An Imperfect Production Inventory Problem with Inspection Errors

Nirmal Kumar Mandal

Department of Mathematics, Silda Chandrasekhar College,
Silda, Paschim Medinipur – 721515, India. E-mail: mandal_n_k@yahoo.co.in

Abstract. Classical inventory models are formulated with the assumptions that the items are produced with perfect quality. But in reality, a proportion of the produced items may be defective. Produced items are screened by inspectors. Non-defective items are sold to the customers at the selling price and defective items are sold at salvage value. But due to screening errors customer may receive defective items as non-defective items. Also non-defective items may be sold at a salvage value to the customer. Customer will return the defective item to the manufacturer. In this paper, we formulate a multi-item imperfect quality inventory model with inspection errors in fuzzy environment. Cost parameters and storage space are considered as generalized trapezoidal fuzzy numbers. A numerical example is provided to illustrate the proposed model. Sensitivity analysis is given for the screening rate.

1. Introduction
Inventory models are generally formulated by considering that only the perfect quality items are produced. But in reality, product quality is not always perfect and is usually a function of the production process. The process may deteriorate and produce defective or poor quality items. So, a proportion of the produced items can be found to be defective. Porteus [9] incorporated the effect of defective items into the inventory problem. Rosenblatt and Lee [11] studied the effect of substandard quality, due to deterioration process on lot sizing decisions. Cheng [4] proposed a classical inventory model with demand dependent unit production cost and imperfect production process. He formulated an inventory model with this idea and solved by geometric programming method. Salameh and Jaber [12] developed an inventory problem where all received items are not perfect quality and after 100% screening process imperfect quality items are withdrawn from the inventory and sold at a discounted price. Hayek and Salameh [6] formulated a finite production inventory model and studied the effect of imperfect quality items on it. Rouf et al. [10] developed an inventory model by considering imperfect inspection process. Khan et al. [7] assumed two types of inspection errors with known probabilistic function. Liu et al. [8] studied imperfect production inventory model with inspection errors.

In real life, it is not always possible to obtain the precise information about inventory parameters. This type of imprecise data is not always well represented by random variables selected from probability distribution. So decision making methods under uncertainty are needed. To deal with this uncertainty and imprecise data, the concept of fuzziness can be applied. The inventory cost parameters such as holding cost, set up cost, production cost, reworking cost are assumed to be flexible i.e. fuzzy in nature. These parameters can be represented by fuzzy numbers. An efficient method of ranking fuzzy numbers has a very important role to handle the fuzzy numbers in a fuzzy decision-making problem. Again, in real life situation, it is almost impossible to predict the total inventory cost precisely. These are also imprecise in nature. Decision maker may change these quantities within some limits as per demand of the situation. Hence, these quantities may be assumed uncertain in non-stochastic sense but fuzzy in nature. In this situation, the inventory problem along with constraints can be developed with the fuzzy set theory. Asady [1] used ranking method of a fuzzy numbers by distant minimization technique. Chen [3] developed the theory and applications of Generalized Fuzzy Number and proposed the function principle for fuzzy arithmetic operations. Wang et. al. [15] revised the method of ranking fuzzy numbers with an area between the centroid and original points. Thorani et al. [13] describes a ranking method for ordering fuzzy numbers based on area, mode, spreads and weights of generalized trapezoidal fuzzy number.

In this paper, a multi-item economic order quantity problem with imperfect production without shortage is formulated along with total available storage space restriction. The unit cost of production is considered as demand dependent. If the manufacturer produces a large number of items then the unit cost of items will be reduced. The model is formulated with the assumptions that screening process may contain two types of errors. Defective items may be classified as a non-defective item and passes to the customer. As soon as the customer sees the defective item, he will return the defective item to the manufacturer. Non-defective items may be classified as the defective item and will be sold at a salvage value which will incur a loss to the manufacturer. Scrap items are also sold at a salvage value. Due to volatile nature of the market, the cost parameters and storage space of items are represented here by generalized trapezoidal fuzzy numbers. The objective goal cannot be predicted precisely in real life. The authority may allow the flexibility of these goals to some extent. In this context, the objective functions are considered here in fuzzy environment by giving some tolerance value. The problem is solved by non-linear programming technique. Sensitivity analysis for the screening rate is done.

