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Improvement of Point Spread Function Estimation Method for Motion Blurred Image and Image Restoration

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Abstract. The estimation of point spread function (PSF) of motion-blurred images is the key to image restoration, which is mainly affected by two physical quantities: blur angle and blur length. The traditional point spread function estimation method is easy to be disturbed by the bright cross in the centre of the blurred image spectrum when estimating the fuzzy angle, which affects the quality of image restoration. In addition, when using the traditional Radon transform to detect fuzzy angles, the area of bright spots in the picture is large, which will cause large fluctuations in the detection results. Therefore, this paper proposes a cepstrum analysis method for motion blurred images. Firstly, the gradient cepstrum estimation of the blurred image is given, and the cepstrum is binarized and further refined by setting the gray threshold of the cepstrum. Finally, the line fitting is performed on the refined binary image, and the slope of the fitting line is calculated, so that the point spread function of the blurred image can be accurately and directly estimated, and better image restoration effect can be obtained.

Keywords: image processing, image restoration, motion blur, point spread function, blur direction

AMS Mathematics Subject Classification (2010): 68U10

1. Introduction

As a carrier of human information exchange, images contain a large number of information elements. Due to various factors, clear images may degrade and degenerate into blurry images. Common types of blurred images include Gaussian blur, defocus blur, motion blur, etc. Motion blur occurs during the process of image acquisition and is a phenomenon of image degradation caused by camera shake, object displacement, and other factors. The restoration of motion blurred images covers multiple aspects such as astronomical images, medical images, remote sensing technology, transportation systems, and public security criminal investigation, and is currently a hot topic in the field of computer vision and image processing. The overall idea of existing motion blurred image restoration is mainly to establish a degradation model based on the blurred image, obtain the blur parameters through Radon transform [1,2], that is, point spread function (PSF), and finally restore the image using Wiener filter from the estimated PSF. Therefore, the estimation of point spread function of motion blurred images is the key to image restoration. Only by accurately estimating the point spread function can the ill conditioned problem of blurred image restoration be benign, so as to obtain high-quality restored images [3,4,5,6,7]. There are two methods to solve the problem of image blur caused by motion: one is to change the physical conditions of imaging; The second is to use motion blurred image restoration algorithms to restore degraded images. The former often requires a significant investment of manpower and financial resources, making it difficult to widely promote in practical applications, while the latter is an economically applicable solution. Motion blurred image restoration includes the following two processes: one is to estimate the point spread function (PSF) of the motion blurred image, and the other is image deconvolution. For the estimation of point spread function, many algorithms have been applied to how to accurately determine the PSF of motion blurred images. However, these algorithms often ignore the influence of noise and the cross bright line in the center of the spectrum on the spectrum of motion blurred images, resulting in a large error in the estimation of PSF. For image deconvolution, traditional algorithms mainly include inverse filtering, Wiener filtering, etc. However, when using traditional algorithms to process real blurred images, they are often affected by various estimation errors and the lack of prior knowledge in the image, which often leads to secondary degradation of the restored image. One of the more significant degradation is the ringing effect. Therefore, designing an algorithm to accurately estimate the point spread function of motion blurred images has important value in practical applications.

In recent years, scholars have conducted in-depth and extensive research on PSF estimation algorithms for blurred images. Reference [8] uses image enhancement processing and morphological transformation to remove cross bright lines and noise interference in spectral images, and obtains stripe images with appropriate morphology for Radon transform detection. Reference [9] uses the characteristics of directional differentiation and bilinear interpolation method to automatically identify the direction of motion blur. Reference [10] uses Radon transform and Sobel operator to perform first-order differential calculation on fuzzy image, and the absolute error of fuzzy direction obtained is controlled within 2 units. Reference [11] uses Radon transform principle to solve the motion blur direction in point spread function PSF, and the motion blur direction error is controlled below 1 unit. Reference [12] applied sparse representation to fuzzy angle estimation, pointing out that the optimization degree of sparse representation in restored images is positively correlated with the accuracy of fuzzy angle estimation. Reference [13] studies a main stripe width measurement method based on segmented bilateral and membership functions, which integrates the angle information between each edge segmentation segment and the horizontal axis to improve the accuracy of fuzzy angle estimation.

There are still many problems that need to be solved in the existing restoration of motion blurred images, such as being easily affected by the bright cross line in the center of the spectrum when estimating the blur angle, resulting in low estimation accuracy, and having a large range of fluctuations in the Radon blur angle detection results. In response to the above issues, the article proposes improvements on the existing restoration algorithm. Experiments on livingroom, cameraman and other images verify that the algorithm can accurately estimate the point spread function and improve the restoration effect.

