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Spanning Hypertrees and Spanning SuperHypertrees

Takaaki Fujita

Independent Researcher, Tokyo, Japan.
Email: **takaaki.fujita060@gmail.com**

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Abstract. Graph theory provides a rigorous foundation for representing relationships and connectivity through vertices and edges. Hypergraphs extend this framework by introducing *hyperedges* that connect more than two vertices. Superhypergraphs further enhance the model via iterated powerset constructions, capturing hierarchical and self-referential structures among hyperedges. A spanning tree is a connected, acyclic subgraph that covers all vertices of a graph with exactly $|V| - 1$ edges. A spanning hypertree is a connected, Berge-acyclic subhypergraph of a uniform hypergraph that spans all vertices with hypertree structure. In this paper, we study the notion of a *spanning superhypertree* as the natural spanning tree concept within superhypergraphs. We also discuss several concrete real-world examples of spanning superhypertrees and analyze their structural properties.

Keywords: SuperHyperGraph, HyperGraph, Spanning Superhypertree, Spanning Tree, Spanning Hypertree

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1. Introduction

1.1. SuperHyperGraphs

Graphs provide a widely used abstraction for relational structures: vertices encode entities and edges encode pairwise interactions [1]. In many applications, however, the fundamental dependency is not binary. Examples include co-authorship groups, multi-protein complexes, and multi-party transactions, where interactions naturally involve three or more participants. To represent such *higher-order* relations, a finite *hypergraph* replaces edges by hyperedges that may join any nonempty subset of vertices [2, 3, 42, 43].

Higher-order incidence alone is often insufficient when the data exhibit *nested or multilevel* organization. Modern systems routinely contain communities inside communities, modules inside subsystems, and teams within departments. A convenient set-theoretic mechanism to encode such layering is to iterate the powerset operator. This motivates finite *SuperHyperGraphs*, in which set-valued objects obtained at one stage can be treated as “vertices” at the next stage, producing explicit nesting that captures hierarchical aggregation in addition to multiway interaction [4–6]. In this sense, SuperHyperGraphs combine two complementary features: (i) hypergraph-style multi-arity and (ii) multi-level structure (groups, groups of groups, and so forth), a viewpoint that aligns well with layered networks, modular architectures, and multiscale relational data. Hierarchical encodings of related flavor have also been studied in complex-network and applied-science contexts [7, 8].

Unless explicitly stated otherwise, the iteration index n in $\mathrm{P}^{n}(\cdot)$ and the level n of an n–SuperHyperGraph are taken to be nonnegative integers. For quick reference, Table 1 summarizes the main distinctions among graphs, hypergraphs, and n-SuperHyperGraphs.

Table 1: A compact comparison of graphs, hypergraphs, and n-SuperHyperGraphs.

Aspect	*Graph*	*Hypergraph*	*n-SuperHyperGraph*
Basic objects	Vertices and edges.	Vertices and hyperedges.	n -supervertices and superhyperedges.
Incidence type	Each edge joins exactly two vertices.	A hyperedge may join any nonempty subset of vertices.	A superhyperedge joins any nonempty subset of supervertices (allowing nested supervertices).
Canonical notation	$G = (V, E)$, $E \subseteq \binom{V}{2}$.	$H = (V, E)$, $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.	$\mathrm{SHG}^{(n)} = (V, E)$, $V \subseteq \mathcal{P}^{n}(V_0)$, $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$.
Primary emphasis	Pairwise relations.	Higher-order relations.	Higher-order *and* hierarchical (multi-level) relations.

1.2. Spanning trees and spanning hypertrees

In an ordinary graph $G = (V, E)$, a *spanning tree* is a connected acyclic subgraph that contains all vertices; equivalently, it has exactly $|V| - 1$ edges [9, 10]. For hypergraphs, several non-equivalent notions of "tree-likeness" exist. A commonly used one is based on *Berge-acyclicity*: informally, a *spanning hypertree* is a connected Berge-acyclic spanning subhypergraph (often studied for uniform hypergraphs) that plays the role of a tree in the hypergraph setting [11, 12].

1.3. Our contributions

The theory of spanning structures is central in graph and hypergraph theory, and hierarchical generalizations such as SuperHyperGraphs provide a natural language for multilevel relational data. Nevertheless, spanning-tree-type concepts tailored to SuperHyperGraphs have not been widely developed. To address this gap, this paper introduces the notion of a *spanning SuperHypertree*. We formalize the definition and investigate its basic structural properties, aiming to connect classical spanning tree ideas with the higher-order and hierarchical expressive power of SuperHyperGraphs. For reference, Table 2 presents a concise comparison of spanning trees, spanning hypertrees, and spanning superhypertrees (under Berge-style acyclicity).

Table 2: Concise comparison of spanning trees, spanning hypertrees, and spanning superhypertrees (under Berge-style acyclicity).

Aspect	*Spanning tree*	*Spanning hypertree*	*Spanning superhypertree*
Ambient structure	Graph $G = (V, E)$.	(Typically uniform) hypergraph $H = (V, \mathcal{E})$.	n –SuperHyperGraph $\mathrm{SHG}^{(n)} = (V, E, \partial)$ with $V \subseteq \mathcal{P}^n(V_0)$.
Edges / incidences	Edges join exactly two vertices.	Hyperedges join any nonempty subset of vertices (often fixed size h).	Superedges incident to nonempty subsets of *supervertices* (nested objects allowed).
Spanning condition	Uses all vertices: $V(T) = V(G)$.	Uses all vertices: $V(T) = V(H)$.	Uses all supervertices: $V(T) = V(\mathrm{SHG}^{(n)})$.
Connectedness notion	Graph-theoretic connectedness (paths).	Hypergraph connectedness (e.g. via Berge paths / incidence graph).	Superconnectedness via superpaths (Berge-style alternating vertex–edge sequences).
Acyclicity notion	No graph cycle.	Berge-acyclic (no Berge cycle), or other hypertree axioms depending on the chosen definition.	Berge-style superacyclic (no supercycle as defined for SuperHyperGraphs).
Typical size law	$\lvert E(T)\rvert = \lvert V\rvert - 1$.	No universal single formula (depends on hypertree notion and uniformity).	No universal single formula (depends on the superlevel n and acyclicity notion).
Reduction / consistency	Base case.	For $n = 0$, superpaths/supercycles reduce to Berge paths/cycles.	For $n = 0$ and 2-uniform, it reduces to spanning trees; for $n = 0$ and h-uniform, it matches spanning Berge-hypertrees.

