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# Fixed Point Theorems for Compatible Mappings of Type $(A)$ in Intuitionistic Fuzzy Metric Space

***Syed Shahnawaz Ali*[1], *Niharika Kumari*[2] and *Rajesh Kumar*[3]**

Department of Mathematics
Sri Satya Sai University of Technology and Medical Sciences
Sehore (M.P.) India
*Corresponding author. Email: drsyedshahnawazali@gmail.com

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***Abstract.*** The principal motivation of this paper is to relate some results in the literature by discussing the existence and uniqueness of fixed points for new classes of mappings defined on a complete metric space. In particular, we prove common fixed point theorems for four self-mappings under the conditions of compatible mappings of type $(A\text{-}1)$ and type $(A\text{-}2)$ in complete intuitionistic fuzzy metric space.

***Keywords:*** Fuzzy metric space, intuitionistic fuzzy metric space, compatible mappings of type A, type (A-1) and type (A-2), fixed point, common fixed point theorem

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## 1. Introduction

Fuzzy mathematics commenced with the introduction of the notion of fuzzy sets by Zadeh [22] as a new way to represent vagueness in everyday life. In mathematical programming, problems are expressed as optimizing some goal function given certain constraints, and real-life problems consider multiple objectives. Generally, it is very difficult to get a feasible solution that brings us to the optimum of all objective functions. A possible method of resolution that is quite useful is the one using fuzzy sets by Turkoglu and Rhoades [20]. Atanassov [5] introduced the notion of intuitionistic fuzzy sets by generalizing the notion of fuzzy set by treating membership as a fuzzy logical value rather than a single truth value. In 2004, Park [17] defined the notion of intuitionistic fuzzy metric space with the help of continuous $t$ −norms and continuous $t$ −conorms. George and Veeramani [9] had showed that every metric induces an intuitionistic fuzzy metric and found a necessary and sufficient conditions for an intuitionistic fuzzy metric space to be complete. Choudhary [7] introduced mutually contractive sequence of self maps and proved a fixed point theorem. Kramaosil and Michalek [13] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach [6]. Turkoglu et al [21] gave the generalization of Jungck's [11,12] common fixed point theorem to intuitionistic fuzzy metric spaces. Subsequently, several authors like Coker [8], Saadati and park [19], Gregori et al [10], Manro et al [15], Alaca et al. [2], Rashmi and

Manro [18], Manro [14], Abu-Donia and Nase [1], Alaca et al. [3,4], and Muthuraj and Pandiselvi [16] derived fixed point theorems in intuitionistic fuzzy metric space.

In view of the considerations given by various authors, the principal motivation of this paper is to relate some results in the literature by discussing the existence and uniqueness of fixed points for new classes of mappings defined on a complete metric space. In particular, we prove common fixed point theorems for four self-mappings under the conditions of compatible mappings of type $(A$-1) and type $(A$-2) in complete intuitionistic fuzzy metric space.

## 2. Preliminaries

Some basic definitions are given here.

**Definition 2.1.** A binary operation $*: [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous $t$ −norm if $*$ satisfies the following conditions

(a) $*$ is commutative and associative;
(b) $*$ is continuous;
(c) $a * 1 = a$ for all $a \in [0,1]$;
(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

**Definition 2.2.** A binary operation $\Diamond: [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous $t$ −conorm if $\Diamond$ is satisfying the following conditions

(a) $\Diamond$ is commutative and associative;
(b) $\Diamond$ is continuous;
(c) $a \Diamond 0 = a$ for all $a \in [0,1]$;
(d) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

**Definition 2.3.** A 5−tuple $(X, \mathcal{M}, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if $X$ is an arbitrary set, $*$ is a continuous $t-$ norm, $\Diamond$ is a continuous $t$ −conorm and $\mathcal{M}, N$ are fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$:

(a) $\mathcal{M}(x, y, z, t) + N(x, y, z, t) \leq 1$;
(b) $\mathcal{M}(x, y, z, t) > 0$;
(c) $\mathcal{M}(x, y, z, t) = 1$ if and only if $x = y = z$;
(d) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$ where $p$ is a permutation function;
(e) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$;
(f) $\mathcal{M}(x, y, z, .): (0, \infty) \longrightarrow [0,1]$ is continuous;
(g) $N(x, y, z, t) < 1$;
(h) $N(x, y, z, t) = 0$ if and only if $x = y = z$;
(i) $N(x, y, z, t) = N(p\{x, y, z\}, t)$ where $p$ is a permutation function;
(j) $N(x, y, a, t) \Diamond N(a, z, z, s) \geq N(x, y, z, t + s)$;
(k) $N(x, y, z, .): (0, \infty) \longrightarrow [0,1]$ is continuous;

Then $(\mathcal{M}, N)$ is called an intuitionistic fuzzy metric on $X$. The functions $\mathcal{M}(x, y, z, t)$ and $N(x, y, z, t)$ denote respectively the degree of nearness and the degree of non nearness between $x, y$ and $z$ with respect to $t$.

**Remark 2.1.** It is to be noted that
$(i)$ The intuitionistic fuzzy setting provides both a membership degree and a nonmembership degree for an element, whereas the fuzzy settings provide only the membership degree alone and thus the space considered here will definitely provide a better environment than the latter to work with the applications.
$(ii)$ Every fuzzy setting can be generalized to intuitionistic fuzzy setting but not the converse

**Example 2.1.** Let $X = R$ and $\mathcal{M}(x, y, z, t) = \frac{t}{t+|x-y|+|y-z|+|z-x|}$, $N(x, y, z, t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}$ for every $x, y, z$ and $t > 0$, let $A$ and $B$ defined as $Ax = 2x + 1, Bx = x + 2$, consider the sequence $x_n = \frac{1}{n} + 1, n = 1, 2, \dots$ .
Thus we have
$\lim_{n\to\infty} \mathcal{M}(Ax_n, 3, 3, t) = \lim_{n\to\infty} \mathcal{M}(Bx_n, 3, 3, t) = 1$ and
$\lim_{n\to\infty} N(Ax_n, 3, 3, t) = \lim_{n\to\infty} N(Bx_n, 3, 3, t) = 0$ for every $t > 0$.
Then $A$ and $B$ satisfying the property $(E)$.

