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Reduced Augmented Sombor Index of Certain Networks

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Abstract: In this study, we introduce the reduced augmented Sombor and reciprocal reduced augmented Sombor indices of a graph. Furthermore, we compute these augmented Sombor indices of certain networks.

Keywords: reduced augmented Sombor index, reciprocal reduced augmented Sombor index, network.

AMS Mathematics Subject Classification (2010): 05C10, 05C69

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

The augmented Sombor index [3] of a graph is defined as

$$ASO(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)^2+d_G(v)^2}{d_G(u)+d_G(v)-2}}.$$

Recently, the augmented Nirmala index was studied in [4].

We define the reduced augmented Sombor index of a graph G as

$$RASO(G)=\sum_{uv\in E(G)}\sqrt{\frac{(d_G(u)-1)^2+(d_G(v)-1)^2}{(d_G(u)-1)+(d_G(v)-1)-2}}$$

where $d_G(u)+d_G(u)\geq 5$.

We define the reciprocal reduced augmented Sombor index of a graph G as

$$RRASO(G)=\sum_{uv\in E(G)}\sqrt{\frac{(d_G(u)-1)+(d_G(v)-1)-2}{(d_G(u)-1)^2+(d_G(v)-1)^2}}.$$

Recently some Sombor indices were studied in [5, 6, 7].

In this paper, the reduced augmented Sombor and reciprocal reduced augmented Sombor indices of certain networks are computed.

2. Results for silicate networks

Silicates are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional silicate network is presented in Figure 1.

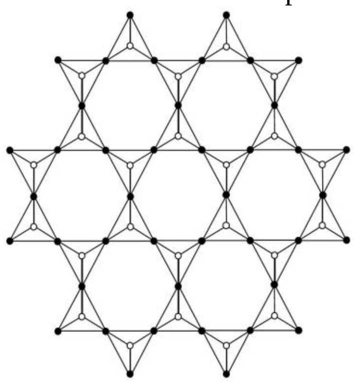

Figure 1: A 2-dimensional silicate network

Let G be the graph of a silicate network SL_n. From Figure 1, it is easy to see that $\Delta(G) = 6$. Clearly we have $c_u = \Delta(G) - d_G(u) + 1 = 7 - d_G(u)$. The graph G has $15n^2 + 3n$ vertices and $36n^2$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6n$.
$E_{36} = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_{36}| = 18n^2 + 6n$.
$E_{66} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{66}| = 18n^2 - 12n$.

We compute the reduced augmented Sombor index of SL_n.

Theorem 1. The reduced augmented Sombor index of a silicate network SL_n is

$$RASO(SL_n) = 18\sqrt{\frac{29}{5}}n^2 + 45n^2 + 6\sqrt{\frac{29}{5}}n - 18n.$$

Proof: Let G be the graph of a silicate network SL_n. We obtain

$$RASO(SL_n) = \sum_{uv \in E(G)} \sqrt{\frac{(d_G(u)-1)^2 + (d_G(v)-1)^2}{(d_G(u)-1) + (d_G(v)-1) - 2}}$$

$$= 6n\sqrt{\frac{(3-1)^2 + (3-1)^2}{(3-1) + (3-1) - 2}} + \left(18n^2 + 6n\right)\sqrt{\frac{(3-1)^2 + (6-1)^2}{(3-1) + (6-1) - 2}}$$

$$+ \left(18n^2 - 12n\right)\sqrt{\frac{(6-1)^2 + (6-1)^2}{(6-1) + (6-1) - 2}}$$

$$=18\sqrt{\frac{29}{5}}n^2+45n^2+6\sqrt{\frac{29}{5}}n-18n.$$

We compute the reciprocal reduced augmented Sombor index of SL_n.