2. Trapezoidal fuzzy number and its ranking function

Definition 2.1. A Trapezoidal Fuzzy Number (TrFN) $\tilde{A}(=(a_1, a_2, a_3, a_4))$ is defined by the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & if \quad a_1 \leq x < a_2 \\ 1 & if \quad a_2 \leq x < a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & if \quad a_3 \leq x < a_4 \\ 0 & otherwise \end{cases}$$

Definition 2.2. Generalized Fuzzy Number: A fuzzy number $\tilde{A}(= a_1, a_2, a_3, a_4; w)$ is said to be a generalized fuzzy number if its membership function satisfies the following characteristics

(i) $\mu_{\tilde{A}}(x) : R \to [0.1]$ is continuous

(ii) $\mu_{\tilde{A}}(x) = 0$ for $-\infty < x \le a_1, a_4 \le x < \infty$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $a_1 \le x < a_2$ and strictly decreasing on $a_3 \le x < a_4$

(iv) $\mu_{\tilde{A}}(x) = w,$ for all $a_3 \le x < a_4$ where $0 < w \le 1$.

Definition 2.3. Generalized Trapezoidal Fuzzy Number (GTrFN): A fuzzy number $\tilde{A}(= a_1, a_2, a_3, a_4; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function satisfies the following characteristics

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{A}}^L(x) = w \dfrac{x - a_1}{a_2 - a_1} & if \quad a_1 \le x < a_2 \\ w & if \quad a_2 \le x < a_3 \\ \mu_{\tilde{A}}^R(x) = w \dfrac{a_4 - x}{a_4 - a_3} & if \quad a_3 \le x < a_4 \\ 0 & otherwise \end{cases}$$

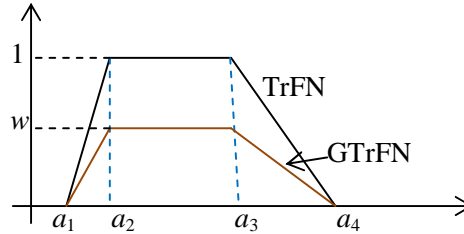**Figure 1:** TrFN and GTrFN

Remark 2.1. Here $\mu_{\tilde{A}}^L(x) : [a_1, a_2] \to [0, w]$ and $\mu_{\tilde{A}}^R(x) : [a_3, a_4] \to [0, w]$ are continuous and strictly monotonic functions. Inverse functions $(\mu_{\tilde{A}}^L)^{-1}(y) : [0, w] \to [a_1, a_2]$ and $(\mu_{\tilde{A}}^R)^{-1}(y) : [0, w] \to [a_3, a_4]$ are also continuous and strictly monotonic.

Remark 2.2. If $w = 1$, then $\tilde{A} = (a_1, a_2, a_3, a_4; 1)$ is a normalized TrFN, otherwise $\tilde{A}$ is said to be a GTrFN.

Remark 2.3. If $a_2 = a_3$ then $\tilde{A} = (a_1, a_2, a_4; w)$ is known as generalized triangular fuzzy number.

Remark 2.4. If $a_2 = a_3$ and $w = 1$ then $\tilde{A} = (a_1, a_2, a_4; 1)$ is known as normalized triangular fuzzy number.

Definition 2.4. Two GTrFNs $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_2)$ are said to be equal if $a_1 = b_1,\ a_2 = b_2,\ a_3 = b_3,\ a_4 = b_4,\ w_1 = w_2$.

Definition 2.5. If $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; w_2)$ then

Addition: $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; w)$ where $w = \min(w_1, w_2)$

Subtraction: $\tilde{A}(-)\tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_4; w)$ where $w = \min(w_1, w_2)$

Scalar multiplication: $k\tilde{A} = (ka_1, ka_2, ka_3, ka_4; w_1)$ when $k > 0$,
$$= (ka_4, ka_3, ka_2, ka_1; w) \text{ when } k < 0.$$

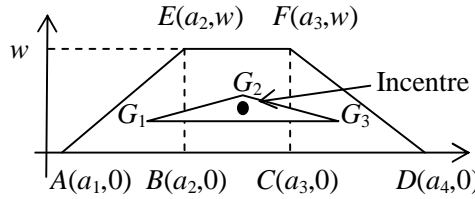**Ranking method of TrFN:** Let $\tilde{A} = (a_1, a_2, a_3, a_4; w)$ be a GTrFN.