**Definition 2.4.** Let $(X, \mathcal{M}, N, *, \Diamond)$ be an intuitionistic fuzzy metric space and $\{x_n\}$ be a sequence in $X$

(i) $\{x_n\}$ is said to be convergent to a point $x \in X$, if $\lim_{n\to\infty} \mathcal{M}(x, x, x_n, t) = 1$, $\lim_{n\to\infty} N(x, x, x_n, t) = 0$ for all $t > 0$.
(ii) $\{x_n\}$ in $X$ is said to be Cauchy sequence if
$$\lim_{n\to\infty} \mathcal{M}\left(x_{n+p}, x_{n+p}, x_n, t\right) = 1, \lim_{n\to\infty} N\left(x_{n+p}, x_{n+p}, x_n, t\right) = 0$$
for all $t > 0$ and $p > 0$.
(iii) An intuitionstic fuzzy metric space$(X, \mathcal{M}, N, *, \Diamond)$ is said to be complete. if and only if every Cauchy sequence in $X$ is convergent.

**Lemma 2.1.** Let $(X, \mathcal{M}, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then $\mathcal{M}(x, y, z, t)$ and $N(x, y, z, t)$ are non decreasing with respect to $t$, for all $x, y, z$ in $X$.

**Definition 2.5.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \Diamond)$ into itself. Then the mappings are said to be compatible if
$\lim_{n\to\infty} \mathcal{M}(ASx_n, SAx_n, SAx_n, t) = 1$, and $\lim_{n\to\infty} N(ASx_n, SAx_n, SAx_n, t) = 0$,
for every $t > 0$, whenever $\{x_n\}$ is a sequence in $X$ such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

**Definition 2.6.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \Diamond)$ into itself. Then the mappings are said to be compatible of type $(A)$, if
$\lim_{n\to\infty} \mathcal{M}(AAx_n, SSx_n, SSx_n, t) = 1$, and $\lim_{n\to\infty} N(AAx_n, SSx_n, SSx_n, t) = 0$,
for every $t > 0$, whenever $\{x_n\}$ is a sequence in $X$ such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

**Definition 2.7.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ into itself. Then the mappings are said to be $R$ −weakly compatible of type $(A)$, if there exists some $R > 0$, such that
$\lim_{n\to\infty} \mathcal{M}(AAx, SSx, SSx, t) \geq \mathcal{M}\left(Ax, Sx, Sx, \frac{t}{R}\right)$, and $\lim_{n\to\infty} N(AAx, SSx, SSx, t) \leq N\left(Ax, Sx, Sx, \frac{t}{R}\right)$, for every $t > 0$, and $x \in X$.

**Definition 2.8.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ into itself. Then the mappings are said to be compatible of type $(A - 1)$, if $\lim_{n\to\infty} \mathcal{M}(SAx_n, AAx_n, AAx_n, t) = 1$, and $\lim_{n\to\infty} N(SAx_n, AAx_n, AAx_n, t) = 0$, for every $t > 0$, whenever $\{x_n\}$ is a sequence in $X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

**Definition 2.9.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ into itself. Then the mappings are said to be compatible of type $(A - 2)$, if $\lim_{n\to\infty} \mathcal{M}(AAx_n, SSx_n, SSx_n, t) = 1$, and $\lim_{n\to\infty} N(AAx_n, SSx_n, SSx_n, t) = 0$, for every $t > 0$, whenever $\{x_n\}$ is a sequence in $X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

**Proposition 2.1.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ into itself.

(a) If $A$ is continuous map then the pair of mappings $(A, S)$ is compatible of type $(A - 1)$, if and only if $A$ and $S$ are compatible.

(b) If $S$ is continuous map then the pair of mappings $(A, S)$ is compatible of type $(A - 2)$, if and only if $A$ and $S$ are compatible.

**Proof:** (a) Let $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$, and let the pair $(A, S)$ be compatible of type $(A - 1)$. Since $A$ is continuous, we have $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} AAx_n = Az$. Therefore it follows that

$\mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq \mathcal{M}\left(SAx_n, AAx_n, AAx_n, \frac{t}{2}\right) * \mathcal{M}\left(AAx_n, ASx_n, ASx_n, \frac{t}{2}\right)$

and $N(SAx_n, ASx_n, ASx_n, t) \leq N\left(SAx_n, AAx_n, AAx_n, \frac{t}{2}\right) \lozenge N\left(AAx_n, ASx_n, ASx_n, \frac{t}{2}\right)$

yields $\lim_{n\to\infty} \mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq 1 * 1 = 1$ and

$\lim_{n\to\infty} N(SAx_n, ASx_n, ASx_n, t) \geq 0 \lozenge 0 = 0$ and so the mappings compatible.

Now, let $A$ and $S$ be compatible.

Therefore it follows that

$\mathcal{M}(SAx_n, AAx_n, AAx_n, t) \geq \mathcal{M}\left(SAx_n, ASx_n, ASx_n, \frac{t}{2}\right) * \mathcal{M}\left(ASx_n, AAx_n, AAx_n, \frac{t}{2}\right)$

and

$$N(SAx_n, AAx_n, AAx_n, t) \leq N\left(SAx_n, ASx_n, ASx_n, \frac{t}{2}\right) \lozenge N\left(ASx_n, AAx_n, AAx_n, \frac{t}{2}\right)$$

yields $\lim_{n\to\infty} \mathcal{M}(SAx_n, AAx_n, AAx_n, t) \geq 1 * 1 = 1$ and

$\lim_{n\to\infty} N(SAx_n, AAx_n, AAx_n, t) \leq 0 \lozenge 0 = 0$ and so that pair of mappings $(A, S)$ are compatible of type $(A-1)$.