2. Preliminaries

We collect the basic terminology and notation used in what follows. Unless explicitly stated otherwise, all graphs considered are finite.

2.1. SuperHyperGraphs

Graph theory provides a rigorous foundation for representing relationships and connectivity through vertices and edges [1, 13]. A classical hypergraph generalizes an ordinary graph by permitting an edge to connect an arbitrary (finite) number of vertices, which makes it suitable for representing multiway relationships [3, 14, 15]. A SuperHyperGraph carries this idea further by forming vertices and edges from iterated powersets of a base set; this viewpoint has appeared in several recent contexts [16–18].

Reported applications include, among others, molecular structure modeling, complex network analysis, and signal processing [7,19–21]. Throughout, the level n is a fixed nonnegative integer.

Definition 1. (Base set). A *base (ground) set* is a fixed finite set S from which higher-level objects are generated:

$$S = \{x \mid x \text{ belongs to the chosen domain}\}.$$

All structures introduced below ultimately draw their elements from S.

Definition 2. (Powerset) [22,23]. Given a set X, its powerset is

$$\mathcal{P}(X) = \{A \subseteq X\}.$$

We also use the *nonempty* powerset $\mathcal{P}^*(X) := \mathcal{P}(X)\setminus\{\varnothing\}$.

Definition 3. (Iterated powerset) [24-27]. For $k \in \mathbb{N}_0$ define

$$\mathcal{P}^0(X) := X, \qquad \mathcal{P}^{k+1}(X) := \mathcal{P}\big(\mathcal{P}^k(X)\big).$$

For the nonempty version set

$$(\mathcal{P}^*)^0(X) := X, \qquad (\mathcal{P}^*)^{k+1}(X) := \mathcal{P}^*\big((\mathcal{P}^*)^k(X)\big).$$

Definition 4. (Hypergraph) [15,28]. A *hypergraph* is a pair $H = (V(H), E(H))$ with $V(H) \neq \varnothing$ and $E(H) \subseteq \mathcal{P}^*(V(H))$. Throughout this paper both $V(H)$ and $E(H)$ are finite.

Definition 5. (n-SuperHyperGraph) [4,29]. Fix a finite base set V_0 and a level $n \in \mathbb{N}_0$. An *n-SuperHyperGraph over V_0* is a triple

$$\mathrm{SHG}^{(n)} = (V,\ E,\ \partial),$$

where

- $V \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-supervertices*;
- E is a finite set of *(super)edge identifiers*;
- $\partial : E \to \mathcal{P}^*(V)$ is an *incidence map* sending each edge to a nonempty finite subset of V. For $e \in E$, the set $\partial(e) \subseteq V$ is called the *(super)edge incidence set*.

For reference, Figure 1 presents a schematic illustration of a HyperGraph and a SuperHyperGraph.

Remark 6 (Simple, uniform, and nonempty-tier options). (i) *Simple*: ∂ is injective (no parallel superedges). (ii) *k-uniform*: $|\partial(e)| = k$ for all $e \in E$. (iii) To exclude empties at every tier, one may require $V \subseteq (\mathcal{P}^*)^n(V_0)$.

Remark 7. (Subset presentation). If parallel superedges are unnecessary, one may identify each edge with its incidence set and work with a pair $(V, \mathcal{E})$ where $\mathcal{E} \subseteq \mathcal{P}^*(V)$. This is equivalent to Definition 5 by taking $E := \mathcal{E}$ and $\partial := \mathrm{id}$.

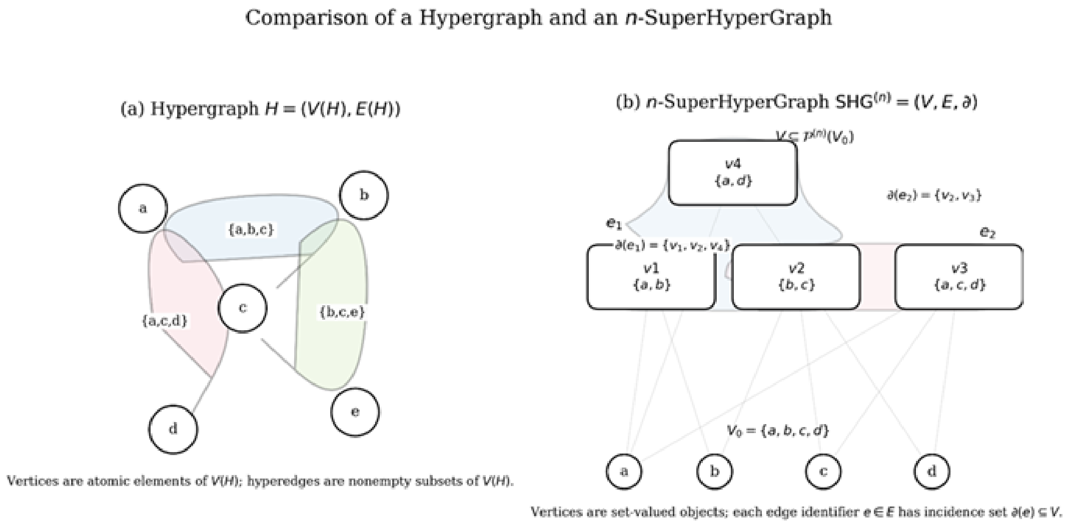

Figure 1: HyperGraph and SuperHyperGraph

2.2. Spanning tree and hypertree

A spanning tree is a connected, acyclic subgraph that includes all vertices of the graph with exactly $|V|-1$ edges [9, 10]. A spanning hypertree is a connected, Berge-acyclic subhypergraph of a uniform hypergraph that spans all vertices with hypertree structure[11, 12].

Definition 8. (Spanning tree of a graph) [9, 10]. Let $G=(V,E)$ be a finite (simple, undirected) graph. A subgraph $T=(V,E_T)$ of G is a *spanning tree* of G if T is connected and acyclic. Equivalently, $|E_T|=|V|-1$ and T is connected.

Example 9. (Spanning tree in a simple graph). Let $G=(V,E)$ with

$$V=\{1,2,3,4\}, \qquad E=\{\{1,2\},\{2,3\},\{3,4\},\{1,4\},\{1,3\}\}.$$

Consider the subgraph $T=(V,E_T)$ where

$$E_T=\{\{1,2\},\{2,3\},\{3,4\}\}.$$

Then $|E_T|=3=|V|-1$ and T is connected (the unique simple path $1-2-3-4$ joins any two vertices). Since a connected simple graph on $|V|$ vertices with $|V|-1$ edges is acyclic, T is a spanning tree of G. Equivalently, T is a tree (no cycles) and uses all vertices of G.

