Journal of Mathematics and Informatics
Vol. 26, 2024, 27-40
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 16 February 2024
www.researchmathsci.org
DOI:http://dx.doi.org/10.22457/jmi.v26a03232

Journal of
Mathematics and Informatics

Extension of Weak Compatible Maps in Fuzzy Metric Space

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Received 10 July 2023; accepted 22 August 2023

Abstract. The principal motive of this paper is to establish a common fixed point theorem for seven self-mappings in fuzzy metric space using weak compatibility without continuity. Our results extend and generalize several known results of fixed point theory in different spaces.

Keywords: Fixed Point, Fixed Point Theorem, Weak Compatibility, Fuzzy Metric Space and Compatible Maps

AMS Mathematics Subject Classification (2010): 54E35

1. Introduction

The foundations of fuzzy set theory and fuzzy mathematics were laid down by Zadeh [15] in 1965 with the introduction of the notion of fuzzy sets. The theory of fuzzy sets has vast applications in applied sciences and engineering such as neural network theory, stability theory, mathematical programming, genetics, nervous systems, image processing, control theory etc. to name a few. The theory of fixed points is one of the basic tools to handle physical formulations. This has led to the development and fuzzification of several concepts of analysis and topology. In 1975, Kramosil and Michalek [7] introduced the concept of a fuzzy metric space by generalizing the concept of a probabilistic metric space to the fuzzy situation. The concept of Kramosil and Michalek of a fuzzy metric space was later modified by George and Veeramani [2] in 1994. In 1988, Grabeic [4] followed the concept of Kramosil and Michalek [7] and obtained the fuzzy version of Banach's fixed point theorem. Using the notion of weak commuting property, Sessa [9] improved commutative conditions in fixed point theorems. Jungck [5] introduced the concept of compatibility in metric spaces. The concept of compatibility in fuzzy metric space was proposed by Mishra et al. [8]. In 1996, Jungck [6] introduced the concept of weakly compatible maps which was the generalization of the concept of compatible maps. Singh and Chauhan [10] and Cho [1] provided fixed point theorems in fuzzy metric space for four self-maps using the concept of compatibility where two mappings needed to be continuous. In 2017 Govery and Singh [3] proved a common fixed point theorem for six self mappings in fuzzy metric space using the concept of compatibility and weak compatibility where one map needs to be continuous. In this paper, we prove a common fixed point theorem for seven self-mappings in fuzzy metric space using weak compatibility without continuity. Our results extend and generalize several known results of fixed point theory in different spaces.

2. Preliminaries

Definition 2.1. Let X be any set. A fuzzy set A in X is a function with domain in X and values in [0,1].

Definition 2.2. A $t-norm$ or more precisely triangular norm $*$ is a binary operation defined on $[0,1]$ such that for all $a,b,c,d \in [0,1]$, following conditions are satisfied:

(1) $a * 1 = 1$;
(2) $a * b = b * a$;
(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$;
(4) $a * (b * c) = (a * b) * c$.

Definition 2.3. The $3-tuple$ $(X, \mathcal{M}, *)$ is called a fuzzy metric space if X is an arbitrary non-empty set, $*$ is a continuous $t-norm$ and $\mathcal{M}$ is a fuzzy set in $X^2 \times (0,\infty)$ satisfying the following conditions, for all $x,y,z \in X$ and $s,t > 0$:

(1) $\mathcal{M}(x,y,0) > 0$;
(2) $\mathcal{M}(x,y,t) = 1$ for all $t > 0$, iff $x = y$;
(3) $\mathcal{M}(x,y,t) = \mathcal{M}(y,x,t)$;
(4) $\mathcal{M}(x,y,t) * \mathcal{M}(y,z,s) \leq \mathcal{M}(x,z,t+s)$;
(5) $\mathcal{M}(x,y,.) : (0,\infty) \to [0,1]$ is continuous.

Example 2.1. Let (X,d) be a metric space. Define $a * b = \min(a,b)$, and

$$\mathcal{M}(x,y,t) = \frac{t}{t + d(x,y)}$$

induced by the metric d is often called the standard fuzzy metric.

Definition 2.4. A sequence $\{x_n\}$ in a fuzzy metric space $(X, \mathcal{M}, *)$ is said to be a Cauchy sequence if, for each $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$\mathcal{M}(x_n, x_m, t) > 1 - \varepsilon$ for all $n,m \geq n_0$.

A sequence $\{x_n\}$ in a fuzzy metric space $(X, \mathcal{M}, *)$ is said to be convergent to $x \in X$ if there exists $n_0 \in \mathbb{N}$ such that $\lim_{n\to\infty} \mathcal{M}(x_n, x, t) > 1 - \varepsilon$ for all $t > 0$ & $n \geq n_0$. A fuzzy metric space $(X, \mathcal{M}, *)$ is said to be complete if every Cauchy sequence in X converges to a point in X.

Lemma 2.1. $\mathcal{M}(x,y,.)$ is non-decreasing for all $x,y \in X$.
Proof: Suppose $\mathcal{M}(x,y,t) > M(x,y,s)$ for some $0 < t < s$,
then $\mathcal{M}(x,y,t) * \mathcal{M}(y,y,s-t) \leq \mathcal{M}(x,y,s) < M(x,y,t)$
Since $\mathcal{M}(y,y,s-t) = 1$, therefore, $\mathcal{M}(x,y,t) \leq \mathcal{M}(x,y,s) < M(x,y,t)$, which is a contradiction. Thus, $\mathcal{M}(x,y,.)$ is non-decreasing for all $x,y \in X$.

Lemma 2.2. Let $(X, \mathcal{M}, *)$ be a fuzzy metric space then $\mathcal{M}$ is a continuous function on $X^2 \times (0, \infty)$ throughout this paper $(X, \mathcal{M}, *)$ will denote the fuzzy metric space with the following condition $\lim_{n \to \infty} \mathcal{M}\,(x, y, t) = 1$ for all $x, y\ \in X$ and $t > 0$.