(b) Let $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = z$ for some $z \in X$, and let the pair $(A, S)$ be compatible of type $(A-2)$. Since $S$ is continuous, we have

$$\lim_{n\to\infty} SAx_n = Sz \text{ and } \lim_{n\to\infty} SSx_n = Sz.$$

Therefore it follows that

$\mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq \mathcal{M}\left(SAx_n, SSx_n, SSx_n, \frac{t}{2}\right) * \mathcal{M}\left(SSx_n, ASx_n, ASx_n, \frac{t}{2}\right)$ and

$N(SAx_n, ASx_n, ASx_n, t) \leq N\left(SAx_n, SSx_n, SSx_n, \frac{t}{2}\right) \lozenge N\left(SSx_n, ASx_n, ASx_n, \frac{t}{2}\right)$

yields $\lim_{n\to\infty} \mathcal{M}(SAx_n, ASx_n, ASx_n, t) \geq 1 * 1 = 1$ and

$\lim_{n\to\infty} N(SAx_n, ASx_n, ASx_n, t) \geq 0 \lozenge 0 = 0$ and so the mappings $A$ and $S$ are compatible. Now, let $A$ and $S$ be compatible. Therefore it follows that

$\mathcal{M}(ASx_n, SSx_n, SSx_n, t) \geq \mathcal{M}\left(ASx_n, SAx_n, SAx_n, \frac{t}{2}\right) * \mathcal{M}\left(SAx_n, SSx_n, SSx_n, \frac{t}{2}\right)$ and

$$N(ASx_n, SSx_n, SSx_n, t) \leq N\left(ASx_n, SAx_n, SAx_n, \frac{t}{2}\right) \lozenge N\left(SAx_n, SSx_n, SSx_n, \frac{t}{2}\right)$$

yields $\lim_{n\to\infty} \mathcal{M}(ASx_n, SSx_n, SSx_n, t) \geq 1 * 1 = 1$

and $\lim_{n\to\infty} N(ASx_n, SSx_n, SSx_n, t) \leq 0 \lozenge 0 = 0$ and so that pair of mappings $(A, S)$ are compatible of type $(A-2)$.

**Proposition 2.2.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ into itself. If the pair $(A, S)$ are compatible of type $(A-2)$ and $Sz = Az$ for some $z \in X$. Then $ASz = SSz$

**Proof:** Let $\{x_n\}$ be a sequence in $X$ define by $x_n = z$ for $n = 1,2, \ldots$ and let $z = Sz$ . Then we have $\lim_{n\to\infty} Sx_n = Sz$ and $\lim_{n\to\infty} Ax_n = Az$. Since the pair $(A, S)$ is compatible of type $(A-2)$, we have

$$\mathcal{M}(ASz, SSz, SSz, t) = \lim_{n\to\infty} \mathcal{M}(ASx_n, SSx_n, SSx_n, t) = 1 \text{ and}$$

$$N(ASz, SSz, SSz, t) = \lim_{n\to\infty} N(ASx_n, SSx_n, SSx_n, t) = 0$$

Hence $ASz = SSz$.

**Proposition 2.3.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq 1-t$ for all $t \in [0,1]$ if the pair $(A, S)$ are compatible of type $(A-1)$ and $Ax_n, Sx_n \to z$ for some $z \in X$ and a sequence $\{x_n\}$ in $X$. Then $AAx_n \to Sz$, if $S$ is continuous at $z$

**Proof:** Since $S$ is continuous at $z$, we have $SAx_n \to Sz$. Since the pair $(A, S)$ are compatible of type $(A-1)$, we have $\mathcal{M}(SAx_n, AAx_n, AAx_n, t) \to 1$ as $n \to \infty$. It follows that

$$\mathcal{M}(Sz, AAx_n, AAx_n, t) \geq \mathcal{M}\left(Sz, SAx_n, SAx_n, \frac{t}{2}\right) * \mathcal{M}\left(SAx_n, AAx_n, AAx_n, \frac{t}{2}\right)$$

and

$N(Sz, AAx_n, AAx_n, t) \leq N\left(Sz, SAx_n, SAx_n, \frac{t}{2}\right) \lozenge N\left(SAx_n, AAx_n, AAx_n, \frac{t}{2}\right)$

yield $\mathcal{M}\ (Sz, AAx_n, AAx_n, t) \geq\ 1 * 1 = 1$ and
$N\ (Sz, AAx_n, AAx_n, t) \leq 0 \lozenge 0 = 0$
and so we have $AAx_n \to Sz$ as $n \to \infty$.

**Proposition 2.4.** Let $A$ and $S$ be self mappings from an intuitionistic fuzzy metric space $(X, \mathcal{M}, N, *, \lozenge)$ with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq 1-t$ for all $t \in [0,1]$ if the pair $(A, S)$ are compatible of type $(A-2)$ and $Ax_n, Sx_n \to z$ for some $z \in X$ and a sequence $\{x_n\}$ in $X$. Then $SSx_n \to Az$, if $A$ is continuous at $z$
**Proof:** Since $A$ is continuous at $z$, we have $ASx_n \to Az$. Since the pair $(A, S)$ are compatible of type $(A-2)$, we have $\mathcal{M}\ (ASx_n, SSx_n, SSx_n, t) \to 1$ as $n \to \infty$.
It follows that

$$\mathcal{M}\ (Az, SSx_n, SSx_n, t) \geq \mathcal{M}\ \left(Az, ASx_n, ASx_n, \frac{t}{2}\right) * \mathcal{M}\ \left(ASx_n, SSx_n, SSx_n, \frac{t}{2}\right)$$

and

$$N\ (Az, SSx_n, SSx_n, t) \leq N\ \left(Az, ASx_n, ASx_n, \frac{t}{2}\right) \lozenge\ N\ \left(ASx_n, SSx_n, SSx_n, \frac{t}{2}\right)$$

yield $\mathcal{M}\ (Az, SSx_n, SSx_n, t) \geq\ 1 * 1 = 1$
and $N\ (Az, SSx_n, SSx_n, t) \leq 0 \lozenge 0 = 0$
and so we have $SSx_n \to Az$ as $n \to \infty$.