Definition 10. (Hypertrees in uniform hypergraphs: recursive h-hypertrees). (cf. [11,12,30]) Fix $h\geq 2$. An *h-hypertree* is an h-uniform hypergraph $T=(X,\mathcal{E})$ defined recursively as follows:

1. If $|X|=h$, then T has the unique edge X.
2. If $|X|\geq h+1$, there exists a vertex $x\in X$ such that, writing $\mathcal{E}(x)=\{E_1,\dots,E_q\}$ for the edges of T containing x, the family $\{E_1\backslash\{x\},\dots,E_q\backslash\{x\}\}$ induces an $(h-1)$-hypertree on $X\backslash\{x\}$ and the remaining edges (those not containing x) induce an h-hypertree on $X\backslash\{x\}$.

(For $h = 2$, this coincides with the usual notion of a tree.) *Spanning h-hypertree in H.* If $H = (V, E)$ is an h-uniform hypergraph, a subhypergraph T of H is a *spanning h-hypertree of H* if T is an h-hypertree and $V(T) = V(H)$ (i.e., T spans all vertices of H and uses only edges of H).

Example 11. (A 3-hypertree and a spanning 3-hypertree). Fix $h = 3$. Let $X = \{a, b, c, d\}$ and define the 3-uniform hypergraph

$$T = (X,\ \mathcal{E}_T), \qquad \mathcal{E}_T = \{\{a, b, c\},\ \{a, c, d\},\ \{b, c, d\}\}.$$

We verify that T is a 3-hypertree by the given recursion. Choose $x = a$. The edges of T containing a are $\{a, b, c\}$ and $\{a, c, d\}$. Removing a from these gives the 2-edges

$$\{b, c\},\ \{c, d\} \subseteq X \backslash \{a\} = \{b, c, d\}.$$

These two 2-edges form a (connected, acyclic) graph on $\{b, c, d\}$, hence an $(h - 1) = 2$-hypertree on $X \backslash \{a\}$. The remaining edges of T that do not contain a form the family

$$\{\{b, c, d\}\},$$

which, on the vertex set $X \backslash \{a\} = \{b, c, d\}$, is exactly the base case of a 3-hypertree (a single 3-edge on 3 vertices). Thus the recursive conditions are satisfied, and T is a 3-hypertree.

To exhibit a *spanning* 3-hypertree inside a larger 3-uniform hypergraph, enlarge T to

$$H = (X,\ \mathcal{E}_H), \qquad \mathcal{E}_H = \mathcal{E}_T \cup \{\{a, b, d\}\}.$$

Then T is a subhypergraph of H with $V(T) = V(H) = X$, hence T is a spanning 3-hypertree of H.

Example 12. (Spanning 3-hypertree — handoff teams in a project (real life)). **Formal instance.** Let $h = 3$. Take the vertex set

$$X = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$$

and the 3-uniform hyperedge family

$$\mathcal{E} = \{\ E_1 = \{w_1, w_2, w_3\},\ \ E_2 = \{w_3, w_4, w_5\},\ \ E_3 = \{w_5, w_6, w_7\}\ \}.$$

Let the host hypergraph be $H = (X, \mathcal{E})$. Then $T = (X, \mathcal{E})$ is a spanning 3-hypertree of H:

- *Spanning:* $V(T) = X = V(H)$.
- *Connectivity:* E_1 meets E_2 at w_3, and E_2 meets E_3 at w_5, so the incidence graph is a path E_1—E_2—E_3.
- *Berge-acyclicity:* Any Berge cycle would require $E_1 \cap E_3 \neq \varnothing$, but $E_1 \cap E_3 = \varnothing$; hence no cycle exists.

Thus T is a connected, Berge-acyclic 3-uniform subhypergraph spanning all vertices, i.e., a spanning 3-hypertree.

Each hyperedge models a *handoff team* of three people working a project stage. Consecutive stages share exactly one member (w_3 then w_5) to transfer know-how. All seven workers are covered (spanned), and the pipeline has no loops.

Example 13. (Spanning 4-hypertree — co-requisite course blocks across terms (real life)). **Formal instance.** Let $h = 4$. Take

$$X = \{a, b, c, d, e, f, g, h, i, j\}, \qquad \mathcal{E} = \{\ E_1 = \{a, b, c, d\},\ E_2 = \{d, e, f, g\},\ E_3 = \{g, h, i, j\}\ \}.$$

Let $H = (X, \mathcal{E})$. Then $T = (X, \mathcal{E})$ is a spanning 4-hypertree of H:

- *Spanning:* $V(T) = X = V(H)$.
- *Connectivity:* $E_1 \cap E_2 = \{d\}$ and $E_2 \cap E_3 = \{g\}$, so the incidence graph is the path E_1—E_2—E_3.
- *Berge-acyclicity:* A Berge cycle would force $E_1 \cap E_3 \neq \varnothing$, but $E_1 \cap E_3 = \varnothing$; contradiction.

Hence, T is a connected, Berge-acyclic 4-uniform subhypergraph that spans all vertices—i.e., a spanning 4-hypertree.

Each hyperedge represents a *term block* of four co-requisite courses. Successive terms share exactly one course (d then g) to ensure curricular continuity. All courses in the program are covered without cycles.

3. Main results: spanning SuperHyperTree

A spanning superhypertree is a connected, Berge-acyclic substructure of a superhypergraph, covering all supervertices, generalizing spanning trees and hypertrees.

Definition 14. (Berge-style superpaths and supercycles). Let $\mathrm{SHG}^{(n)} = (V, E, \partial)$ be an n–SuperHyperGraph (so $\partial: E \to \mathcal{P}^*(V)$).

- A *superpath of length* $\ell \geq 1$ is a sequence
$$v_0, e_1, v_1, e_2, \dots, e_\ell, v_\ell$$
with $v_i \in V$, $e_j \in E$, such that $v_{j-1}, v_j \in \partial(e_j)$ for all j, the vertices $v_0, \dots, v_\ell$ are pairwise distinct, and the edges $e_1, \dots, e_\ell$ are pairwise distinct.
- A *supercycle of length* $\ell \geq 2$ is a sequence
$$v_0, e_1, v_1, e_2, \dots, e_\ell, v_\ell$$
with $v_\ell = v_0$, the intermediate vertices $v_1, \dots, v_{\ell-1}$ pairwise distinct, and the edges $e_1, \dots, e_\ell$ pairwise distinct, and $v_{j-1}, v_j \in \partial(e_j)$ for all j (indices modulo ℓ).