Lemma 2.3. If for all $x, y\ \in X,\ t > 0$ and $0 < k < 1$, $\mathcal{M}\,(x, y, kt)\ \geq\ \mathcal{M}\,(x, y, t)$, then $x = y$.

Proof: Suppose that there exists $0 < k < 1$ such that $\mathcal{M}\,(x, y, kt)\ \geq\ \mathcal{M}\,(x, y, t)$ for all $x, y\ \in X$ and $t > 0$. Then $\mathcal{M}\,(x, y, t)\ \geq\ \mathcal{M}\,\left(x, y, \frac{t}{k}\right)$, and

so $\mathcal{M}\,(x, y, t)\ \geq\ \mathcal{M}\,\left(x, y, \frac{t}{k^n}\right)$ for positive integer n.

Taking the limit as $n\ \to\ \infty\ \ \mathcal{M}\,(x, y, t)\ \geq\ 1$ and hence $x = y$.

Definition 2.5. Two self mappings A and B of a fuzzy metric space $(X, \mathcal{M}, *)$ are said to be weakly commuting if $\mathcal{M}\,(ABz, BAz, t) \geq\ \mathcal{M}\,(Az, Bz, t)$ for all $z\ \in X$ and $t > 0$.

Definition 2.6. Let A and B be mappings from a fuzzy metric space $(X, \mathcal{M}, *)$ into itself. Then the mappings are said to be compatible if $\lim_{n \to \infty} \mathcal{M}(ABx_n, BAx_n, t) = 1$, for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \to \infty} Ax_n\ =\ \lim_{n \to \infty} Bx_n\ = x$ for some $x \in X$.

Definition 2.7. If A and B are two self-mappings of a fuzzy metric space $(X, \mathcal{M}, *)$, then a point $x\ \in X$ is called the coincidence point of A and B if and only if $Ax = Bx$.

Definition 2.8. Two self-mappings A and B of a fuzzy metric space $(X, \mathcal{M}, *)$ are said to be weakly compatible or coincidently commuting if they commute at their coincidence points, that is if $ABx = BAx$ whenever $Ax = Bx$ for some $x\ \in X$.

Remark 2.1. It can be easily verified that compatible mappings are also weakly compatible but the converse is not necessarily true.

Definition 2.9. Two self-mappings A and B of a fuzzy metric space $(X, \mathcal{M}, *)$ are said to be occasionally weakly compatible if and only if there exists a point $x\ \in X$ which is the coincidence point of A and B at which A and B commute.

Definition 2.10. A pair (A, B) of self-mappings of a fuzzy metric space $(X, \mathcal{M}, *)$ is said to be semi-compatible if there exists a sequence $\{x_n\}$ in X such that

$\lim_{n \to \infty} AB\,x_n = Bx$ whenever $\lim_{n \to \infty} A\,x_n = \lim_{n \to \infty} B\,x_n = x$ for some $x\ \in X$.

3. The main results

Theorem 3.1. Let A, B, R, S, T, P and Q be seven self-maps of a complete fuzzy metric space $(X, \mathcal{M}, *)$ such that the following conditions are satisfied:

(1) $P(X) \subset STR(X), Q(X) \subset ABR(X)$;
(2) $AB = BA,\ ST = TS,\ PB = BP,\ QT = TQ,\ PR = RP,\ TR = RT$ and $BR = RB$;
(3) (P, ABR) and (Q, STR) are weakly compatible;

(4)

$$\mathcal{M}(Px, Qy, kt) \geq min \left\{ \begin{array}{c} \mathcal{M}(ABRx, STRy, t), \mathcal{M}(Qy, Px, t), \mathcal{M}(ABRx, Px, t), \\ \mathcal{M}(STRy, Qy, t), \mathcal{M}(Px, STRy, t), \\ \dfrac{a\,\mathcal{M}(Px, Qy, t) + b\,\mathcal{M}(Qy, STRy, t)}{a\,\mathcal{M}(Px, STRy, t) + b}, \\ \dfrac{c\,\mathcal{M}(Px, Qy, t) + d\,\mathcal{M}(Px, STRy, t)}{c\,\mathcal{M}(Qy, STRy, t) + d} \end{array} \right\}$$

for all $x, y \in X$ and $t > 0$, where $k \in (0,1)$ and $a, b, c, d \geq 0$ with $a \,\&\, b$, and $c \,\&\, d$ cannot be simultaneously 0.
Then A, B, R, S, T, P and Q have a unique common fixed point in X.
Proof: Let x_0 be any arbitrary point. As $P(X) \subset STR(X)$, $Q(X) \subset ABR(X)$ so, there exists $x_1, x_2 \in X$ such that
$Px_0 = STRx_1 = y_0$ and $Qx_1 = ABRx_2 = y_1$.
Inductively we construct the sequences $\{y_n\}$ and $\{x_n\}$ in X such that
$y_{2n} = Px_{2n} = STRx_{2n+1}$
and $y_{2n+1} = Qx_{2n+1} = ABRx_{2n+2}$ for $n = 0, 1, 2, \cdots$.