## 3. The main results

**Theorem 3.1.** Let $(X, \mathcal{M}, N, *, \lozenge)$ be a complete generalized intuitionistic fuzzy metric spaces and let $A, B, S$, and $T$ be self mappings of $X$ satisfying the following conditions:

(1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,
(2) $A$ and $S$ are continuous,
(3) The pairs $(A, S)$and $(B, T)$ are compatible mappings of type $(A)$,
(4) There exist $q \in (0,1)$ such that

$$\mathcal{M}(Ax, Bz, Bz, qt)\ \geq min \begin{Bmatrix} \mathcal{M}\ (Sx, Ay, Tz, t), \mathcal{M}\ (Sx, Ay, By, t), \\ \mathcal{M}\ (Sx, By, Tz, t), \mathcal{M}(Sx, By, By, t), \\ \mathcal{M}(Sx, Ay, Ay, t), \mathcal{M}\ (Sx, Tz, Tz, t), \\ \mathcal{M}\ (By, Tz, Tz, t), \mathcal{M}(By, By, Tz, t), \\ \mathcal{M}\ (By, Tz, Ay, t), \mathcal{M}\ (Tz, Ay, Ay, t) \end{Bmatrix} \text{ and}$$

$$N(Ax, Bz, Bz, qt)\ \leq max \begin{Bmatrix} N\ (Sx, Ay, Tz, t), N(Sx, Ay, By, t), \\ N(Sx, By, Tz, t), N(Sx, By, By, t), \\ N(Sx, Ay, Ay, t), N\ (Sx, Tz, Tz, t), \\ N\ (By, Tz, Tz, t), N(By, By, Tz, t), \\ N(By, Tz, Ay, t), N\ (Tz, Ay, Ay, t) \end{Bmatrix}$$

For every $x, y \in X$ and $t > 0$. Then $A, B, S$ and $T$ have a unique common fixed point in $X$.
**Proof:** Let $x_0$ be any arbitrary point in $X$. Thus we construct a sequence $\{y_n\}$ in $X$ such that $y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$.
Put $x = x_{2n-1}, y = x_{2n-1}$ and $z = x_{2n}$

$$\mathcal{M}(Ax_{2n-1}, Bx_{2n}, Bx_{2n}, qt) \geq \min \left\{ \begin{array}{c} \mathcal{M}\,(Sx_{2n-1}, Ax_{2n-1}, Tx_{2n}, t), \mathcal{M}\,(Sx_{2n-1}, Ax_{2n-1}, Bx_{2n-1}, t), \\ \mathcal{M}\,(Sx_{2n-1}, Bx_{2n-1}, Tx_{2n}, t), \mathcal{M}(Sx_{2n-1}, Bx_{2n-1}, Bx_{2n-1}, t), \\ \mathcal{M}(Sx_{2n-1}, Ax_{2n-1}, Ax_{2n-1}, t), \mathcal{M}\,(Sx_{2n-1}, Tx_{2n}, Tx_{2n}, t), \\ \mathcal{M}\,(Bx_{2n-1}, Tx_{2n}, Tx_{2n}, t), \mathcal{M}(Bx_{2n-1}, Bx_{2n-1}, Tx_{2n}, t), \\ \mathcal{M}\,(Bx_{2n-1}, Tx_{2n}, Ax_{2n-1}, t), \mathcal{M}\,(Tx_{2n}, Ax_{2n-1}, Ax_{2n-1}, t) \end{array} \right\}$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \geq \min \left\{ \begin{array}{c} \mathcal{M}\,(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{M}\,(y_{2n-1}, y_{2n}, y_{2n}, t), \\ \mathcal{M}\,(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t), \\ \mathcal{M}(y_{2n-1}, y_{2n}, y_{2n}, t), \mathcal{M}\,(y_{2n-1}, y_{2n}, y_{2n}, t), \\ \mathcal{M}\,(y_{2n}, y_{2n}, y_{2n}, t), \mathcal{M}(y_{2n}, y_{2n}, y_{2n}, t), \\ \mathcal{M}\,(y_{2n}, y_{2n}, y_{2n}, t), \mathcal{M}\,(y_{2n}, y_{2n}, y_{2n}, t) \end{array} \right\}$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \geq \mathcal{M}\,(y_{2n-1}, y_{2n}, y_{2n}, t)$$

This implies that $\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$ is an increasing sequence of positive real numbers and

$$N(Ax_{2n-1}, Bx_{2n}, Bx_{2n}, qt) \leq \max \left\{ \begin{array}{c} N\,(Sx_{2n-1}, Ax_{2n-1}, Tx_{2n}, t), N\,(Sx_{2n-1}, Ax_{2n-1}, Bx_{2n-1}, t), \\ N\,(Sx_{2n-1}, Bx_{2n-1}, Tx_{2n}, t), N(Sx_{2n-1}, Bx_{2n-1}, Bx_{2n-1}, t), \\ N(Sx_{2n-1}, Ax_{2n-1}, Ax_{2n-1}, t), N\,(Sx_{2n-1}, Tx_{2n}, Tx_{2n}, t), \\ N\,(Bx_{2n-1}, Tx_{2n}, Tx_{2n}, t), N(Bx_{2n-1}, Bx_{2n-1}, Tx_{2n}, t), \\ N\,(Bx_{2n-1}, Tx_{2n}, Ax_{2n-1}, t), N\,(Tx_{2n}, Ax_{2n-1}, Ax_{2n-1}, t) \end{array} \right\}$$

$$N(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \leq \max \left\{ \begin{array}{c} N\,(y_{2n-1}, y_{2n}, y_{2n}, t), N\,(y_{2n-1}, y_{2n}, y_{2n}, t), \\ N\,(y_{2n-1}, y_{2n}, y_{2n}, t), N(y_{2n-1}, y_{2n}, y_{2n}, t), \\ N(y_{2n-1}, y_{2n}, y_{2n}, t), N\,(y_{2n-1}, y_{2n}, y_{2n}, t), \\ N\,(y_{2n}, y_{2n}, y_{2n}, t), N(y_{2n}, y_{2n}, y_{2n}, t), \\ N\,(y_{2n}, y_{2n}, y_{2n}, t), N\,(y_{2n}, y_{2n}, y_{2n}, t) \end{array} \right\}$$

$$N(y_{2n}, y_{2n+1}, y_{2n+1}, qt) \leq N\,(y_{2n-1}, y_{2n}, y_{2n}, t)$$

This implies that $N(y_{2n}, y_{2n+1}, y_{2n+1}, t)$ is an decreasing sequence of positive real numbers.