Theorem 2. The reciprocal reduced augmented Sombor index of a silicate network SL_n is

$$RRASO(SL_n)=18\sqrt{\frac{5}{29}}n^2+\frac{36}{5}n^2+3n+6\sqrt{\frac{5}{29}}n-\frac{24}{5}n.$$

Proof: Let G be the graph of a silicate network SL_n. We obtain

$$RRASO(SL_n)=\sum_{uv\in E(G)}\sqrt{\frac{(d_G(u)-1)+(d_G(v)-1)-2}{(d_G(u)-1)^2+(d_G(v)-1)^2}}$$

$$=6n\sqrt{\frac{(3-1)+(3-1)-2}{(3-1)^2+(3-1)^2}}+\left(18n^2+6n\right)\sqrt{\frac{(3-1)+(6-1)-2}{(3-1)^2+(6-1)^2}}$$

$$+\left(18n^2-12n\right)\sqrt{\frac{(6-1)+(6-1)-2}{(6-1)^2+(6-1)^2}}$$

$$=18\sqrt{\frac{5}{29}}n^2+\frac{36}{5}n^2+3n+6\sqrt{\frac{5}{29}}n-\frac{24}{5}n.$$

3. Results for Oxide networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 2.

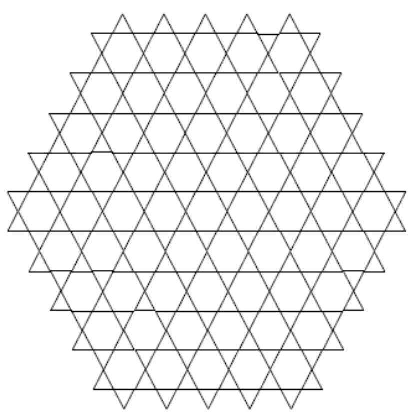

Figure 2: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. From Figure 2, it is easy to see that $\Delta(G)=4$. Thus $c_u = \Delta(G) - d_G(u) + 1 = 5 - d_G(u)$. By calculation, we obtain that G has $9n^2+3n$ vertices and $18n^2$ edges. In G, by algebraic method, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{24} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $|E_{24}| = 12n$.

$E_{44} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_{44}| = 18n^2 - 12n$.

We compute the reduced augmented Sombor index of OX_n.

Theorem 3. The reduced augmented Sombor index of an oxide network OX_n is

$$RASO(OX_n) = 54\sqrt{\frac{1}{2}}n^2 + 12\sqrt{5}n - 36\sqrt{\frac{1}{2}}n.$$

Proof: Let G be the graph of an oxide network OX_n. We obtain

$$RASO(OX_n) = \sum_{uv\in E(G)} \sqrt{\frac{(d_G(u)-1)^2 + (d_G(v)-1)^2}{(d_G(u)-1) + (d_G(v)-1) - 2}}$$

$$= 12n\sqrt{\frac{(2-1)^2 + (4-1)^2}{(2-1) + (4-1) - 2}} + \left(18n^2 - 12n\right)\sqrt{\frac{(4-1)^2 + (4-1)^2}{(4-1) + (4-1) - 2}}$$

$$= 54\sqrt{\frac{1}{2}}n^2 + 12\sqrt{5}n - 36\sqrt{\frac{1}{2}}n.$$

We compute the reciprocal reduced augmented Sombor index of OX_n.

Theorem 4. The reciprocal reduced augmented Sombor index of an oxide network OX_n is

$$RRASO(OX_n) = 6\sqrt{2}n^2 + 12\sqrt{\frac{1}{5}}n - 4\sqrt{2}n.$$

Proof: Let G be the graph of an oxide network OX_n. We obtain

$$RRASO(OX_n) = \sum_{uv\in E(G)} \sqrt{\frac{(d_G(u)-1) + (d_G(v)-1) - 2}{(d_G(u)-1)^2 + (d_G(v)-1)^2}}$$

$$= 12n\sqrt{\frac{(2-1) + (4-1) - 2}{(2-1)^2 + (4-1)^2}} + \left(18n^2 - 12n\right)\sqrt{\frac{(4-1) + (4-1) - 2}{(4-1)^2 + (4-1)^2}}$$

$$= 6\sqrt{2}n^2 + 12\sqrt{\frac{1}{5}}n - 4\sqrt{2}n.$$

4. Conclusion

In this paper, the reduced augmented Sombor and reciprocal reduced augmented Sombor indices of a graph are defined. Also the reduced augmented Sombor and reciprocal reduced augmented Sombor indices of certain networks are determined.

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Conflicts of Interest. The author declares that there are no conflicts of interest.

Author's Contribution. The author solely carried out all aspects of this work.

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