Now, we first show that $\{y_n\}$ is a Cauchy sequence in X.
Using condition (4) we get

$$\mathcal{M}(y_{2n+1}, y_{2n}, kt) = \mathcal{M}(y_{2n}, y_{2n+1}, kt) = \mathcal{M}(Px_{2n}, Qx_{2n+1}, kt)$$

$$\geq min \left\{ \begin{array}{c} \mathcal{M}(ABRx_{2n}, STRx_{2n+1}, t), \mathcal{M}(Qx_{2n+1}, Px_{2n}, t), \mathcal{M}(ABRx_{2n}, Px_{2n}, t), \\ \mathcal{M}(STRx_{2n+1}, Qx_{2n+1}, t), \mathcal{M}(Px_{2n}, STRx_{2n+1}, t), \\ \dfrac{a\,\mathcal{M}(Px_{2n}, Qx_{2n+1}, t) + b\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t)}{a\,\mathcal{M}(Px_{2n}, STRx_{2n+1}, t) + b}, \\ \dfrac{c\,\mathcal{M}(Px_{2n}, Qx_{2n+1}, t) + d\,\mathcal{M}(Px_{2n}, STRx_{2n+1}, t)}{c\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t) + d} \end{array} \right\}$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq min \left\{ \begin{array}{c} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n+1}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, t), \mathcal{M}(y_{2n}, y_{2n}, t), \\ \dfrac{a\,\mathcal{M}(y_{2n}, y_{2n+1}, t) + b\,\mathcal{M}(y_{2n+1}, y_{2n}, t)}{a\,\mathcal{M}(y_{2n}, y_{2n}, t) + b}, \\ \dfrac{c\,\mathcal{M}(y_{2n}, y_{2n+1}, t) + d\,\mathcal{M}(y_{2n}, y_{2n}, t)}{c\,\mathcal{M}(y_{2n+1}, y_{2n}, t) + d} \end{array} \right\}$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n+1}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, t), 1, \mathcal{M}(y_{2n+1}, y_{2n}, t), 1 \end{array} \right\}$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq \mathcal{M}(y_{2n-1}, y_{2n}, t)$$

Similarly $\mathcal{M}(y_{2n+1}, y_{2n+2}, kt) \geq \mathcal{M}(y_{2n}, y_{2n+1}, t)$
Therefore for all n and $t > 0$, we have

$$\mathcal{M}(y_n, y_{n+1}, kt) \geq \mathcal{M}(y_n, y_{n-1}, t)$$
$$\mathcal{M}(y_n, y_{n+1}, kt) \geq \mathcal{M}\left(y_n, y_{n-1}, {}^{t}/_{k}\right) \geq \mathcal{M}\left(y_{n-1}, y_{n-2}, {}^{t}/_{k^2}\right) \geq \cdots$$
$$\geq \mathcal{M}\left(y_1, y_0, {}^{t}/_{k^n}\right)$$

on taking $n \to \infty$, we get

$$\lim_{n\to\infty} \mathcal{M}(y_{n+1}, y_n, t) = 1, \forall\, t > 0$$

now for any integer p we have

$$\mathcal{M}(y_n, y_{n+p}, t) \geq \mathcal{M}\left(y_n, y_{n+1}, {}^{t}/_{p}\right) * \mathcal{M}\left(y_{n+1}, y_{n+2}, {}^{t}/_{p}\right) * \cdots$$
$$* \mathcal{M}\left(y_{n+p-1}, y_{n+p}, {}^{t}/_{p}\right)$$

Therefore $\lim_{n\to\infty} \mathcal{M}(y_n, y_{n+p}, t) \geq 1 * 1 * 1 * \cdots * 1 = 1$

Hence $\{y_n\}$ is a Cauchy sequence in X which is complete

Therefore $y_n \to z$ in X; so its subsequences Px_{2n}, $STRx_{2n+1}$, $ABRx_{2n+2}$ and Qx_{2n+1} also converge to z.

$$\lim_{n\to\infty} Px_{2n} = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} STRx_{2n+1} = \lim_{n\to\infty} ABRx_{2n+2} = z$$

Case (1)

Since $P(X) \subset STR(X)$ and $\lim_{n\to\infty} Px_{2n} = z$

then there exist $u \in X$ such that $STRu = z$ ... (i)

putting $x = x_{2n}$ and $y = u$ in condition (4)

$$\mathcal{M}(Px_{2n}, Qu, kt) \geq min \left\{ \begin{array}{c} \mathcal{M}(ABRx_{2n}, STRu, t), \mathcal{M}(Qu, Px_{2n}, t), \mathcal{M}(ABRx_{2n}, Px_{2n}, t), \\ \mathcal{M}(STRu, Qu, t), \mathcal{M}(Px_{2n}, STRu, t), \\ \dfrac{a\, \mathcal{M}(Px_{2n}, Qu, t) + b\, \mathcal{M}(Qu, STRu, t)}{a\, \mathcal{M}(Px_{2n}, STRu, t) + b}, \\ \dfrac{c\, \mathcal{M}(Px_{2n}, Qu, t) + d\, \mathcal{M}(Px_{2n}, STRu, t)}{c\, \mathcal{M}(Qu, STRu, t) + d} \end{array} \right\}$$

$$\mathcal{M}(Px_{2n}, Qu, kt) \geq min \left\{ \begin{array}{c} \mathcal{M}(ABRx_{2n}, z, t), \mathcal{M}(Qu, Px_{2n}, t), \mathcal{M}(ABRx_{2n}, Px_{2n}, t), \\ \mathcal{M}(z, Qu, t), \mathcal{M}(Px_{2n}, z, t), \\ \dfrac{a\, \mathcal{M}(Px_{2n}, Qu, t) + b\, \mathcal{M}(Qu, z, t)}{a\, \mathcal{M}(Px_{2n}, z, t) + b}, \\ \dfrac{c\, \mathcal{M}(Px_{2n}, Qu, t) + d\, \mathcal{M}(Px_{2n}, z, t)}{c\, \mathcal{M}(Qu, z, t) + d} \end{array} \right\}$$

