reducible element of N at point x. Then N and N_K generate the same lower approximation operator. That is, $\underline{N}(X) = \underline{N_K}(X)$ for all $X \subseteq U$.

Proof. Let $X \subseteq U$. Obviously, $\underline{N_K}(X) \subseteq \underline{N}(X)$ by $\underline{N_K}(x) \subseteq \underline{N}(x)$ for all $x \in X$. Conversely, let $x \in \underline{N}(X)$. By definition 2.3, there exists an $M \in N(x)$ such that $M \subseteq X$. If $M \neq K$ then we have $M \in N_K(x)$, and hence $x \in \underline{N_K}(X)$. If $M = K$, since K is a reducible element of N at point x, then there exists an $V \in N(x)$ such that $V \subset K = M$, which means $V \in N_K(x)$, and so $x \in \underline{N_K}(X)$. Thus $\underline{N_K}(x) \supseteq \underline{N}(x)$.

By Lemma 3.1.1, we get the following corollary.

Corollary 3.1.1. Let N be a generalized neighborhood system operator of a universe U. Then N and $reduct(N)$ generate the same lower approximation operator.

Proposition 3.1.1. Let N_1, N_2 be two irreducibly generalized neighborhood system operators of a universe U generating the same lower approximation operator. Then $N_1 = N_2$.

Proof. For any $K \in N_1(x)$, by definition 2.3, we have $x \in \underline{N_1}(K) = \underline{N_2}(K)$, then there exists an $K' \in N_2(x)$ such that $K' \subseteq K$. Similar to the above proof, there exists an $K'' \in N_1(x)$ such that $K'' \subseteq K' \subseteq K$. Since N_1 is irreducible, then we get $K'' = K$, and then $K = K' \in N_2(x)$. It follows immediately that $K \in N_1 \Leftrightarrow K \in N_2$. Hence $N_1 = N_2$.

By Corollary 3.1.1 and Proposition 3.1.1, we get the following theorem.

Theorem 3.1.1. Let N_1, N_2 be two generalized neighborhood system operators of a universe U. Then N_1, N_2 generate the same lower approximation operator if and only if $reduct(N_1) = reduct(N_2)$.

3.2. For upper approximation operator

By dualizing the results on lower approximation operator we get the following results on upper approximation operator. We omit the similar proofs.

Lemma 3.2.1. Let N be a generalized neighborhood system operator of a universe U and K be a reducible element of N at point x. Then N and N_K generate the same upper approximation operator. That is, $\overline{N} = \overline{N_K}$ for all $X \subseteq U$.

Corollary 3.2.1. Let N be a generalized neighborhood system operator of a universe U. Then N and $reduct(N)$ generate the same upper approximation operator.

By Corollary 3.1.1 and 3.2.1, we get the following corollary.

Corollary 3.2.2. Let N be a generalized neighborhood system operator of a universe U. Then N and $reduct(N)$ generate the same upper and lower approximation operators.

Proposition 3.2.1. Let N_1, N_2 be two irreducibly generalized neighborhood system operators of a universe U generating the same upper approximation operator. Then $N_1 = N_2$.

By Corollary 3.2.1 and Proposition 3.2.1, we get the following theorem.

Theorem 3.2.1. Let N_1, N_2 be two generalized neighborhood system operators of a universe U . Then N_1, N_2 generate the same upper approximation operator if and only if $reduct(N_1) = reduct(N_2)$.

From Theorem 3.1.1 and 3.2.1, we get the following corollary.

Corollary 3.2.3. Let N_1, N_2 be two generalized neighborhood system operators of a universe U. Then N_1, N_2 generate the same upper approximation operator if and only if they generate the same lower approximation operator.

4. Conclusions

In this paper, we discuss the theory of reduction of rough set based on generalized neighborhood system operator, and present the conditions for two generalized neighborhood system operator to generate the same generalized neighborhood system-based lower or upper approximation operator.

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