Now to prove that $\mathcal{M}(y_{2n}, y_{2n+1}, y_{2n+1}, t)$ converges to 1 as $n \to \infty$ and $N(y_{2n}, y_{2n+1}, y_{2n+1}, t)$ converges to 0 as $n \to \infty$ by lemma 2.1

$$\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}\left(y_{n-1}, y_n, y_n, \frac{t}{q}\right) \geq \mathcal{M}\left(y_{n-2}, y_{n-1}, y_{n-1}, \frac{t}{q^2}\right) \cdots \geq \mathcal{M}\left(y_0, y_1, y_1, \frac{t}{q^n}\right)$$

Thus $\mathcal{M}(y_n, y_{n+1}, y_{n+1}, t) \geq \mathcal{M}\left(y_0, y_1, y_1, \frac{t}{q^n}\right)$

and

$$N(y_n, y_{n+1}, y_{n+1}, t) \leq N\left(y_{n-1}, y_n, y_n, \frac{t}{q}\right) \leq N\left(y_{n-2}, y_{n-1}, y_{n-1}, \frac{t}{q^2}\right) \cdots \leq N\left(y_0, y_1, y_1, \frac{t}{q^n}\right)$$

Thus $N(y_n, y_{n+1}, y_{n+1}, t) \leq N\left(y_0, y_1, y_1, \frac{t}{q^n}\right)$ then by the definition of intuitionistic fuzzy metric space

$$\mathcal{M}(y_n, y_{n+p}, y_{n+p}, t)$$
$$\geq \mathcal{M}\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{p}\right) * \ldots p \text{ times} \ldots * \mathcal{M}\left(y_{n+p-1}, y_{n+p}, y_{n+p}, \frac{t}{p}\right)$$

$$\geq \mathcal{M}\left(y_0, y_1, y_1, \frac{t}{q^n}\right) * \ldots p \text{ times} \ldots * \mathcal{M}\left(y_0, y_1, y_1, \frac{t}{pq^{n+p-1}}\right)$$

and

$$N(y_n, y_{n+p}, y_{n+p}, t)$$
$$\leq N\left(y_n, y_{n+1}, y_{n+1}, \frac{t}{p}\right) \Diamond \ldots p \text{ times} \ldots \Diamond N\left(y_{n+p-1}, y_{n+p}, y_{n+p}, \frac{t}{p}\right)$$
$$\leq N\left(y_0, y_1, y_1, \frac{t}{q^n}\right) \Diamond \ldots p \text{ times} \ldots \Diamond N\left(y_0, y_1, y_1, \frac{t}{pq^{n+p-1}}\right)$$
$$\lim_{n\to\infty} \mathcal{M}(y_n, y_{n+p}, y_{n+p}, t) \geq 1 * 1 * \ldots * p \text{ times} \ldots * 1 \lim_{n\to\infty} \mathcal{M}(y_n, y_{n+p}, y_{n+p}, t) = 1$$

and

$$\lim_{n\to\infty} N(y_n, y_{n+p}, y_{n+p}, t) \leq 0 \Diamond 0 \Diamond \ldots \Diamond p \text{ times} \ldots \Diamond 0$$
$$\lim_{n\to\infty} N(y_n, y_{n+p}, y_{n+p}, t) = 0.$$

Therefore $\{y_n\}$ is a Cauchy sequence in intuitionistic fuzzy metric space $X$ and since $X$ is complete there exists a point $u \in X$ fsuch that $y_n \to u$. Therefore $\{Sx_{2n}\}, \{Bx_{2n-1}\}, \{Tx_{2n-1}\}$ and $Ax_{2n-2}$ are cauchy sequence converge to $u$.
Put $x = Sx_{2n}, y = u$ and $z = Tx_{2n-1}$ in condition (4), we get

$$\mathcal{M}(ASx_{2n}, BTx_{2n-1}, BTx_{2n-1}, qt) \geq$$
$$min \left\{ \begin{array}{c} \mathcal{M}(SSx_{2n}, Au, TTx_{2n-1}, t), \mathcal{M}(SSx_{2n}, Au, Bu, t), \\ \mathcal{M}(SSx_{2n}, Bu, TTx_{2n-1}, t), \mathcal{M}(SSx_{2n}, Bu, Bu, t), \\ \mathcal{M}(SSx_{2n}, Au, Au, t), \mathcal{M}(SSx_{2n}, TTx_{2n-1}, TTx_{2n-1}, t), \\ \mathcal{M}(Bu, TTx_{2n-1}, TTx_{2n-1}, t), \mathcal{M}(Bu, Bu, TTx_{2n-1}, t), \\ \mathcal{M}(Bu, TTx_{2n-1}, Au, t), \mathcal{M}(TTx_{2n-1}, Au, Au, t) \end{array} \right\} \text{ and}$$

$$N(ASx_{2n}, BTx_{2n-1}, BTx_{2n-1}, qt)$$
$$\leq max \left\{ \begin{array}{c} N(SSx_{2n}, Au, TTx_{2n-1}, t), N(SSx_{2n}, Au, Bu, t), \\ N(SSx_{2n}, Bu, TTx_{2n-1}, t), N(SSx_{2n}, Bu, Bu, t), \\ N(SSx_{2n}, Au, Au, t), N(SSx_{2n}, TTx_{2n-1}, TTx_{2n-1}, t), \\ N(Bu, TTx_{2n-1}, TTx_{2n-1}, t), N(Bu, Bu, TTx_{2n-1}, t), \\ N(Bu, TTx_{2n-1}, Au, t), N(TTx_{2n-1}, Au, Au, t) \end{array} \right\}$$