We say $\mathrm{SHG}^{(n)}$ is *connected* if every two vertices can be joined by a superpath, and it is *Berge-acyclic* if it contains no supercycle.

Example 15. (Real-World Illustration: Multi-Tier Supply Contracts). Consider a 3–SuperHyperGraph modeling a supply chain with atomic parts
$$V_0 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}.$$

Level 1 (kits) in $\mathcal{P}^1(V_0)$:
$$K_1 = \{\mathrm{A}, \mathrm{B}\}, \quad K_2 = \{\mathrm{B}, \mathrm{C}\}, \quad K_3 = \{\mathrm{C}, \mathrm{D}\}.$$

Level 2 (bundles) in $\mathcal{P}^2(V_0)$:
$$B_1 = \{K_1, K_2\}, \qquad B_2 = \{K_2, K_3\}.$$

Level 3 (contracts) in $\mathcal{P}^3(V_0)$:
$$C_1 = \{B_1\}, \qquad C_2 = \{B_1, B_2\}, \qquad C_3 = \{B_2\}.$$
Define the 3–supervertex set and edges
$$V = \{C_1, C_2, C_3\}, \qquad E = \{e_{12}, e_{23}, e_{31}\}.$$
The incidence map $\partial: E \to \mathcal{P}^*(V)$ encodes cooperative agreements:
$$\partial(e_{12}) = \{C_1, C_2\}, \qquad \partial(e_{23}) = \{C_2, C_3\}, \qquad \partial(e_{31}) = \{C_3, C_1\}.$$

Then $\mathrm{SHG}^{(3)} = (V, E, \partial)$ models three contract clusters (C_1, C_2, C_3) with superedges expressing shared logistics or financing channels between them.

Berge–style superpath. The sequence

$$C_1,\ e_{12},\ C_2,\ e_{23},\ C_3$$

is a superpath of length 2 since $C_1, C_2 \in \partial(e_{12})$ and $C_2, C_3 \in \partial(e_{23})$, with distinct vertices and edges.

Berge–style supercycle. The sequence

$$C_1,\ e_{12},\ C_2,\ e_{23},\ C_3,\ e_{31},\ C_1$$

is a supercycle of length 3 because each consecutive pair lies in the incidence of the corresponding edge and all edges are distinct. Hence the structure is connected (every two vertices are joined by a superpath) but not Berge–acyclic (it contains a supercycle).

Definition 16. (Spanning superhypertree). Let $\mathrm{SHG}^{(n)} = (V, E, \partial)$ be an $n-$SuperHyperGraph. A substructure

$$T := \left(V,\ E_T,\ \partial \restriction_{E_T}\right), \qquad \varnothing \neq E_T \subseteq E,$$

is called a *spanning superhypertree* of $\mathrm{SHG}^{(n)}$ if T is connected and Berge-acyclic. Equivalently, T spans all vertices of $\mathrm{SHG}^{(n)}$, uses only edges of $\mathrm{SHG}^{(n)}$, has no supercycle, and connects every vertex pair via a superpath.

Remark 17. (Minimality by edge deletion). If T is a spanning superhypertree and $e \in E_T$, then $T - e$ is disconnected. Indeed, removing any edge on some superpath between two vertices breaks all superpaths between them; if it did not, e would lie on a supercycle, contradicting acyclicity.

Example 18. (Graph case $n = 0$ (reduces to a spanning tree)). Let $V = \{1,2,3,4\}$ and consider the 2-uniform $\mathrm{SHG}^{(0)} = (V, E, \partial)$ with edges

$$\partial(e_{12}) = \{1,2\},\ \partial(e_{23}) = \{2,3\},\ \partial(e_{34}) = \{3,4\},\ \partial(e_{14}) = \{1,4\}.$$

Take

$$E_T = \{e_{12}, e_{23}, e_{34}\}.$$

Then $T = (V, E_T, \partial|_{E_T})$ is connected (paths $1-2-3-4$) and has no supercycle (three edges form a simple path), hence a spanning superhypertree. By Theorem 24, G_T is the usual spanning tree on V with edge set $\{\{1,2\},\{2,3\},\{3,4\}\}$.

Example 19. (Proper super case $n = 1$ (supervertices are subsets of a base set)). Let the base set be $V_0 = \{a, b, c, d\}$ and set

$$V = \{v_1, v_2, v_3\} = \{\{a,b\},\ \{b,c\},\ \{c,d\}\} \subseteq \mathcal{P}^1(V_0).$$

Define the 2-uniform 1–SuperHyperGraph $\mathrm{SHG}^{(1)} = (V, E, \partial)$ by

$$\partial(e_{12}) = \{v_1, v_2\}, \qquad \partial(e_{23}) = \{v_2, v_3\}, \qquad \partial(e_{13}) = \{v_1, v_3\}.$$

Take the edge subset $E_T = \{e_{12}, e_{23}\}$. Then $T = (V, E_T, \partial|_{E_T})$ is connected (superpath $v_1, e_{12}, v_2, e_{23}, v_3$) and Berge-acyclic (no supercycle since only two edges are used), hence a spanning superhypertree in the level-1 setting. Note that the “vertices” here are *subsets* of V_0, so this example is genuinely beyond ordinary graphs/hypergraphs.

Example 20. (Spanning superhypertree ($n = 1$) — overlapping project teams (real life)). **Formal instance.** Let the ground set of people be

$$V_0 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}.$$

Work at level $n = 1$ so vertices are nonempty subsets of V_0. Define the supervertex set

$$V = \{v_1 = \{\mathrm{A}, \mathrm{B}\},\ v_2 = \{\mathrm{B}, \mathrm{C}, \mathrm{D}\},\ v_3 = \{\mathrm{D}, \mathrm{E}\},\ v_4 = \{\mathrm{E}, \mathrm{F}\}\} \subseteq \mathcal{P}^*(V_0).$$

Let the edge set be $E = \{e_{12}, e_{23}, e_{34}\}$ with incidence map

$$\partial(e_{12}) = \{v_1, v_2\}, \qquad \partial(e_{23}) = \{v_2, v_3\}, \qquad \partial(e_{34}) = \{v_3, v_4\}.$$

Consider the substructure $T = (V, E_T, \partial|_{E_T})$ with $E_T = E$. Then:

- *Spanning:* $V(T) = V$ (all supervertices are included).
- *Connectedness:* for any v_i, v_j there is a superpath along the chain
$$v_1, e_{12}, v_2, e_{23}, v_3, e_{34}, v_4.$$
- *Berge-acyclicity:* the incidence graph on $V \cup E$ is a path, hence contains no supercycle.