Figure 2: Centroid method for ranking of $\tilde{A}$

AEFD is represents a GTrFN $\tilde{A}$. The centroids of the area *ABE*, *EBCF* and *CDF* are $G_1 = \left(\dfrac{a_1 + 2a_2}{3}, \dfrac{w}{3}\right)$, $G_2 = \left(\dfrac{a_2 + a_3}{2}, \dfrac{w}{2}\right)$ and $G_3 = \left(\dfrac{2a_3 + a_4}{3}, \dfrac{w}{3}\right)$ respectively. The incentre

of triangle $G_1 G_2 G_3$ is $I_{\tilde{A}}(\bar{x}, \bar{y}) = \left(\dfrac{\alpha.\dfrac{a_1 + 2a_2}{3} + \beta.\dfrac{a_2 + a_3}{2} + \gamma.\dfrac{2a_3 + a_4}{3}}{\alpha + \beta + \gamma}, \dfrac{\alpha.\dfrac{w}{3} + \beta.\dfrac{w}{2} + \gamma.\dfrac{w}{3}}{\alpha + \beta + \gamma}\right)$

where $\alpha = \dfrac{\sqrt{(a_3 - 3a_2 + 2a_4)^2 + w^2}}{6}$, $\beta = \dfrac{\sqrt{(2a_3 + a_4 - a_1 - 2a_2)^2}}{3}$, $\gamma = \dfrac{\sqrt{(3a_3 - 2a_1 - a_2)^2 + w^2}}{6}$.

Ranking function of GTrFN $\tilde{A}$ is defined as

$$R(\tilde{A}) = \bar{x}.\bar{y} = \frac{\left(\alpha.\dfrac{a_1 + 2a_2}{3} + \beta.\dfrac{a_2 + a_3}{2} + \gamma.\dfrac{2a_3 + a_4}{3}\right)\left(\alpha.\dfrac{w}{3} + \beta.\dfrac{w}{2} + \gamma.\dfrac{w}{3}\right)}{(\alpha + \beta + \gamma)^2}$$

Remark 2.5. If $a_2 = a_3$ i.e., $\tilde{A} = (a_1, a_2, a_4; w)$ be a generalized triangular fuzzy number then its ranking function will be

$$R(\tilde{A}) = \frac{\left(\alpha.\dfrac{a_1 + 2a_2}{3} + \beta.a_2 + \gamma\dfrac{2a_2 + a_4}{3}\right)\left(\alpha.\dfrac{w}{3} + \beta.\dfrac{w}{2} + \gamma.\dfrac{w}{3}\right)}{(\alpha + \beta + \gamma)^2}$$

where $\alpha = \dfrac{\sqrt{(2a_4 - 2a_2)^2 + w^2}}{6}$, $\beta = \dfrac{\sqrt{(a_4 - a_1)^2}}{3}$, $\gamma = \dfrac{\sqrt{(2a_2 - 2a_1)^2 + w^2}}{6}$.

3. Notations and assumptions

A multi-item inventory model is developed under the following notations and assumptions.

Notations

D_i	demand rate per year
Q_i	lot size
C_{1i}	unit production cost

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C_{2i}	holding cost
C_{3i}	set up cost
C_{4i}	screening cost
C_{5i}	selling price of non-defective item
C_{6i}	selling price of defective item
C_{7i}	cost of accepting a defective item
C_{8i}	cost of rejecting a non-defective item
α_i	percentage of non-defective items are classified as defective
β_i	percentage of defective items are classified as non-defective
γ_i	percentage of defective items in Q_i
δ_i	screening rate
t_{1i}	inspection time in a cycle
t_{2i}	time after defective items are screened out
T_i	cycle time
τ_i	space per unit item
H	total available space

Assumptions
1. Production rate is instantaneous,
2. Unit production cost is taken here as inversely related to the demand of the item. For i-th item, unit price $c_{1i} = v_i D_i^{-\sigma_i}$ where scaling constant of c_{1i} be v_i (> 0) and degree of economies of scale be σ_i (>1).
3. Screening of finished goods is done to classify the items. Inspection errors are occurred due to misclassification.
4. Lead time is zero.