Let $n \to \infty$ and using the above result we get

$$\mathcal{M}(z,Qu,kt) \geq \ min \left\{ \begin{array}{c} \mathcal{M}(z,z,t), \mathcal{M}(Qu,z,t), \mathcal{M}(z,z,t), \\ \mathcal{M}(z,Qu,t), \mathcal{M}(z,z,t), \\ \dfrac{a\,\mathcal{M}(z,Qu,t) + b\,\mathcal{M}(Qu,z,t)}{a\,\mathcal{M}(z,z,t) + \ b}, \\ \dfrac{c\,\mathcal{M}(z,Qu,t) + \ d\,\mathcal{M}(z,z,t)}{c\,\mathcal{M}(Qu,z,t) + \ d} \end{array} \right\}$$

$$\mathcal{M}(z,Qu,kt) \geq min \left\{ \begin{array}{c} 1, \mathcal{M}(Qu,z,t), 1, \mathcal{M}(z,Qu,t), \\ 1, \mathcal{M}(z,Qu,t), 1 \end{array} \right\}$$

$\mathcal{M}(z,Qu,kt) \geq \ \mathcal{M}(Qu,z,t)$
by lemma (2.3) $Qu = z$

$$Qu = STu = z \qquad \dots(ii)$$

u is the coincident point of X such that $Qu = STu = z$ and (Q,STR) is weakly compatible mappings
$QSTRu = STRQu$
$Qu = z \rightarrow STRQu = STRz$ and $STRu = z \rightarrow QSTRu = Qz$
$Qz = STRz$
putting $x = x_{2n}$ and $y = z$ in condition (4)

$$\begin{aligned} &\mathcal{M}(Px_{2n},Qz,kt) \\ &\geq \ min \left\{ \begin{array}{c} \mathcal{M}(ABRx_{2n},STRz,t), \mathcal{M}(Qz,Px_{2n},t), \mathcal{M}(ABRx_{2n},Px_{2n},t), \\ \mathcal{M}(STRz,Qz,t), \mathcal{M}(Px_{2n},STRz,t), \\ \dfrac{a\,\mathcal{M}(Px_{2n},Qz,t) + b\,\mathcal{M}(Qz,STRz,t)}{a\,\mathcal{M}(Px_{2n},STRz,t) + \ b}, \\ \dfrac{c\,\mathcal{M}(Px_{2n},Qz,t) + \ d\,\mathcal{M}(Px_{2n},STRz,t)}{c\,\mathcal{M}(Qz,STRz,t) + \ d} \end{array} \right\} \end{aligned}$$

$$\mathcal{M}(Px_{2n},Qz,kt) \geq \ min \left\{ \begin{array}{c} \mathcal{M}(ABRx_{2n},Qz,t), \mathcal{M}(Qz,Px_{2n},t), \mathcal{M}(ABRx_{2n},Px_{2n},t), \\ \mathcal{M}(Qz,Qz,t), \mathcal{M}(Px_{2n},Qz,t), \\ \dfrac{a\,\mathcal{M}(Px_{2n},Qz,t) + b\,\mathcal{M}(Qz,Qz,t)}{a\,\mathcal{M}(Px_{2n},Qz,t) + \ b}, \\ \dfrac{c\,\mathcal{M}(Px_{2n},Qz,t) + \ d\,\mathcal{M}(Px_{2n},Qz,t)}{c\,\mathcal{M}(Qz,Qz,t) + \ d} \end{array} \right\}$$

Let $n \rightarrow \infty$ and using above result we get

$$\mathcal{M}(z,Qz,kt) \geq \ min \left\{ \begin{array}{c} \mathcal{M}(z,Qz,t), \mathcal{M}(Qz,z,t), \mathcal{M}(z,z,t), \\ \mathcal{M}(Qz,Qz,t), \mathcal{M}(z,Qz,t), \\ \dfrac{a\,\mathcal{M}(z,Qz,t) + b\,\mathcal{M}(Qz,Qz,t)}{a\,\mathcal{M}(z,Qz,t) + \ b}, \\ \dfrac{c\,\mathcal{M}(z,Qz,t) + \ d\,\mathcal{M}(z,Qz,t)}{c\,\mathcal{M}(Qz,Qz,t) + \ d} \end{array} \right\}$$

$$\mathcal{M}(z,Qz,kt) \geq \ min \left\{ \begin{array}{c} \mathcal{M}(z,Qz,t), \mathcal{M}(Qz,z,t), 1, 1, \\ \mathcal{M}(z,Qz,t), 1, \mathcal{M}(z,Qz,t) \end{array} \right\}$$

$$\mathcal{M}(z, Qz, kt) \geq \mathcal{M}(z, Qz, t)$$

By lemma (2.3) we get $Qz = z$. Therefore $Qz = STRz = z$ ... (iii)

Case $(\mathbf{2})$

Since $Q(X) \subset ABR(X)$ and $\lim_{n\to\infty} Qx_{2n+1} = z$

then there exist $v \in X$ such that $ABRv = z$ ... (iv)

putting $x = v$ and $y = x_{2n+1}$ in condition (4)

$$\mathcal{M}(Pv, Qx_{2n+1}, kt) \geq min\left\{\begin{array}{c}\mathcal{M}(ABRv, STRx_{2n+1}, t), \mathcal{M}(Qx_{2n+1}, Pv, t), \mathcal{M}(ABRv, Pv, t), \\ \mathcal{M}(STRx_{2n+1}, Qx_{2n+1}, t), \mathcal{M}(Pv, STRx_{2n+1}, t), \\ \dfrac{a\,\mathcal{M}(Pv, Qx_{2n+1}, t) + b\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t)}{a\,\mathcal{M}(Pv, STRx_{2n+1}, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pv, Qx_{2n+1}, t) + d\,\mathcal{M}(Pv, STRx_{2n+1}, t)}{c\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t) + d}\end{array}\right\}$$

$$\mathcal{M}(Pv, Qx_{2n+1}, kt) \geq min\left\{\begin{array}{c}\mathcal{M}(z, STRx_{2n+1}, t), \mathcal{M}(Qx_{2n+1}, Pv, t), \mathcal{M}(z, Pv, t), \\ \mathcal{M}(STRx_{2n+1}, Qx_{2n+1}, t), \mathcal{M}(Pv, STRx_{2n+1}, t), \\ \dfrac{a\,\mathcal{M}(Pv, Qx_{2n+1}, t) + b\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t)}{a\,\mathcal{M}(Pv, STRx_{2n+1}, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pv, Qx_{2n+1}, t) + d\,\mathcal{M}(Pv, STRx_{2n+1}, t)}{c\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t) + d}\end{array}\right\}$$