Thus T is a spanning superhypertree of the 1–SuperHyperGraph $\mathrm{SHG}^{(1)} = (V, E, \partial)$.

Each supervertex v_i is a *team pod* (subset of people). Edges record handoffs between pods that share a member (e.g., $v_1 \cap v_2 = \{\mathrm{B}\}$). The chain covers all pods (spanning) and has no loop, modeling a linear, non-cyclic delivery pipeline.

Example 21. (Spanning superhypertree ($n = 2$) — compliance dossier bundles (real life)). **Formal instance.** Let the base set of atomic documents be

$$V_0 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}.$$

Work at level $n = 2$, so supervertices are *families of document sets*, i.e. elements of $\mathcal{P}(\mathcal{P}(V_0))$. Define

$$\begin{aligned} v_1 &= \{\{\mathrm{A}\}, \{\mathrm{A}, \mathrm{B}\}\}, \\ v_2 &= \{\{\mathrm{B}\}, \{\mathrm{B}, \mathrm{C}\}\}, \\ v_3 &= \{\{\mathrm{C}\}, \{\mathrm{C}, \mathrm{D}\}\}, \\ v_4 &= \{\{\mathrm{D}\}\}, \end{aligned} \qquad V = \{v_1, v_2, v_3, v_4\} \subseteq \mathcal{P}_2(V_0).$$

Let $E = \{e_{12}, e_{23}, e_{34}\}$ and define the incidence map

$$\partial(e_{12}) = \{v_1, v_2\}, \qquad \partial(e_{23}) = \{v_2, v_3\}, \qquad \partial(e_{34}) = \{v_3, v_4\}.$$

Set $E_T = E$ and $T = (V, E_T, \partial|_{E_T})$. Then:

- *Spanning:* $V(T) = V$.
- *Connectedness:* there is a superpath $v_1, e_{12}, v_2, e_{23}, v_3, e_{34}, v_4$ joining any endpoints.
- *Berge-acyclicity:* with only the three edges arranged in a chain, no supercycle can occur.

Therefore, T is a spanning superhypertree of $\mathrm{SHG}^{(2)} = (V, E, \partial)$.

Each supervertex v_i is a *dossier bundle*: a collection of related document-sets (policies, reports). Consecutive bundles share thematic subsets (e.g., $\{\mathrm{B}\}$ or $\{\mathrm{B}, \mathrm{C}\}$), so review proceeds linearly across quarters. All bundles are covered (spanned) without circular dependencies.

Example 22. (Spanning SuperHypertree (n=3): Multi-Agency Disaster Response). Let the atomic resources be

$$V_0 = \{\mathrm{Med},\ \mathrm{Log},\ \mathrm{Comms},\ \mathrm{Safety}\}.$$

Level 1 (taskforces; subsets of resources) in $\mathcal{P}^1(V_0)$:

$$T_1 = \{\mathrm{Med}, \mathrm{Comms}\}, \quad T_2 = \{\mathrm{Med}, \mathrm{Log}\}, \quad T_3 = \{\mathrm{Log}, \mathrm{Safety}\}, \quad T_4 = \{\mathrm{Comms}, \mathrm{Safety}\}.$$

Level 2 (operations; sets of taskforces) in $\mathcal{P}^2(V_0)$:

$$O_A = \{T_1, T_2\}, \qquad O_B = \{T_2, T_3\}, \qquad O_C = \{T_1, T_4\}.$$

Level 3 (incident plans; sets of operations) in $\mathcal{P}^3(V_0)$:

$$L_1 = \{O_A\}, \qquad L_2 = \{O_A, O_B\}, \qquad L_3 = \{O_B\}, \qquad L_4 = \{O_C\}.$$

Define the 3–supervertex set and an edge set by

$$V = \{L_1, L_2, L_3, L_4\} \subseteq \mathcal{P}^3(V_0), \qquad E = \{e_{12}, e_{23}, e_{34}, e_{14}\}.$$

Define the incidence map $\partial: E \to \mathcal{P}^*(V)$ as

$$\partial(e_{12}) = \{L_1, L_2\}, \quad \partial(e_{23}) = \{L_2, L_3\}, \quad \partial(e_{34}) = \{L_3, L_4\}, \quad \partial(e_{14}) = \{L_1, L_4\}.$$

Consider the substructure

$$T := \left(V,\ E_T,\ \partial \restriction_{E_T}\right) \quad \text{with} \quad E_T = \{e_{12}, e_{23}, e_{34}\} \subseteq E.$$

Then T spans all vertices V and is connected via the superpaths

$$L_1 \overset{e_{12}}{\leftrightarrow} L_2 \overset{e_{23}}{\leftrightarrow} L_3 \overset{e_{34}}{\leftrightarrow} L_4$$

Moreover, T is Berge–acyclic: the edges in E_T form a simple chain and there is no sequence $v_0, e_1, v_1, e_2, \dots, e_\ell, v_\ell = v_0$ with pairwise distinct edges e_i witnessing a supercycle. Hence T is a *spanning superhypertree* of the 3–SuperHyperGraph $\mathrm{SHG}^{(3)} = (V, E, \partial)$.

Theorem 23. (Restriction is an n–SuperHyperGraph). *If* $\mathrm{SHG}^{(n)} = (V, E, \partial)$ *is an* n*–SuperHyperGraph and* $E_T \subseteq E$ *is nonempty, then* $T = \left(V, E_T, \partial \restriction_{E_T}\right)$ *is an* n*–SuperHyperGraph. In particular, every spanning superhypertree is (by definition) an* n*–SuperHyperGraph.*

Proof: By hypothesis, $\partial: E \to \mathcal{P}^*(V)$ takes edges to nonempty subsets of V. Hence its restriction $\partial \restriction_{E_T}: E_T \to \mathcal{P}^*(V)$ does the same. The vertex set remains V, which is finite, so T satisfies the axioms of an n–SuperHyperGraph. □

Theorem 24. (Generalization of spanning trees in graphs). *Let* $n = 0$ *and suppose* $\mathrm{SHG}^{(0)} = (V, E, \partial)$ *is 2-uniform, i.e.* $|\partial(e)| = 2$ *for all* $e \in E$*. Identify each* e *with the (unordered) pair* $\partial(e) \subseteq V$ *to obtain a simple graph* $G = (V, E')$ *with* $E' = \{\partial(e): e \in E\}$*. Then:*