4. Mathematical formulation

Consider that lot size of i-th item be Q_i. Screening process is done at the rate of δ_i to classify the defective and non-defective items.
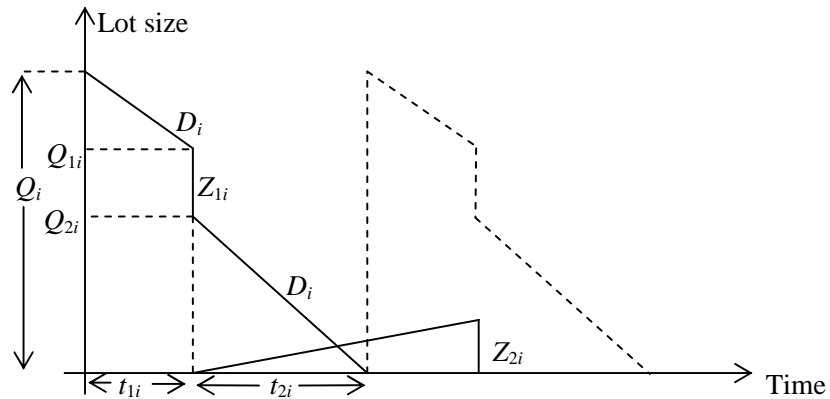

Figure 3: Inventory level

As the inspectors are human beings so human error may occur which may lead to inspection/classification error as
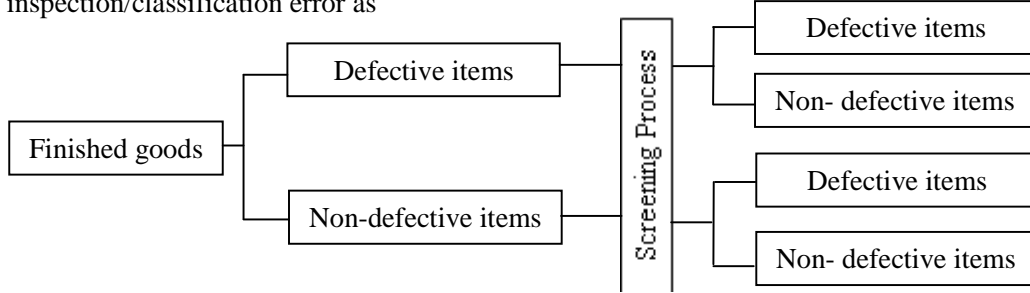

Figure 4: Schematic diagram of screening process

There are four types of classifications occurred in a screening process.

Type 1: no of defective items are classified defective $= Q_i \gamma_i (1 - \beta_i)$

Type 2: no of defective items are classified non-defective $= Q_i \beta_i \gamma_i$

Type 3: no of non-defective items are classified defective $= Q_i \alpha_i (1 - \gamma_i)$

Type 4: no of non-defective items are classified non-defective $= Q_i (1 - \gamma_i)(1 - \alpha_i)$.

So, no of misclassified items as per type 2 and type 3 is $Z_{1i} = Q_i \alpha_i (1 - \gamma_i) + Q_i \beta_i \gamma_i$

No of defective items that are returned from the customers as per type 2 is $Z_{2i} = Q_i \beta_i \gamma_i$

Therefore selling price of non-defective items $= C_{5i} Q_i (1 - \gamma_i)(1 - \alpha_i)$

Salvage value $= C_{6i} Q_i \{ \gamma_i (1 - \beta_i) + \beta_i \gamma_i + \gamma_i (1 - \beta_i) \}$

Total average revenue $(RC) = \sum_{i=1}^{n} \frac{1}{T_i} \left[C_{5i} Q_i (1 - \gamma_i)(1 - \alpha_i) + C_{6i} Q_i \{ \gamma_i (1 - \beta_i) + \beta_i \gamma_i + \gamma_i (1 - \beta_i) \} \right]$

Production cost $= C_{1i} Q_i = v_i D_i^{-\sigma_i} Q_i$

Holding cost for defective, non-defective and returned items

$$C_{2i} \left\{ \frac{1}{2}(Q_i - Q_{2i})t_{1i} + Q_{1i}t_{1i} + \frac{1}{2}Q_{2i}t_{2i} + \frac{1}{2}Z_{2i}T_i \right\}$$

where $Q_{1i} = Q_i - D_i t_{1i}$, $t_{1i} = D_i / \delta_i$, $Q_{2i} = Q_{1i} - Z_{1i}$, $t_{2i} = Q_{2i} / D_i$, $T_i = Q_i (1 - \gamma_i)(1 - \alpha_i) / D_i$