Let $n \to \infty$ and using above result we get

$$\mathcal{M}(Pv, z, kt) \geq min\left\{\begin{array}{c}\mathcal{M}(z, z, t), \mathcal{M}(z, Pv, t), \mathcal{M}(z, Pv, t), \\ \mathcal{M}(z, z, t), \mathcal{M}(Pv, z, t), \\ \dfrac{a\,\mathcal{M}(Pv, z, t) + b\,\mathcal{M}(z, z, t)}{a\,\mathcal{M}(Pv, z, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pv, z, t) + d\,\mathcal{M}(Pv, z, t)}{c\,\mathcal{M}(z, z, t) + d}\end{array}\right\}$$

$$\mathcal{M}(Pv, z, kt) \geq min\left\{\begin{array}{c}1, \mathcal{M}(z, Pv, t), \mathcal{M}(z, Pv, t), 1, \\ \mathcal{M}(z, Pv, t), 1, \mathcal{M}(z, Pv, t),\end{array}\right\}$$

$$\mathcal{M}(Pv, z, kt) \geq \mathcal{M}(Pv, z, t)$$

By lemma (2.3) we get $Pv = z$. Therefore $Pv = ABRv = z$ ... (v)

v is the coincident point of X such that $Pv = ABRv = z$ and (P, ABR) is weakly compatible mappings.

$PABRv = ABRPv$

$Pv = z \to ABRPv = ABRz$ and $ABRv = z \to PABRv = Pz$

$Pz = ABRz$

Now putting $x = z$ and $y = x_{2n+1}$ in condition (4)

$$\mathcal{M}(Pz, Qx_{2n+1}, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(ABRz, STRx_{2n+1}, t), \mathcal{M}(Qx_{2n+1}, Pz, t), \mathcal{M}(ABRz, Pz, t), \\ \mathcal{M}(STRx_{2n+1}, Qx_{2n+1}, t), \mathcal{M}(Pz, STRx_{2n+1}, t), \\ \dfrac{a\,\mathcal{M}(Pz, Qx_{2n+1}, t) + b\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t)}{a\,\mathcal{M}(Pz, STRx_{2n+1}, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz, Qx_{2n+1}, t) + d\,\mathcal{M}(Pz, STRx_{2n+1}, t)}{c\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t) + d} \end{array} \right\}$$

$$\mathcal{M}(Pz, Qx_{2n+1}, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(Pz, STRx_{2n+1}, t), \mathcal{M}(Qx_{2n+1}, Pz, t), \mathcal{M}(Pz, Pz, t), \\ \mathcal{M}(STRx_{2n+1}, Qx_{2n+1}, t), \mathcal{M}(Pz, STRx_{2n+1}, t), \\ \dfrac{a\,\mathcal{M}(Pz, Qx_{2n+1}, t) + b\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t)}{a\,\mathcal{M}(Pz, STRx_{2n+1}, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz, Qx_{2n+1}, t) + d\,\mathcal{M}(Pz, STRx_{2n+1}, t)}{c\,\mathcal{M}(Qx_{2n+1}, STRx_{2n+1}, t) + d} \end{array} \right\}$$

Let $n \to \infty$ and using above result we get

$$\mathcal{M}(Pz, z, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(Pz, z, t), \mathcal{M}(z, Pz, t), \mathcal{M}(Pz, Pz, t), \\ \mathcal{M}(z, z, t), \mathcal{M}(Pz, z, t), \\ \dfrac{a\,\mathcal{M}(Pz, z, t) + b\,\mathcal{M}(z, z, t)}{a\,\mathcal{M}(Pz, z, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz, z, t) + d\,\mathcal{M}(Pz, z, t)}{c\,\mathcal{M}(z, z, t) + d} \end{array} \right\}$$

$$\mathcal{M}(Pz, z, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(Pz, z, t), \mathcal{M}(z, Pz, t), 1, 1, \\ \mathcal{M}(Pz, z, t), 1, \mathcal{M}(Pz, z, t), \end{array} \right\}$$

$$\mathcal{M}(Pz, z, kt) \geq \mathcal{M}(Pz, z, t)$$

By lemma (2.3) we get $Pz = z$. Therefore $Pz = ABRv = z$ ... (vi)

again putting $x = Rz$ and $y = z$ in condition (4)

$$\mathcal{M}(PRz, Qz, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(ABRRz, STRz, t), \mathcal{M}(Qz, PRz, t), \mathcal{M}(ABRRz, PRz, t), \\ \mathcal{M}(STRz, Qz, t), \mathcal{M}(PRz, STRz, t), \\ \dfrac{a\,\mathcal{M}(PRz, Qz, t) + b\,\mathcal{M}(Qz, STRz, t)}{a\,\mathcal{M}(PRz, STRz, t) + b}, \\ \dfrac{c\,\mathcal{M}(PRz, Qz, t) + d\,\mathcal{M}(PRz, STRz, t)}{c\,\mathcal{M}(Qz, STRz, t) + d} \end{array} \right\}$$

$$\mathcal{M}(RPz, Qz, kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(ARBRz, STRz, t), \mathcal{M}(Qz, RPz, t), \mathcal{M}(ARBRz, RPz, t), \\ \mathcal{M}(STRz, Qz, t), \mathcal{M}(RPz, STRz, t), \\ \dfrac{a\,\mathcal{M}(RPz, Qz, t) + b\,\mathcal{M}(Qz, STRz, t)}{a\,\mathcal{M}(RPz, STRz, t) + b}, \\ \dfrac{c\,\mathcal{M}(RPz, Qz, t) + d\,\mathcal{M}(RPz, STRz, t)}{c\,\mathcal{M}(Qz, STRz, t) + d} \end{array} \right\}$$