A substructure $T = (V, E_T, \partial|_{E_T})$ *is a spanning superhypertree of* $\mathrm{SHG}^{(0)}$ *if and only if the simple graph* $G_T = (V, \{\partial(e): e \in E_T\})$ *is a spanning tree of* G*.*

Proof: In the 2-uniform, $n = 0$ case, a superpath is exactly a usual graph path (each hyperedge joins precisely two vertices), and a supercycle is exactly a graph cycle. Thus "connected and Berge-acyclic" coincides with "connected and acyclic" in graph theory. Spanning means the vertex set is V in both settings. Hence the equivalence. □

Theorem 25. (Generalization of spanning hypertrees). *Let* $n = 0$ *and assume* $\mathrm{SHG}^{(0)} = (V, E, \partial)$ *is an* h*–uniform hypergraph (i.e.* $|\partial(e)| = h \geq 2$ *for all* e*). Identify each* e *with its incidence set* $\partial(e) \subseteq V$ *to obtain the* h*–uniform hypergraph* $H = (V, \mathcal{E})$, $\mathcal{E} = \{\partial(e): e \in E\}$*. Then:*

A substructure $T = (V, E_T, \partial|_{E_T})$ is a spanning superhypertree of SHG$^{(0)}$ *if and only if the hypergraph $H_T = (V, \{\partial(e): e \in E_T\})$ is a spanning Berge-acyclic hypertree of H.*

Proof: For $n = 0$, our superpaths and supercycles are exactly the classical Berge paths and Berge cycles in hypergraphs (alternating vertex–edge sequences with the prescribed incidences). Therefore, "connected and Berge-acyclic" in T is equivalent to "connected and Berge-acyclic" in H_T. Spanning again means V is the full vertex set. Hence the equivalence. □

1. Additional Results: Hierarchical spanning n-superhypertree

A hierarchical superhypergraph permits vertices from several powerset levels and allows edges to join *mixed-level* vertices, while enforcing a downward-closure coherence principle.

Definition 26. (Hierarchical SuperHyperGraph of height r). Let V_0 be a finite nonempty base set and fix $r \in \mathbb{N}_0$. Define a *nonempty powerset tower* $(\mathcal{P}^{\langle k \rangle}(V_0))_{k=0}^{r}$ by

$$\mathcal{P}^{\langle 0 \rangle}(V_0) := V_0, \qquad \mathcal{P}^{\langle k+1 \rangle}(V_0) := \mathcal{P}\big(\mathcal{P}^{\langle k \rangle}(V_0)\big) \backslash \{\emptyset\} \quad (0 \leq k < r).$$

Set the *hierarchical universe*

$$\mathcal{U}_r(V_0) \ := \ \bigcup_{k=0}^{r} \mathcal{P}^{\langle k \rangle}(V_0).$$

For $x \in \mathcal{U}_r(V_0)$, define its *level* by

$$\ell(x) \ := \ \min\{\, k \in \{0,1, \dots, r\}: x \in \mathcal{P}^{\langle k \rangle}(V_0)\,\}.$$

A *hierarchical SuperHyperGraph of height* r on V_0 is a pair

$$\mathbb{H}^{\langle r \rangle} = (V, E)$$

satisfying:

1. *(Hierarchical vertex set).* V is a finite nonempty set with
$$V \subseteq \mathcal{U}_r(V_0).$$
Elements of V are called *hierarchical supervertices*.
2. *(Cross-level edges).* E is a finite family of nonempty subsets of V, i.e.,
$$E \subseteq \mathcal{P}(V) \backslash \{\emptyset\}.$$
Elements of E are called *hierarchical superhyperedges*. In particular, a single edge may contain supervertices of different levels.
3. *(Coherence / downward closure).* If $X \in V$ and $\ell(X) \geq 1$, then every immediate constituent of X is also a vertex:
$$X \subseteq V.$$

For each $k \in \{0, \dots, r\}$ we define the *k-th layer* by

$$V_k \ := \ \{\, x \in V\colon\ \ell(x) = k\,\}, \qquad \text{so that} \qquad V = \dot{\bigcup}_{k=0}^{r} V_k.$$

Remark (level ambiguity). If the ground set V_0 contains subsets of itself, then the same object may belong to multiple levels in principle. The definition of $\ell(\cdot)$ selects the smallest admissible level; an alternative is to work with tagged copies of each level to avoid collisions.

We now refine spanning superhypertrees by explicitly incorporating the internal *hierarchy* carried by iterated powerset vertices.

Definition 27. (Level-k constituents of an iterated-powerset object). Let V_0 be a finite nonempty set and let $n \in \mathbb{N}_0$. For $m \in \{0,1,\dots,n\}$ and $x \in \mathcal{P}^m(V_0)$, define the family of *level-k constituents* $\mathrm{Con}_k^{(m)}(x)$ for $k = 0,1,\dots,m$ recursively by

$$\mathrm{Con}_m^{(m)}(x) \ := \ \{x\}, \qquad \mathrm{Con}_k^{(m)}(x) \ := \bigcup_{y \in \mathrm{Con}_{k+1}^{(m)}(x)} y \quad (0 \leq k < m).$$

When $m = n$ we abbreviate $\mathrm{Con}_k(x) := \mathrm{Con}_k^{(n)}(x)$.

If $\mathrm{SHG}^{(n)} = (V, E, \partial)$ is an n–SuperHyperGraph over V_0 (so $V \subseteq \mathcal{P}^n(V_0)$), we define the *level-$k$ projection of V* by

$$V^{[k]} \ := \ \bigcup_{v \in V} \mathrm{Con}_k(v) \qquad (k = 0,1,\dots,n).$$

Thus $V^{[n]} = V$, while $V^{[0]} \subseteq V_0$ is the set of ground elements appearing anywhere in the chosen n-supervertices.