Set up cost $= C_{3i}$

Screening cost $= C_{4i} Q_i$

Misclassification cost for type 2 error $= C_{7i} Q_i \beta_i \gamma_i$

Misclassification cost for type 3 error $= C_{8i} Q_i \alpha_i (1 - \gamma_i)$

Total average cost (TC) = production cost + holding cost + set up cost + screening cost + misclassification cost for type 2 error + misclassification cost for type 3 error

$$= \sum_{i=1}^{n} \frac{1}{T_i} \left[v_i D_i^{-\sigma_i} Q_i + \frac{1}{2} C_{2i} \left\{ (Q_i - Q_{2i})t_{1i} + 2Q_{1i}t_{1i} + Q_{2i}t_{2i} + Z_{2i}T_i \right\} + C_{3i} + C_{4i} Q_i + C_{7i} Q_i \beta_i \gamma_i + C_{8i} Q_i \alpha_i (1 - \gamma_i) \right]$$

Therefore total average profit is $PF(D,Q) = RC - TC$

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$$= \sum_{i=1}^{n} \left[\left\{ C_{5i} + C_{6i} \frac{\gamma_i(1-\beta_i) + \beta_i\gamma_i + \gamma_i(1-\alpha_i)}{(1-\gamma_i)(1-\alpha_i)} - C_{4i} \frac{1}{(1-\gamma_i)(1-\alpha_i)} - C_{7i} \frac{\beta_i\gamma_i}{(1-\gamma_i)(1-\alpha_i)} - C_{8i} \frac{\alpha_i}{1-\alpha_i} \right\} D_i \right.$$

$$- v_i \frac{D_i^{1-\sigma_i}}{(1-\gamma_i)(1-\alpha_i)} + C_{2i} \frac{\gamma_i(\alpha_i+\beta_i) - (\alpha_i+\gamma_i)}{\delta_i(1-\gamma_i)(1-\alpha_i)} D_i Q_i - \frac{1}{2} C_{2i} \left\{ \beta_i\gamma_i + \frac{(1-(\alpha_i+\gamma_i) + \gamma_i(\alpha_i+\beta_i))^2}{(1-\gamma_i)(1-\alpha_i)} \right\} Q_i$$

$$\left. - C_{3i} \frac{1}{(1-\gamma_i)(1-\alpha_i)} \frac{D_i}{Q_i} \right]$$

The problem to maximize $PF(D, Q)$ subject to the total available storage space constraint
$SS(Q) = \sum_{i=1}^{n} \tau_i Q_i \le H, D, Q > 0.$

5. Fuzzy inventory model with imprecise parameters

As the cost parameters $(C_{ji}, j = 1,2 \ldots, 8; i = 1, 2, \ldots, n)$ are not precise in nature, they are taken here as generalized trapezoidal fuzzy numbers (GTrFNs) $(\tilde{C}_{ji} = (C_{ji1}, C_{ji2}, C_{ji3}, C_{ji4}; w_{ji}))$. In case of production cost we take the scaling constant is taken as GTrFN i.e., $\tilde{C}_{1i} = \tilde{v}_i D^{-\sigma_i}$ where $\tilde{v}_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4}; w_{vi})$. We also consider here that the space per unit item ($\tilde{\tau}_i$) as GTrFN i.e. $\tilde{\tau}_i = (\tau_{i1}, \tau_{i2}, \tau_{i3}, \tau_{i4}; w_{\tau i})$. As the parameters are imprecise, the objective goals and constraint goals may fluctuate to some extent which can be represented as fuzzy goals.

$$Max\ P\tilde{F}(D,Q) = \sum_{i=1}^{n} \left[\left\{ \tilde{C}_{5i} + \tilde{C}_{6i} \frac{\gamma_i(1-\beta_i) + \beta_i\gamma_i + \gamma_i(1-\alpha_i)}{(1-\gamma_i)(1-\alpha_i)} - \frac{\tilde{C}_{4i}}{(1-\gamma_i)(1-\alpha_i)} - \frac{\tilde{C}_{7i}\beta_i\gamma_i}{(1-\gamma_i)(1-\alpha_i)} - \frac{\tilde{C}_{8i}\alpha_i}{1-\alpha_i} \right\} D_i \right.$$