from condition (2)

$$\mathcal{M}(Rz,z,kt) \geq \min \begin{Bmatrix} \mathcal{M}(RABRz,z,t), \mathcal{M}(z,Rz,t), \mathcal{M}(RABRz,Rz,t), \\ \mathcal{M}(z,z,t), \mathcal{M}(Rz,z,t), \\ \dfrac{a\,\mathcal{M}(Rz,z,t) + b\,\mathcal{M}(z,z,t)}{a\,\mathcal{M}(Rz,z,t) + b}, \\ \dfrac{c\,\mathcal{M}(Rz,z,t) + d\,\mathcal{M}(Rz,z,t)}{c\,\mathcal{M}(z,z,t) + d} \end{Bmatrix}$$

from equation (iii) & (vi) and condition (2)

$$\mathcal{M}(Rz,z,kt) \geq \min \begin{Bmatrix} \mathcal{M}(Rz,z,t), \mathcal{M}(z,Rz,t), \mathcal{M}(Rz,Rz,t), \\ \mathcal{M}(z,z,t), \mathcal{M}(Rz,z,t), \\ \dfrac{a\,\mathcal{M}(Rz,z,t) + b\,\mathcal{M}(z,z,t)}{a\,\mathcal{M}(Rz,z,t) + b}, \\ \dfrac{c\,\mathcal{M}(Rz,z,t) + d\,\mathcal{M}(Rz,z,t)}{c\,\mathcal{M}(z,z,t) + d} \end{Bmatrix}$$

$$\mathcal{M}(Rz,z,kt) \geq \min \begin{Bmatrix} \mathcal{M}(Rz,z,t), \mathcal{M}(z,Rz,t), 1,1, \\ \mathcal{M}(Rz,z,t), 1, \mathcal{M}(Rz,z,t) \end{Bmatrix}$$

$$\mathcal{M}(Rz,z,kt) \geq \mathcal{M}(Rz,z,t)$$

By lemma (2.3) we get $Rz = z$.

Therefore $STRz = z \to STz = z$ & $ABRz = z \to ABz = z$

$$STz = ABz = z \qquad \ldots (vii)$$

again putting $x = Bz$ and $y = z$ in condition (4)

$$\mathcal{M}(PBz,Qz,kt) \geq \min \begin{Bmatrix} \mathcal{M}(ABRBz,STRz,t), \mathcal{M}(Qz,PBz,t), \mathcal{M}(ABRBz,PBz,t), \\ \mathcal{M}(STRz,Qz,t), \mathcal{M}(PBz,STRz,t), \\ \dfrac{a\,\mathcal{M}(PBz,Qz,t) + b\,\mathcal{M}(Qz,STRz,t)}{a\,\mathcal{M}(PBz,STRz,t) + b}, \\ \dfrac{c\,\mathcal{M}(PBz,Qz,t) + d\,\mathcal{M}(PBz,STRz,t)}{c\,\mathcal{M}(Qz,STRz,t) + d} \end{Bmatrix}$$

$$\mathcal{M}(BPz,Qz,kt) \geq \min \begin{Bmatrix} \mathcal{M}(BARBz,STRz,t), \mathcal{M}(Qz,BPz,t), \mathcal{M}(BARBz,BPz,t), \\ \mathcal{M}(STRz,Qz,t), \mathcal{M}(BPz,STRz,t), \\ \dfrac{a\,\mathcal{M}(BPz,Qz,t) + b\,\mathcal{M}(Qz,STRz,t)}{a\,\mathcal{M}(BPz,STRz,t) + b}, \\ \dfrac{c\,\mathcal{M}(BPz,Qz,t) + d\,\mathcal{M}(BPz,STRz,t)}{c\,\mathcal{M}(Qz,STRz,t) + d} \end{Bmatrix}$$

from condition (2)

$$\mathcal{M}(Bz,z,kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(BARBz,z,t), \mathcal{M}(z,Bz,t), \mathcal{M}(BARBz,Bz,t), \\ \mathcal{M}(z,z,t), \mathcal{M}(Bz,z,t), \\ \dfrac{a\,\mathcal{M}(Bz,z,t) + b\,\mathcal{M}(z,z,t)}{a\,\mathcal{M}(Bz,z,t) + b} \\ \dfrac{c\,\mathcal{M}(Bz,z,t) + d\,\mathcal{M}(Bz,z,t)}{c\,\mathcal{M}(z,z,t) + d} \end{array} \right\}$$

from equation (iii) & (vi)

$$\mathcal{M}(Bz,z,kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(BARBz,z,t), \mathcal{M}(z,Bz,t), \mathcal{M}(BARBz,Bz,t), \\ \mathcal{M}(z,z,t), \mathcal{M}(Bz,z,t), \\ \dfrac{a\,\mathcal{M}(Bz,z,t) + b\,\mathcal{M}(z,z,t)}{a\,\mathcal{M}(Bz,z,t) + b}, \\ \dfrac{c\,\mathcal{M}(Bz,z,t) + d\,\mathcal{M}(Bz,z,t)}{c\,\mathcal{M}(z,z,t) + d} \end{array} \right\}$$

$$\mathcal{M}(Bz,z,kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(Bz,z,t), \mathcal{M}(z,Bz,t), \mathcal{M}(Bz,Bz,t), \\ \mathcal{M}(z,z,t), \mathcal{M}(Bz,z,t), \\ \dfrac{a\,\mathcal{M}(Bz,z,t) + b\,\mathcal{M}(z,z,t)}{a\,\mathcal{M}(Bz,z,t) + b}, \\ \dfrac{c\,\mathcal{M}(Bz,z,t) + d\,\mathcal{M}(Bz,z,t)}{c\,\mathcal{M}(z,z,t) + d} \end{array} \right\}$$