Definition 28. (Hierarchical incidence graph of a substructure). Let $\mathrm{SHG}^{(n)} = (V, E, \partial)$ be an n–SuperHyperGraph over a finite base set V_0, and let $E_T \subseteq E$ be nonempty. Write

$$T \ := \ (V, E_T, \partial \restriction_{E_T}).$$

Choose a tag symbol $* \notin \{0,1,\dots,n\}$. The *hierarchical incidence graph* of T is the (simple, undirected) graph

$$\mathsf{HInc}(T) \ := \ (\mathsf{N}(T),\ \mathsf{L}(T)),$$

whose node set is the disjoint union

$$\mathsf{N}(T) \ := \ \left(\bigcup_{k=0}^{n} \left(\{k\} \times V^{[k]}\right) \right) \ \dot{\cup} \ (\{*\} \times E_T),$$

and whose link set $\mathsf{L}(T)$ consists of the following two kinds of links:

1. *(Top-level incidence links).* For each $e \in E_T$ and each $v \in \partial(e)$, include the link
$$\{(*, e),\ (n, v)\} \ \in \ \mathsf{L}(T).$$
2. *(Vertical membership links).* For each $k \in \{0,1,\dots,n-1\}$, each $u \in V^{[k+1]}$, and each $x \in u$ (so $x \in V^{[k]}$), include the link
$$\{(k+1, u),\ (k, x)\} \ \in \ \mathsf{L}(T).$$

Definition 29. (Hierarchical spanning n-superhypertree). Let $\mathrm{SHG}^{(n)} = (V, E, \partial)$ be an n–SuperHyperGraph over a finite base set V_0. A substructure $T = (V, E_T, \partial|_{E_T})$ with $\varnothing \neq E_T \subseteq E$ is called a *hierarchical spanning n-superhypertree* of $\mathrm{SHG}^{(n)}$ if:

1. T is a spanning superhypertree of $\mathrm{SHG}^{(n)}$, i.e., T is connected and Berge-acyclic in the sense of Definition (Berge-style superpaths and supercycles) already adopted in this paper;
2. the hierarchical incidence graph $\mathsf{HInc}(T)$ (Definition 28) is a tree (connected and acyclic).

Theorem 30. (Well-definedness). *Let V_0 be finite and let $\mathrm{SHG}^{(n)} = (V, E, \partial)$ be an n–SuperHyperGraph over V_0. For every nonempty $E_T \subseteq E$, the hierarchical incidence graph $\mathsf{HInc}(T)$ of $T = (V, E_T, \partial|_{E_T})$ is a well-defined finite simple graph. Consequently, Definition 29 is well-defined.*

Proof:

Step 1 (finiteness of iterated-powerset objects). Since V_0 is finite, every set in $\mathcal{P}^m(V_0)$ is finite for every $m \in \{0,1,\dots,n\}$. (Indeed, $\mathcal{P}(X)$ is finite whenever X is finite; apply induction on m.)

Step 2 (finiteness of constituent projections). Fix $v \in V \subseteq \mathcal{P}^n(V_0)$. By construction, $\mathrm{Con}_n(v) = \{v\}$ is finite, and for $0 \leq k < n$,

$$\mathrm{Con}_k(v) \;=\; \bigcup_{y \in \mathrm{Con}_{k+1}(v)} y$$

is a finite union of finite sets (by Step 1), hence finite. Because V is finite, each level-k projection

$$V^{[k]} \;=\; \bigcup_{v \in V} \mathrm{Con}_k(v)$$

is a finite union of finite sets, hence finite.

Step 3 (node set is finite and collision-free). By Step 2, each $\{k\} \times V^{[k]}$ is finite, and E_T is finite because E is finite. Therefore

$$\mathsf{N}(T) \;=\; \left(\bigcup_{k=0}^{n} \left(\{k\} \times V^{[k]}\right) \right) \;\dot{\cup}\; (\{*\} \times E_T)$$

is finite. Moreover, the level-tags k and the edge-tag $*$ enforce a disjoint union, so nodes coming from different levels (or from edges) cannot accidentally coincide even if the underlying set-theoretic objects happen to be equal.

Step 4 (link set is finite and defines a simple graph). For (L1), there are finitely many incidence links, at most $\sum_{e \in E_T} |\partial(e)| < \infty$. For (L2), for each fixed k there are at most $\sum_{u \in V^{[k+1]}} |u| < \infty$ membership links, because each u is finite (Step 1) and $V^{[k+1]}$ is finite (Step 2). Hence $\mathsf{L}(T)$ is finite. By definition, each element of $\mathsf{L}(T)$ is a 2-element subset of $\mathsf{N}(T)$, so $\mathsf{HInc}(T)$ is an undirected simple graph (multiple occurrences of the same 2-subset are irrelevant).

Thus $\mathsf{HInc}(T)$ is a well-defined finite simple graph for every nonempty $E_T \subseteq E$. Therefore the predicate "$\mathsf{HInc}(T)$ is a tree" is well-defined, and so is the notion of a hierarchical spanning n-superhypertree in Definition 29. □

5. Conclusion

In this paper, we studied the notion of a spanning superhypertree as the natural spanning tree concept within superhypergraphs. In future work, we aim to explore possible extensions based on Fuzzy Sets [31,37,40,41], Neutrosophic Sets [32, 33], Plithogenic Sets [34–36], and HyperFuzzy Sets [38,39]. Such directions may provide richer generalizations and further applications of the theoretical framework developed in this paper.Such directions may provide richer generalizations and further applications of the theoretical framework developed in this paper.

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Authors' Contributions: The author solely conceived, conducted, and completed this work and approved the final version of the manuscript.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

REFERENCES

1. R.Diestel, *Graph Theory*, Springer, 2024.
2. Y.Feng, H.You, Z.Zhang, R.Ji and Y.Gao, Hypergraph neural networks, *Proceedings of the AAAI Conference on Artificial Intelligence*, 33 (2019) 3558-3565.
3. Y.Gao, Z.Zhang, H.Lin, X.Zhao, S.Du and C.Zou, Hypergraph learning: Methods and practices, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(5) (2020) 2548-2566.
4. F.Smarandache, *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*, Infinite Study, 2020.
5. T.Fujita and F.Smarandache, *HyperGraph and SuperHyperGraph Theory with Applications (III): Intersection Graph and Graph Labeling*, HyperGraph and SuperHyperGraph Theory with Applications, Vol. III, Neutrosophic Science International Association (NSIA) Publishing House, 2026.
6. T.Fujita and F.Smarandache, *HyperGraph and SuperHyperGraph Theory with Applications*, Neutrosophic Science International Association (NSIA) Publishing House, 2026.
7. S.M.Berrocal Villegas, W.Montalvo Fritas, C.R.Berrocal Villegas, M.Y.F.Flores Fuentes Rivera, R.Espejo Rivera, D.L.Bautista Puma and D.M.Macazana Fernandez, Using plithogenic n-superhypergraphs to assess the degree of relationship between information skills and digital competencies, *Neutrosophic Sets and Systems*, 84 (2025) 513-524.
8. N.Hodelin Amable, E.E.Vergel De Salazar, M.G.Martinez Isaac, O.C.Olavarria Sanchez and J.M.Solis Palma, Representation of motivational dynamics in school environments through plithogenic n-superhypergraphs with family participation, *Neutrosophic Sets and Systems*, 92 (2025) 570-583.
9. H.Honma, S.Honma and S.Masuyama, An optimal parallel algorithm for constructing a spanning tree on circular permutation graphs, *IEICE Transactions on Information and Systems*, 92-D (2009) 141-148.
10. M.A.Valle, G.A.Ruz and R.Morras, Market basket analysis: Complementing association rules with minimum spanning trees, *Expert Systems with Applications*, 97 (2018) 146-162.
11. I.Tomescu and M.Zimand, Minimum spanning hypertrees, *Discrete Applied Mathematics*, 54 (1994) 67-76.