Now take the limit as $n \to \infty$ and using condition (3), we get

$$\mathcal{M}(Au,Bu,Bu,qt) \geq min \begin{Bmatrix} \mathcal{M}\ (Au,Au,Bu,t), \mathcal{M}\ (Au,Au,Bu,t), \\ \mathcal{M}\ (Au,Bu,Bu,t), \mathcal{M}(Au,Bu,Bu,t), \\ \mathcal{M}(Au,Au,Au,t), \mathcal{M}\ (Au,Bu,Bu,t), \\ \mathcal{M}\ (Bu,Bu,Bu,t), \mathcal{M}(Bu,Bu,Bu,t), \\ \mathcal{M}\ (Bu,Bu,Au,t), \mathcal{M}\ (Bu,Au,Au,t) \end{Bmatrix} \text{ and}$$

$$N(Au,Bu,Bu,qt) \leq max \begin{Bmatrix} N\ (Au,Au,Bu,t), N(Au,Au,Bu,t), \\ N(Au,Bu,Bu,t), N(Au,Bu,Bu,t), \\ N(Au,Au,Au,t), N\ (Au,Bu,Bu,t), \\ N\ (Bu,Bu,Bu,t), N(Bu,Bu,Bu,t), \\ N(Bu,Bu,Au,t), N\ (Bu,Au,Au,t) \end{Bmatrix}$$

Then by lemma 2.1, we get

$$\mathcal{M}(Au,Bu,Bu,qt) \geq \mathcal{M}\ (Au,Bu,Bu,t) \text{ and}$$

$$N(Au,Bu,Bu,qt) \leq N\ (Au,Bu,Bu,t)$$

Therefore $Au = Bu$. Now put $x = Sx_{2n}, y = x_{2n-1}$ and $z = x_{2n-1}$ in condition (4), we get

$$\mathcal{M}(ASx_{2n},Bx_{2n-1},Bx_{2n-1},qt) \geq$$
$$min \begin{Bmatrix} \mathcal{M}\ (SSx_{2n},Ax_{2n-1},Tx_{2n-1},t), \mathcal{M}\ (SSx_{2n},Ax_{2n-1},Bx_{2n-1},t), \\ \mathcal{M}\ (SSx_{2n},Bx_{2n-1},Tx_{2n-1},t), \mathcal{M}(SSx_{2n},Bx_{2n-1},Bx_{2n-1},t), \\ \mathcal{M}(SSx_{2n},Ax_{2n-1},Ax_{2n-1},t), \mathcal{M}\ (SSx_{2n},Tx_{2n-1},Tx_{2n-1},t), \\ \mathcal{M}\ (Bx_{2n-1},Tx_{2n-1},Tx_{2n-1},t), \mathcal{M}(Bx_{2n-1},Bx_{2n-1},Tx_{2n-1},t), \\ \mathcal{M}\ (Bx_{2n-1},Tx_{2n-1},Ax_{2n-1},t), \mathcal{M}\ (Tx_{2n-1},Ax_{2n-1},Ax_{2n-1},t) \end{Bmatrix} \text{ and}$$

$$N(ASx_{2n},Bx_{2n-1},Bx_{2n-1},qt)$$
$$\leq max \begin{Bmatrix} N\ (SSx_{2n},Ax_{2n-1},Tx_{2n-1},t), N(SSx_{2n},Ax_{2n-1},Bx_{2n-1},t), \\ N(SSx_{2n},Bx_{2n-1},Tx_{2n-1},t), N(SSx_{2n},Bx_{2n-1},Bx_{2n-1},t), \\ N(SSx_{2n},Ax_{2n-1},Ax_{2n-1},t), N\ (SSx_{2n},Tx_{2n-1},Tx_{2n-1},t), \\ N\ (Bx_{2n-1},Tx_{2n-1},Tx_{2n-1},t), N(Bx_{2n-1},Bx_{2n-1},Tx_{2n-1},t), \\ N(Bx_{2n-1},Tx_{2n-1},Ax_{2n-1},t), N\ (Tx_{2n-1},Ax_{2n-1},Ax_{2n-1},t) \end{Bmatrix}$$

Now take the limit as $n \to \infty$ and using condition (3), we get

$$\mathcal{M}(Au,u,u,qt) \geq min \begin{Bmatrix} \mathcal{M}\ (Au,Au,u,t), \mathcal{M}\ (Au,Au,u,t), \\ \mathcal{M}\ (Au,u,u,t), \mathcal{M}(Au,u,u,t), \\ \mathcal{M}(Au,Au,Au,t), \mathcal{M}\ (Au,u,u,t), \\ \mathcal{M}\ (u,u,u,t), \mathcal{M}(u,u,u,t), \\ \mathcal{M}\ (u,u,Au,t), \mathcal{M}\ (u,Au,Au,t) \end{Bmatrix} \text{ and}$$

$$N(Au,u,u,qt) \leq max \begin{Bmatrix} N\ (Au,Au,u,t), N(Au,Au,u,t), \\ N(Au,u,u,t), N(Au,u,u,t), \\ N(Au,Au,Au,t), N\ (Au,u,u,t), \\ N\ (u,u,u,t), N(u,u,u,t), \\ N(u,u,Au,t), N\ (u,Au,Au,t) \end{Bmatrix}$$

Then by lemma 2.1, we get

$$\mathcal{M}(Au,u,u,qt) \geq \mathcal{M}\ (Au,u,u,t) \text{ and}$$

$$N(Au,u,u,qt) \leq N(Au,u,u,t)$$

Therefore $Au = u$, which implies $Au = Bu = u$.