from equation (vi)

$$\mathcal{M}(Bz,z,kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(Bz,z,t), \mathcal{M}(z,Bz,t), 1,1, \\ \mathcal{M}(Bz,z,t), 1, \mathcal{M}(Bz,z,t) \end{array} \right\}$$

$$\mathcal{M}(Bz,z,kt) \geq \mathcal{M}(Bz,z,t)$$

By lemma (2.3) we get $Bz = z$.
Therefore $ABz = z \to Az = z$

$$Az = Bz = z \qquad \ldots (viii)$$

putting $x = z$ and $y = Tz$ in condition (4)

$$(Pz,QTz,kt) \geq \min \left\{ \begin{array}{c} \mathcal{M}(ABRz,STRTz,t), \mathcal{M}(QTz,Pz,t), \mathcal{M}(ABRz,Pz,t), \\ \mathcal{M}(STRTz,QTz,t), \mathcal{M}(Pz,STRTz,t), \\ \dfrac{a\,\mathcal{M}(Pz,QTz,t) + b\,\mathcal{M}(QTz,STRTz,t)}{a\,\mathcal{M}(Pz,STRTz,t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz,QTz,t) + d\,\mathcal{M}(Pz,STRTz,t)}{c\,\mathcal{M}(QTz,STRTz,t) + d} \end{array} \right\}$$

$$(Pz, TQz, kt) \geq min \begin{Bmatrix} \mathcal{M}(ABRz, TSRTz, t), \mathcal{M}(TQz, Pz, t), \mathcal{M}(ABRz, Pz, t), \\ \mathcal{M}(TSRTz, TQz, t), \mathcal{M}(Pz, TSRTz, t), \\ \dfrac{a\,\mathcal{M}(Pz, TQz, t) + b\,\mathcal{M}(TQz, TSRTz, t)}{a\,\mathcal{M}(Pz, TSRTz, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz, TQz, t) + d\,\mathcal{M}(Pz, TSRTz, t)}{c\,\mathcal{M}(TQz, TSRTz, t) + d} \end{Bmatrix}$$

from condition (2)

$$(Pz, TQz, kt) \geq min \begin{Bmatrix} \mathcal{M}(ABRz, TSTRz, t), \mathcal{M}(TQz, Pz, t), \mathcal{M}(ABRz, Pz, t), \\ \mathcal{M}(TSTRz, TQz, t), \mathcal{M}(Pz, TSTRz, t), \\ \dfrac{a\,\mathcal{M}(Pz, TQz, t) + b\,\mathcal{M}(TQz, TSTRz, t)}{a\,\mathcal{M}(Pz, TSTRz, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz, TQz, t) + d\,\mathcal{M}(Pz, TSTRz, t)}{c\,\mathcal{M}(TQz, TSTRz, t) + d} \end{Bmatrix}$$

from condition (2)

$$(z, Tz, kt) \geq min \begin{Bmatrix} \mathcal{M}(z, Tz, t), \mathcal{M}(Tz, z, t), \mathcal{M}(z, z, t), \\ \mathcal{M}(Tz, Tz, t), \mathcal{M}(z, Tz, t), \\ \dfrac{a\,\mathcal{M}(z, Tz, t) + b\,\mathcal{M}(Tz, Tz, t)}{a\,\mathcal{M}(z, Tz, t) + b}, \\ \dfrac{c\,\mathcal{M}(z, Tz, t) + d\,\mathcal{M}(z, Tz, t)}{c\,\mathcal{M}(Tz, Tz, t) + d} \end{Bmatrix}$$

from equation (iii) & (vi)

$$(z, Tz, kt) \geq min \begin{Bmatrix} \mathcal{M}(z, Tz, t), \mathcal{M}(Tz, z, t), 1, 1, \\ \mathcal{M}(z, Tz, t), 1, \mathcal{M}(z, Tz, t) \end{Bmatrix}$$

$$(z, Tz, kt) \geq \mathcal{M}(z, Tz, t)$$

By lemma (2.3) we get $Tz = z$. Therefore $STz = z \rightarrow Sz = z$

$$Sz = Tz = z \qquad \dots (ix)$$

Hence $Pz = Qz = Rz = Sz = Tz = Az = Bz = z$

so we get z is a common fixed point of self mappings P, Q, R, S, T, A and B

Uniqueness: Let w be another common fixed point of self-mappings P, Q, R, S, T, A and B

such that $Pz = Qz = Rz = Sz = Tz = Az = Bz = z$

$Pw = Qw = Rw = Sw = Tw = Aw = Bw = w$

putting $x = z$ and $y = w$ in condition (4)

$$\mathcal{M}(Pz, Qw, kt) \geq min \begin{Bmatrix} \mathcal{M}(ABRz, STRw, t), \mathcal{M}(Qw, Pz, t), \mathcal{M}(ABRz, Pz, t), \\ \mathcal{M}(STRw, Qw, t), \mathcal{M}(Pz, STRw, t), \\ \dfrac{a\,\mathcal{M}(Pz, Qw, t) + b\,\mathcal{M}(Qw, STRw, t)}{a\,\mathcal{M}(Pz, STRw, t) + b}, \\ \dfrac{c\,\mathcal{M}(Pz, Qw, t) + d\,\mathcal{M}(Pz, STRw, t)}{c\,\mathcal{M}(Qw, STRw, t) + d} \end{Bmatrix}$$