12. Y.Chen, S.Im and J.Zhang, Unbounded degree spanning hypertrees in Dirac hypergraphs, *arXiv preprint arXiv:2508.06843*, 2025.
13. J.L.Gross, J.Yellen and M.Anderson, *Graph Theory and Its Applications*, Chapman and Hall/CRC, 2018.
14. M.Akram, A.Nagoor Gani and A.Borumand Saeid, Vague hypergraphs, *Journal of Intelligent and Fuzzy Systems*, 26(2) (2014) 647-653.
15. C.Berge, *Hypergraphs: Combinatorics of Finite Sets*, Vol. 45, Elsevier, 1984.
16. J.C.Mendez Bravo, C.J.Bolanos Piedrahita, M.A.Mendez Bravo and L.M.Pilacuan-Bonete, Integrating SMED and Industry 4.0 to optimize processes with plithogenic n-superhypergraphs, *Neutrosophic Sets and Systems*, 84 (2025) 328-340.
17. N.B.Nalawade, M.S.Bapat, S.G.Jakkewad, G.A.Dhanorkar and D.J.Bhosale, Structural properties of zero-divisor hypergraph and superhypergraph over Z_n: Girth and Helly property, *Panamerican Mathematical Journal*, 35(4S) (2025) 485.
18. M.Hamidi, F.Smarandache and E.Davneshvar, Spectrum of superhypergraphs via flows, *Journal of Mathematics*, 2022(1) (2022) 9158912.
19. S.Zhu, Neutrosophic n-superhypernetwork: A new approach for evaluating short video communication effectiveness in media convergence, *Neutrosophic Sets and Systems*, 85 (2025) 1004-1017.
20. M.Hamidi and M.Taghinezhad, *Application of Superhypergraphs-Based Domination Number in Real World*, Infinite Study, 2023.
21. E.J.Mogro, J.R.Molina, G.J.S.Canas and P.H.Soria, Tree tobacco extract (Nicotiana glauca) as a plithogenic bioinsecticide alternative for controlling fruit fly (Drosophila immigrans) using n-superhypergraphs, *Neutrosophic Sets and Systems*, 74 (2024) 57-65.
22. A.Al-Odhari, Neutrosophic power-set and neutrosophic hyper-structure of neutrosophic set of three types, *Annals of Pure and Applied Mathematics*, 31(2) (2025) 125-146.
23. T.Jech, *Set Theory: The Third Millennium Edition, Revised and Expanded*, Springer, 2003.
24. F.Smarandache, The cardinal of the m-powerset of a set of n elements used in the superhyperstructures and neutrosophic superhyperstructures, *Systems Assessment and Engineering Management*, 2 (2024) 19-22.
25. A.K.Das, R.Das, S.Das, B.K.Debnath, C.Granados, B.Shil and R.Das, A comprehensive study of neutrosophic superhyper BCI-semigroups and their algebraic significance, *Transactions on Fuzzy Sets and Systems*, 8(2) (2025) 80.
26. F.Smarandache, *SuperHyperFunction, SuperHyperStructure, Neutrosophic SuperHyperFunction and Neutrosophic SuperHyperStructure: Current understanding and future directions*, Infinite Study, 2023.
27. F.Smarandache, Foundation of superhyperstructure and neutrosophic superhyperstructure, *Neutrosophic Sets and Systems*, 63(1) (2024) 21.
28. A.Bretto, *Hypergraph Theory: An Introduction*, Mathematical Engineering, Springer, 2013.
29. F.Smarandache, *Introduction to the n-SuperHyperGraph—the most general form of graph today*, Infinite Study, 2022.

30. H.S.Aldosari and C.Greenhill, The average number of spanning hypertrees in sparse uniform hypergraphs, *Discrete Mathematics*, 344(1) (2021) 112192.
31. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8(3) (1965) 338-353.
32. H.Wang, F.Smarandache, Y.Zhang and R.Sunderraman, *Single Valued Neutrosophic Sets*, Infinite Study, 2010.
33. S.Broumi, M.Talea, A.Bakali and F.Smarandache, Single valued neutrosophic graphs, *Journal of New Theory*, (10) (2016) 86-101.
34. F.Sultana, M.Gulistan, M.Ali, N.Yaqoob, M.Khan, T.Rashid and T.Ahmed, A study of plithogenic graphs: Applications in spreading coronavirus disease (COVID-19) globally, *Journal of Ambient Intelligence and Humanized Computing*, 14(10) (2023) 13139-13159.
35. W.B.Vasantha Kandasamy, K.Ilanthenral and F.Smarandache, *Plithogenic Graphs*, Infinite Study, 2020.
36. F.Smarandache, *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*, Infinite Study, 2018.
37. S.S.Zhang, A model of based on Z-number and fuzzy analytic hierarchy process, *Journal of Mathematics and Informatics*, 7 (2017) 63-71.
38. T.Fujita, A.Heilat, R.Hatamleh and A.Ghaib, Linear Diophantine HyperFuzzy Set and SuperHyperFuzzy Set, *Statistics, Optimization & Information Computing*, 2026.
39. T.Fujita, I.M.Batiha, N.Anakira, M.S.Hijazi, A.Al-Khateeb and T.Sasa, Hyperfuzzy and SuperHyperfuzzy Weighted Averages, *Statistics, Optimization & Information Computing*, 2026.
40. C.Z.Jia, L.Q.Li and X.R.Li, Enriched fuzzy σ-algebra and fuzzy rough approximation operators, *Journal of Mathematics and Informatics*, 25 (2023) 59-69.
41. S.Maity, Max-norm and square-max norm of fuzzy matrices, *Journal of Mathematics and Informatics*, 3 (2015) 25-40.
42. D.Thakkar and V.Dave, Edge Stable Sets & Edge Independent Sets in Hypergraphs, *Journal of Mathematics and Informatics*, 12 (2018) 33-39.
43. P.Ghosh and A.Pal, Coloring of hypergraphs, *Journal of Mathematics and Informatics*, 8 (2017) 37-44.