$$- \frac{\tilde{v}_i D_i^{1-\sigma_i}}{(1-\gamma_i)(1-\alpha_i)} + \tilde{C}_{2i} \frac{\gamma_i(\alpha_i+\beta_i) - (\alpha_i+\gamma_i)}{\delta_i(1-\gamma_i)(1-\alpha_i)} D_i Q_i - \frac{1}{2} \tilde{C}_{2i} \left\{ \beta_i\gamma_i + \frac{(1-(\alpha_i+\gamma_i) + \gamma_i(\alpha_i+\beta_i))^2}{(1-\gamma_i)(1-\alpha_i)} \right\} Q_i$$

$$\left. - \frac{\tilde{C}_{3i}}{(1-\gamma_i)(1-\alpha_i)} \frac{D_i}{Q_i} \right]$$

subject to $\sum_{i=1}^{n} \tilde{\tau}_i Q_i \le \tilde{H}, \ D, Q > 0.$

As per the section 2, we convert all fuzzy parameters into its ranking functions.

The problem becomes

$$Max\ R(P\tilde{F}(D,Q)) = \sum_{i=1}^{n} \left[\left\{ R(\tilde{C}_{5i}) + R(\tilde{C}_{6i}) \frac{\gamma_i(1-\beta_i) + \beta_i\gamma_i + \gamma_i(1-\alpha_i)}{(1-\gamma_i)(1-\alpha_i)} - \frac{R(\tilde{C}_{4i})}{(1-\gamma_i)(1-\alpha_i)} - \frac{R(\tilde{C}_{7i})\beta_i\gamma_i}{(1-\gamma_i)(1-\alpha_i)} \right. \right.$$

$$\left. - \frac{R(\tilde{C}_{8i})\alpha_i}{1-\alpha_i} \right\} D_i - \frac{R(\tilde{v}_i) D_i^{1-\sigma_i}}{(1-\gamma_i)(1-\alpha_i)} + R(\tilde{C}_{2i}) \frac{\gamma_i(\alpha_i+\beta_i) - (\alpha_i+\gamma_i)}{\delta_i(1-\gamma_i)(1-\alpha_i)} D_i Q_{ii}$$

$$\left. - \frac{1}{2} R\tilde{C}_{2i} \left\{ \beta_i\gamma_i + \frac{(1-(\alpha_i+\gamma_i) + \gamma_i(\alpha_i+\beta_i))^2}{(1-\gamma_i)(1-\alpha_i)} \right\} Q - \frac{R(\tilde{C}_{3i})}{(1-\gamma_i)(1-\alpha_i)} \frac{D_i}{Q_i} \right]$$

subject to $R(SS(Q)) = \sum_{i=1}^{n} R(\tilde{\tau}_i) Q_i \le \tilde{H}, \ D, Q > 0.$

The proposed inventory model is solved in fuzzy environment. In fuzzy set theory, the imprecise objective and constraint function are defined by their membership functions which may be linear/non-linear. For simplicity, we assume here $\mu_{PF}(D,Q)$ and $\mu_{SS}(Q)$ to be linear membership functions for the objective and constraint. Assume that the decision maker's target to achieve his/her profit PF_0 with tolerance limit is PF_t and available storage space is H_0 with tolerance limit H_t. The corresponding membership functions are as follows

$$\mu_{PF}(D,Q) = \begin{cases} 0 & if & R(PF(D,Q)) < PF_0 - PF_t \\ 1 + \dfrac{R(PF(D,Q)) - PF_0}{PF_t} & if & PF_0 - PF_t \le R(PF(D,Q)) < PF_0 \\ 1 & if & PF_0 \le R(PF(D,Q)) \end{cases}$$

and
$$\mu_{SS}(Q) = \begin{cases} 1 & if & R(SS(Q)) < H_0 \\ 1 - \dfrac{R(SS(Q)) - H_0}{H_t} & if & H_0 \le R(SS(Q)) < H_0 + H_t \\ 0 & if & H_0 + H_t \le R(SS(Q)) \end{cases}$$

The DM may give different weights to the objective goal and the constraint goal. If $w_{PF}, w_{SS} \in [0,1]$ are weights imposed to the membership functions $\mu_{PF}(D,Q)$ and $\mu_{SS}(Q)$ then the problem can be formulated (according to Tewari et.al. [13]) with the additive goal programming problem $Max[w_{PF}\mu_{PF}(D,Q) + w_{SS}\mu_{SS}(Q)]$ subject to $D, Q > 0$.