2. Image restoration theory

2.1. Image degradation model

Image restoration is a pathological inverse problem. Image degradation is the convolution process of image and point spread function, and image restoration is the reverse process of this process [14]. To restore a real image, it is necessary to first establish an image degradation model. Modeling degraded images as a function $h(x,y)$, this function, together with an additive noise term $\eta(x,y)$, operates on the input image $f(x,y)$ to generate a degraded image $g(x,y)$ (Figure 1). After knowing some knowledge about the degradation function $h(x,y)$ and the additive noise term $\eta(x,y)$ of $g(x,y)$ and $h(x,y)$, the purpose of image restoration is to obtain an estimate of the original image [15]. Generally speaking, the more information about $h(x,y)$ and $\eta(x,y)$, the closer the obtained $\hat{f}(x,y)$ is to $f(x,y)$.

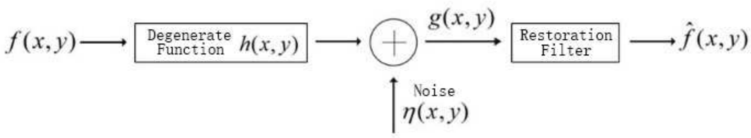


Figure 1: Degraded image degradation model

Generally speaking, the degradation model of an image can be represented as:

$$\hat{f}(x,y) = f(x,y) * h(x,y) + \eta(x,y) \tag{1}$$

In the formula, '$*$'Convolutional operations that cause image degradation, $h(x,y)$ is the point spread function of a blurred image with uniform linear motion. When $\eta(x,y)=0$, that is, without considering noise, the convolution in the spatial domain is transformed into the product form in the frequency domain through discrete Fourier transform:

$$\hat{F}(u,v) = F(u,v)H(u,v) \tag{2}$$

Among them, $\hat{F}(u,v)$, $F(u,v)$, and $H(u,v)$ are the Fourier transforms of image $\hat{f}(x,y)$, $f(x,y)$, and fuzzy kernel function $h(x,y)$, respectively [16].

2.2. PSF estimation of blurred images with uniform linear motion

Assuming image $f(x,y)$ undergoes uniform linear motion, $x_0(t)$ and $y_0(t)$ are components that change over time in the x and y directions, respectively. If the influence of noise is ignored and T is set as the exposure time, then:

$$g(x,y) = \int_0^T f[x - x_0(t), y - y_0(t)]dt \tag{3}$$

where $g(x,y)$ is the blurred image.

The Fourier transform of equation (3) is:

$$G(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)e^{-j2\pi(ux+vy)}dxdy$$
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{0}^{T}f[x-x_0(t),y-y_0(t)]dt\right]e^{-j2\pi(ux+vy)}dxdy$$
$$=\int_{0}^{T}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f\left[x-x_0(t),y-y_0(t)\right]e^{-j2\pi(ux+vy)}dxdy\right]dt \tag{4}$$

According to the properties of Fourier transform, it can be obtained that:

$$G(u,v)=\int_{0}^{T}F(u,v)e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$
$$=F(u,v)\int_{0}^{T}e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \tag{5}$$

If $H(u,v)=\int_{o}^{T}e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$, there is:

$$G(u,v)=F(u,v)H(u,v) \tag{6}$$

According to equation (3), if the motion lengths in the horizontal and vertical directions are obtained, the restored image $f(x,y)$ can be obtained. Assuming that vertical motion is ignored, If $y_0(t)=0$, then equation (3) becomes:

$$g(x,y)=\int_{0}^{T}f\left[x-x_0(t),y\right]dt \tag{7}$$

At this point, the two-dimensional solving problem is transformed into a one-dimensional solving problem. If the displacement in the horizontal direction is L, then there is:

$$x_0(t)=\frac{Lt}{T} \tag{8}$$

Equation (7) becomes:

$$H(u,v)=\int_{0}^{T}e^{-2j\pi ux_0(t)}$$
$$=\int_{0}^{T}e^{-2j\pi u(t)\frac{Lt}{T}}$$
$$=\frac{T}{\pi Lu}\sin(\pi Lu)e^{-j\pi Lu} \tag{9}$$

Perform Fourier transform on the above equation to obtain:

$$h(x,y)=\begin{cases}\frac{1}{L},0<x<L\\ 0,others\end{cases} \tag{10}$$

$h(x,y)$ here is the point spread function of the image blurred by uniform linear motion.

If the vertical motion is taken into account, it can be obtained that:

$$h(x,y)=\begin{cases}\frac{1}{L},\sqrt{x^2+y^2}\leq L\text{且}\frac{x}{y}=-\tan\theta\\ 0,\text{其他}\end{cases} \tag{11}$$

In the equation, $u=0,1,\ldots,M-1,0,1,\ldots,N-1$.

$$|G(u,v)|=T\frac{\sin\left(\pi\left(ua/N+vb/M\right)\right)}{\pi\left(ua/N+vb/M\right)} \tag{12}$$

Among them, point (x,y) is the position of the image pixels, L is the motion blur length, and θ is the motion blur angle, which is the angle between the motion direction and the horizontal direction. It can be seen that the point spread function of uniform linear motion is jointly determined by two physical quantities: blur length and blur angle. The discrete Fourier transform of the point spread function shown in Equation (13) is:

$$G(u,v) = \frac{T\sin(\pi(ua/N + vb/M))}{\pi(ua/N + vb/M)} e^{-j\pi(ua/N+vb/M)} \tag{13}$$

According to Formula 13, $|G(u,v)|$ contains the $\sin c$ function. According to the nature of the $\sin c$ function, $|G(u,v)|$ gets the maximum value at $(ua/N + vb/M) = 0$, and $|G(u,v)|$ is zero when $(ua/N + vb/M)$ gets non-zero. Therefore, the blurred image spectrum will show parallel stripes with alternating light and dark, and the direction of the stripes will also change accordingly with the change of the blurring angle.