Now put $x = Ax_{2n-2}, y = Ax_{2n-2}$ and $z = u$ in condition (4), we get

$$\mathcal{M}(AAx_{2n-2},Bu,Bu,qt) \geq min \begin{Bmatrix} \mathcal{M}(SAx_{2n-2},AAx_{2n-2},Tu,t),\mathcal{M}(SAx_{2n-2},AAx_{2n-2},BAx_{2n-2},t), \\ \mathcal{M}(SAx_{2n-2},BAx_{2n-2},Tu,t),\mathcal{M}(SAx_{2n-2},BAx_{2n-2},BAx_{2n-2},t), \\ \mathcal{M}(SAx_{2n-2},AAx_{2n-2},AAx_{2n-2},t),\mathcal{M}(SAx_{2n-2},Tu,Tu,t), \\ \mathcal{M}(BAx_{2n-2},Tu,Tu,t),\mathcal{M}(BAx_{2n-2},BAx_{2n-2},Tu,t), \\ \mathcal{M}(BAx_{2n-2},Tu,AAx_{2n-2},t),\mathcal{M}(Tu,AAx_{2n-2},AAx_{2n-2},t) \end{Bmatrix} \text{ and}$$

$$N(AAx_{2n-2},Bu,Bu,qt) \leq max \begin{Bmatrix} N(SAx_{2n-2},AAx_{2n-2},Tu,t),N(SAx_{2n-2},AAx_{2n-2},BAx_{2n-2},t), \\ N(SAx_{2n-2},BAx_{2n-2},Tu,t),N(SAx_{2n-2},BAx_{2n-2},BAx_{2n-2},t), \\ N(SAx_{2n-2},AAx_{2n-2},AAx_{2n-2},t),N(SAx_{2n-2},Tu,Tu,t), \\ N(BAx_{2n-2},Tu,Tu,t),N(BAx_{2n-2},BAx_{2n-2},Tu,t), \\ N(BAx_{2n-2},Tu,AAx_{2n-2},t),N(Tu,AAx_{2n-2},AAx_{2n-2},t) \end{Bmatrix}$$

Now take the limit as $n \to \infty$ and using condition (3) and (4), we get

$$\mathcal{M}(Su,u,u,qt) \geq min \begin{Bmatrix} \mathcal{M}(Su,Su,u,t),\mathcal{M}(Su,Su,u,t), \\ \mathcal{M}(Su,u,u,t),\mathcal{M}(Su,u,u,t), \\ \mathcal{M}(Su,Su,Su,t),\mathcal{M}(Su,u,u,t), \\ \mathcal{M}(u,u,u,t),\mathcal{M}(u,u,u,t), \\ \mathcal{M}(u,u,Su,t),\mathcal{M}(u,Su,Su,t) \end{Bmatrix} \text{ and}$$

$$N(Su,u,u,qt) \leq max \begin{Bmatrix} N(Su,Su,u,t),N(Su,Su,u,t), \\ N(Su,u,u,t),N(Su,u,u,t), \\ N(Su,Su,Su,t),N(Su,u,u,t), \\ N(u,u,u,t),N(u,u,u,t), \\ N(u,u,Su,t),N(u,Su,Su,t) \end{Bmatrix}$$

Then by lemma 2.1, we get

$$\mathcal{M}(Su,u,u,qt) \geq \mathcal{M}(Su,u,u,t) \text{ and}$$

$$N(Su,u,u,qt) \leq N(Su,u,u,t)$$

Therefore $Su = u$, which implies $Au = Bu = Su = u$.

Now put $x = u, y = u$ and $z = Bx_{2n-1}$ in condition (4), we get

$$\mathcal{M}(Au,BBx_{2n-1},BBx_{2n-1},qt) \geq min \begin{Bmatrix} \mathcal{M}(Su,Au,TBx_{2n-1},t),\mathcal{M}(Su,Au,Bu,t), \\ \mathcal{M}(Su,Bu,TBx_{2n-1},t),\mathcal{M}(Su,Bu,Bu,t), \\ \mathcal{M}(Su,Au,Au,t),\mathcal{M}(Su,TBx_{2n-1},TBx_{2n-1},t), \\ \mathcal{M}(Bu,TBx_{2n-1},TBx_{2n-1},t),\mathcal{M}(Bu,Bu,TBx_{2n-1},t), \\ \mathcal{M}(Bu,TBx_{2n-1},Au,t),\mathcal{M}(TBx_{2n-1},Au,Au,t) \end{Bmatrix} \text{ and}$$

$$N(Au, BBx_{2n-1}, BBx_{2n-1}, qt) \leq max \left\{ \begin{array}{c} N\,(Su, Au, TBx_{2n-1}, t), N(Su, Au, Bu, t), \\ N(Su, Bu, TBx_{2n-1}, t), N(Su, Bu, Bu, t), \\ N(Su, Au, Au, t), N\,(Su, TBx_{2n-1}, TBx_{2n-1}, t), \\ N\,(Bu, TBx_{2n-1}, TBx_{2n-1}, t), N(Bu, Bu, TBx_{2n-1}, t), \\ N(Bu, TBx_{2n-1}, Au, t), N\,(TBx_{2n-1}, Au, Au, t) \end{array} \right\}$$

Now take the limit as $n \to \infty$ and using condition (3) and (4), we get

$$\mathcal{M}(u, Tu, Tu, qt) \geq min \left\{ \begin{array}{c} \mathcal{M}\,(u, u, Tu, t), \mathcal{M}\,(u, u, u, t), \\ \mathcal{M}\,(u, u, Tu, t), \mathcal{M}(u, u, u, t), \\ \mathcal{M}(u, u, u, t), \mathcal{M}\,(u, Tu, Tu, t), \\ \mathcal{M}\,(u, Tu, Tu, t), \mathcal{M}(u, u, Tu, t), \\ \mathcal{M}\,(u, Tu, u, t), \mathcal{M}\,(Tu, u, u, t) \end{array} \right\} \text{ and}$$

$$N(u, Tu, Tu, qt) \leq max \left\{ \begin{array}{c} N\,(u, u, Tu, t), N(u, u, u, t), \\ N(u, u, Tu, t), N(u, u, u, t), \\ N(u, u, u, t), N\,(u, Tu, Tu, t), \\ N\,(u, Tu, Tu, t), N(u, u, Tu, t), \\ N(u, Tu, u, t), N\,(Tu, u, u, t) \end{array} \right\}$$

Then by lemma 2.1, we get

$$\mathcal{M}(Tu, Tu, u, qt) \geq \mathcal{M}\,(Tu, Tu, u, t) \text{ and}$$
$$N(Tu, Tu, u, qt) \leq N\,(Tu, Tu, u, t)$$

Therefore $Tu = u$, which implies $Au = Bu = Su = Tu = u$.
Hence, $u$ is a common fixed point of $A, B, S$ and $T$.