$$MaxU(D,Q) = \left[w_{PF}\left\{1 + \frac{R(PF(D,Q) - PF_0)}{PF_t}\right\} + w_{SS}\left\{1 - \frac{R(SS(Q) - H_0)}{H_t}\right\}\right]$$

subject to $D, Q > 0$.

which is equivalent to

$$MaxU_0(D,Q) = w_{PF}\frac{R(PF(D,Q))}{PF_t} - w_{SS}\frac{R(SS(Q))}{H_t}$$

where $U(D,Q) = U_0(D,Q) + w_{PF}\left(1 - \dfrac{PF_0}{PF_t}\right) + w_{SS}\left(1 + \dfrac{H_0}{H_t}\right)$.

The problem can be solved by any non-linear programming method.

6. Numerical example
A manufacturing company produces three types of items *A*, *B* and *C* with the following information

Table 1: Input parameters

Parameters	Item *A*	Item *B*	Item *C*
$\tilde{v}_i$	(3000,3500,4000,4200;0.97)	(3400,3800,4200,4400;0.97)	(4400,4800,5000,5500;0.99)
$\tilde{C}_{2i}$	(3.5, 4.2, 5.8, 6.2; 0.98)	(2.1, 2.2, 2.8, 3.0; 0.99)	(3.5, 4.2, 5.8, 6.0; 0.98)
$\tilde{C}_{3i}$	(180, 230, 260, 270; 0.99)	(260, 290, 330, 340; 0.99)	(180, 230, 260, 270; 0.99)

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$\tilde{C}_{4i}$	(0.8, 0.9, 1.3, 1.4; 0.9)	(1.2, 1.3, 1.4, 1.5; 0.9)	(0.8, 0.9, 1.3, 1.4; 0.9)
$\tilde{C}_{5i}$	(200, 240, 260, 270; 0.99)	(380, 400, 450, 490; 0.99)	(440, 460, 500, 550; 0.98)
$\tilde{C}_{6i}$	(30, 40, 60, 70; 0.98)	(54, 56, 65, 70; 0.97)	(55, 58, 67, 70; 0.99)
$\tilde{C}_{7i}$	(1100,1200,1400,1500;0.98)	(1600,1800,2000,2200;0.98)	(1700,1800,2000,2200;0.98)
$\tilde{C}_{8i}$	(220, 250, 280, 300; 0.97)	(250, 270, 290, 310; 0.97)	(210, 230, 260, 300; 0.97)
$\tilde{\tau}_i$	(3.5, 3.8, 3.9, 4.4; 0.98)	(5.5, 5.8, 5.9, 6.4; 0.95)	(6.5, 6.8, 6.9, 7.4; 0.97)
α_i	0.25	0.24	0.2
β_i	0.3	0.35	0.33
γ_i	0.28	0.2	0.22
δ_i	2000	2500	2100
σ_i	1.3	1.2	1.1
$PF_0($	248000		
$PF_t($	1000		
$H_0($	1300		
$H_t($	250		
w_{PF}	0.7		
w_{SS}	0.3		

Table 2: Optimal solution

i	1	2	3
D_i^*	1248.000	1695.000	1462.860
Q_i^*	197.5945	256.6690	175.6714
T_i^*	0.08549762	0.09206771	0.07493470
$PF^*(\$)$	247826.2		
$W^*(m^2)$	1353.484		
μ_{PF}	0.8262228		
μ_{SS}	0.7860643		

7. Sensitivity analysis

Sensitivity analysis is done for the change of screening rate (δ_i)

Table 3: Sensitivity analysis on δ_i

δ_i	0%	10%	20%	30%	40%	50%	60%
PF^*	247826.2	270034.6	298535.5	323903.9	349190.0	374459.3	399727.0

From the above table it is clear that if manufacturer increase the screening rate of items he will gain more profit.

8. Conclusion

Here we have formulated a multi-item profit maximization imperfect production inventory model with limited storage area in crisp and fuzzy environment. The screening process may contain two types of errors. Defective items may be classified as non-defective item and non-defective items may be classified as defective items. Unit cost of production is taken as demand dependent. Cost parameters and storage space of unit items are considered here as generalized trapezoidal fuzzy numbers. These fuzzy numbers are then defuzzified by centroid method. Problem is solved by non-linear programming method. Sensitivity analysis on the screening rate is shown. It is observed that manufacturer will gain more profit if he increase the screening rate of items.

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