$$\mathcal{M}(z,w,kt) \geq \min\left\{\begin{matrix} \mathcal{M}(ABz,STw,t), \mathcal{M}(w,z,t), \mathcal{M}(ABz,z,t), \\ \mathcal{M}(STw,w,t), \mathcal{M}(z,STw,t), \\ \dfrac{a\,\mathcal{M}(z,w,t) + b\,\mathcal{M}(w,STw,t)}{a\,\mathcal{M}(z,STw,t) + b}, \\ \dfrac{c\,\mathcal{M}(z,w,t) + d\,\mathcal{M}(z,STw,t)}{c\,\mathcal{M}(w,STw,t) + d} \end{matrix}\right\}$$

$$\mathcal{M}(z,w,kt) \geq \min\left\{\begin{matrix} \mathcal{M}(Az,Sw,t), \mathcal{M}(w,z,t), \mathcal{M}(Az,z,t), \\ \mathcal{M}(Sw,w,t), \mathcal{M}(z,Sw,t), \\ \dfrac{a\,\mathcal{M}(z,w,t) + b\,\mathcal{M}(w,Sw,t)}{a\,\mathcal{M}(z,Sw,t) + b}, \\ \dfrac{c\,\mathcal{M}(z,w,t) + d\,\mathcal{M}(z,Sw,t)}{c\,\mathcal{M}(w,Sw,t) + d} \end{matrix}\right\}$$

$$\mathcal{M}(z,w,kt) \geq \min\left\{\begin{matrix} \mathcal{M}(z,w,t), \mathcal{M}(w,z,t), \mathcal{M}(z,z,t), \\ \mathcal{M}(w,w,t), \mathcal{M}(z,w,t), \\ \dfrac{a\,\mathcal{M}(z,w,t) + b\,\mathcal{M}(w,w,t)}{a\,\mathcal{M}(z,w,t) + b}, \\ \dfrac{c\,\mathcal{M}(z,w,t) + d\,\mathcal{M}(z,w,t)}{c\,\mathcal{M}(w,w,t) + d} \end{matrix}\right\}$$

$$\mathcal{M}(z,w,kt) \geq \min\left\{\begin{matrix} \mathcal{M}(z,w,t), \mathcal{M}(w,z,t), 1, 1, \\ \mathcal{M}(z,w,t), 1, \mathcal{M}(z,w,t) \end{matrix}\right\}$$

$$\mathcal{M}(z,w,kt) \geq \mathcal{M}(z,w,t)$$

$$\Rightarrow z = w$$

Hence z is the unique common fixed point of A, B, R, S, T, P and Q.
If we take $R = I$($I =$ the identity mapping on X), we have

Corollary 3.1. Let A, B, S, T, P and Q be six self maps of a complete fuzzy metric space $(X, \mathcal{M}, *)$ such that the following conditions are satisfied:

(1) $P(X) \subset ST(X), Q(X) \subset AB(X)$;
(2) $AB = BA$, $ST = TS$, $PB = BP$ and $QT = TQ$,
(3) (P, AB) and (Q, ST) are weakly compatible;
(4) $\mathcal{M}(Px, Qy, kt)$

$$\geq \min\left\{\begin{matrix} \mathcal{M}(ABx,STy,t), \mathcal{M}(Qy,Px,t), \mathcal{M}(ABx,Px,t), \\ \mathcal{M}(STy,Qy,t), \mathcal{M}(Px,STy,t), \\ \dfrac{a\,\mathcal{M}(Px,Qy,t) + b\,\mathcal{M}(Qy,STRy,t)}{a\,\mathcal{M}(Px,STRy,t) + b}, \\ \dfrac{c\,\mathcal{M}(Px,Qy,t) + d\,\mathcal{M}(Px,STRy,t)}{c\,\mathcal{M}(Qy,STRy,t) + d} \end{matrix}\right\}$$

for all $x, y \in X$ and $t > 0$, where $k \in (0,1)$ and $a, b, c, d, e, f \geq 0$ with $a \,\&\, b, c \,\&\, d, e \,\&\, f$ and $a, b \,\&\, c$ cannot be simultaneously 0.

Then A, B, S, T, P and Q have a unique common fixed point in X.
If we take $B = T = I(\ I =$ the identity mapping on $X)$, we have

Corollary 3.2: Let A, S, P and Q be four self-maps of a complete fuzzy metric space $(X, \mathcal{M}, *)$ such that the following conditions are satisfied:

(1) $P(X) \subset S(X), Q(X) \subset A(X)$;

(2) (P, A) and (Q, S) are weakly compatible;

(3)

$$\mathcal{M}(Px, Qy, kt) \geq min \left\{ \begin{array}{c} \mathcal{M}(Ax, Sy, t), \mathcal{M}(Qy, Px, t), \mathcal{M}(Ax, Px, t), \\ \mathcal{M}(Sy, Qy, t), \mathcal{M}(Px, Sy, t), \\ \dfrac{a\,\mathcal{M}(Px, Qy, t) + b\,\mathcal{M}(Qy, Sy, t)}{a\,\mathcal{M}(Px, Sy, t) +\ b}, \\ \dfrac{c\,\mathcal{M}(Px, Qy, t) +\ d\,\mathcal{M}(Px, Sy, t)}{c\,\mathcal{M}(Qy, Sy, t) +\ d} \end{array} \right\}$$

for all $x, y \in X$ and $t > 0$, where $k \in (0, 1)$ and $a, b, c, d, e, f \geq 0$ with $a \,\&\, b, c \,\&\, d, e \,\&\, f$ and $a, b \,\&\, c$ cannot be simultaneously 0. Then A, S, P and Q have a unique common fixed point in X.

4. Conclusion

In the present work, a common fixed point theorem for seven self-mappings in fuzzy metric space using weak compatibility without continuity has been proved. Our result extends and generalizes several known results of fixed point theory in different spaces which is more gripping and useful for other researchers.

Acknowledgement. We also acknowledge anonymous reviewers for their excellent and constructive comments.

Conflicts of Interest. The authors declare no conflicts of interest.

Authors' contributions. All authors contributed equally to this work.

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