**For Uniqueness**

Let $w$ be another common fixed point of $A, B, S$ and $T$. Then put $x = u, y = w$ and $z = w$ in condition (4), we get

$$\mathcal{M}(Au, Bw, Bw, qt) \geq min \left\{ \begin{array}{c} \mathcal{M}\,(Su, Aw, Tw, t), \mathcal{M}\,(Su, Aw, Bw, t), \\ \mathcal{M}\,(Su, Bw, Tw, t), \mathcal{M}(Su, Bw, Bw, t), \\ \mathcal{M}(Su, Aw, Aw, t), \mathcal{M}\,(Su, Tw, Tw, t), \\ \mathcal{M}\,(Bw, Tw, Tw, t), \mathcal{M}(Bw, Bw, Tw, t), \\ \mathcal{M}\,(Bw, Tw, Aw, t), \mathcal{M}\,(Tw, Aw, Aw, t) \end{array} \right\} \text{ and}$$

$$N(Au, Bw, Bw, qt) \leq max \left\{ \begin{array}{c} N\,(Su, Aw, Tw, t), N(Su, Aw, Bw, t), \\ N(Su, Bw, Tw, t), N(Su, Bw, Bw, t), \\ N(Su, Aw, Aw, t), N\,(Su, Tw, Tw, t), \\ N\,(Bw, Tw, Tw, t), N(Bw, Bw, Tw, t), \\ N(Bw, Tw, Aw, t), N\,(Tw, Aw, Aw, t) \end{array} \right\}$$

$$\mathcal{M}(u,w,w,qt) \geq min \begin{Bmatrix} \mathcal{M}(u,w,w,t), \mathcal{M}(u,w,w,t), \\ \mathcal{M}(u,w,w,t), \mathcal{M}(u,w,w,t), \\ \mathcal{M}(u,w,w,t), \mathcal{M}(u,w,w,t), \\ \mathcal{M}(w,w,w,t), \mathcal{M}(w,w,w,t), \\ \mathcal{M}(w,w,w,t), \mathcal{M}(w,w,w,t) \end{Bmatrix} \text{ and}$$

$$N(u,w,w,qt) \leq max \begin{Bmatrix} N(u,w,w,t), N(u,w,w,t), \\ N(u,w,w,t), N(u,w,w,t), \\ N(u,w,w,t), N(u,w,w,t), \\ N(w,w,w,t), N(w,w,w,t), \\ N(w,w,w,t), N(w,w,w,t) \end{Bmatrix}$$

$$\mathcal{M}(u,w,w,qt) \geq \mathcal{M}(u,w,w,t) \text{ and}$$
$$N(u,w,w,qt) \leq N(u,w,w,t)$$

which is a contradiction. Therefore $u = w$.
Hence, $u$ is a unique common fixed point of $A, B, S$ and $T$.

**Corollary 3.1.** Let $(X, \mathcal{M}, N, *, \lozenge)$ be a complete generalized intuitionistic fuzzy metric spaces and let $A, B, S$, and $T$ be self mappings of $X$ satisfying the following conditions:

(1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,
(2) $A$ and $S$ are continuous,
(3) The pairs $(A, S)$and $(B, T)$ are compatible mappings of type $(A)$,
(4) There exist $q \in (0,1)$ such that

$$\begin{aligned} &\mathcal{M}(Ax, Bz, Bz, qt) \\ &\geq \mathcal{M}(Sx, Ay, Tz, t) * \mathcal{M}(Sx, Ay, By, t) * \mathcal{M}(Sx, By, Tz, t) \\ &* \mathcal{M}(Sx, By, By, t) * \mathcal{M}(Sx, Ay, Ay, t) * \mathcal{M}(Sx, Tz, Tz, t) \\ &* \mathcal{M}(By, Tz, Tz, t) * \mathcal{M}(By, By, Tz, t) * \mathcal{M}(By, Tz, Ay, t) \\ &* \mathcal{M}(Tz, Ay, Ay, t) \end{aligned}$$

and

$$\begin{aligned} &N(Ax, Bz, Bz, qt) \\ &\geq N(Sx, Ay, Tz, t) \lozenge N(Sx, Ay, By, t) \lozenge N(Sx, By, Tz, t) \\ &\lozenge N(Sx, By, By, t) \lozenge N(Sx, Ay, Ay, t) \lozenge N(Sx, Tz, Tz, t) \\ &\lozenge N(By, Tz, Tz, t) \lozenge N(By, By, Tz, t) \lozenge N(By, Tz, Ay, t) \\ &\lozenge N(Tz, Ay, Ay, t) \end{aligned}$$

For every $x, y \in X$ and $t > 0$. Then $A, B, S$ and $T$ have a unique common fixed point in $X$.

## 4. Conclusion

In this paper, we prove common fixed point theorems for four self mappings under the conditions of compatible mappings of type $(A\text{-}1)$ and type $(A\text{-}2)$ in complete intuitionistic fuzzy metric space. This work can be easily extended and generalized by various known fixed point theorems in the literature in the setting of fuzzy and intuitionistic fuzzy metric spaces.

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**Conflicts of Interest.** The authors declare no conflicts of interest.

**Authors' contributions.** All authors contributed equally to this work.

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