If l_u and l_v are the components of motion blur length in both vertical and horizontal directions, then:

$$L = \sqrt{l_u^{\,2} + l_v^{\,2}}\ , \quad \tan\theta = \tfrac{l_u}{l_v} \tag{14}$$

If the inclination angle of the light and dark stripes relative to the positive direction of the horizontal axis is φ, then:

$$\tan\varphi = -\frac{l_u N}{l_v M} \tag{15}$$

The relationship between the tilt angle φ of the stripes and the motion blur angle θ can be obtained as follows:

$$\tan(\theta) = -\tan(\varphi)\tfrac{M}{N} \tag{16}$$

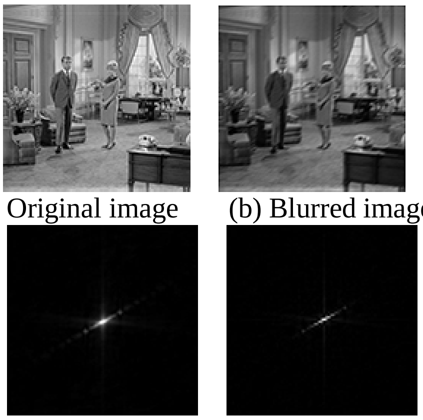
It can be seen from the above formula that the dip angle φ between light and dark stripes in the spectrum image is the motion blur angle θ, so the blur angle of point spread function can be obtained by detecting the dip angle (16).

3. Improved PSF estimation algorithm

The corresponding blur kernel in the image degradation process, namely the point spread function $h(x,y)$ in the convolution operation, describes the camera's dithering path. This path is random, so its corresponding two-dimensional PSF image has randomness and sparsity. The spectrum is not sensitive to the influence of random transmission paths, nor is it affected by other random components during the imaging process. So, using cepstrum analysis can truly separate PSF information. However, due to interference from other factors such as the environment, the center of the cepstrum image obtained by directly performing Fourier transform on the motion blurred image will have a clear cross bright line, which will affect the accuracy of subsequent blur angle measurement. And image gradients provide more useful information than the image itself. Compared to fuzzy image cepstrum, image gradient cepstrum can effectively isolate the information of the image itself, highlight the high-frequency components in the image, suppress low-frequency components, effectively isolate the information of the image itself, effectively eliminate the bright lines in the center of the cepstrum, and accurately estimate PSF. Therefore, this paper proposes a method to estimate the PSF function more accurately and quickly by analyzing the binary image of the gradient cepstrum of the blurred image, fitting the line of the thinned binary image, and directly calculating the slope of the fitting line to obtain a more accurate motion blur direction angle. The specific steps are as follows:

(1) Analyze images and image cepstrums

Figure (2) analyzes the cepstrum features of motion blurred images using optical images as an example. Figure 2 (a) shows the original clear image, Figure 2 (b) shows a motion blurred image with a blur angle of 30 and a blur scale of 10. Figure 2 (c) shows the cepstrum of Figure 2 (b), and Figure 2 (d) shows the gradient cepstrum of Figure 2 (b). By comparison, it can be seen that blurred image cepstrum has richer cepstrum information in the direction of image blur. Fuzzy image gradient cepstrum can effectively isolate the original image cepstrum information, remove the cross bright line in the center of the cepstrum, and preserve the cepstrum information in the fuzzy direction, thus calculating the fuzzy angle more accurately.



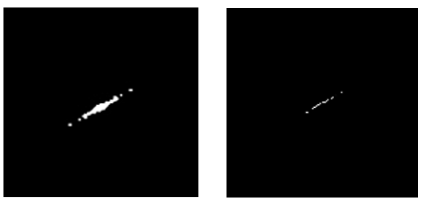
(a) Original image (b) Blurred image

(c) Cepstrum (d) Gradient cepstrum

Figure 2: Analysis of optical image cepstru

(2) Cepstrum image binarization and binary image thinning

By setting the grayscale threshold of the cepstrum, the cepstrum is binarized, but the straight line direction obtained by directly fitting the binary image has a certain deviation from the fuzzy direction. Therefore, in order to more accurately calculate the direction of bright white lines in the cepstrum, the binary image is first refined, and then the refined image is fitted with a straight line to calculate the direction of the white lines. The refinement process can also eliminate the influence of other subsequent interference factors. Figures 3 (a) and 3 (b) show the processing results of cepstrum binarization and binary image thinning respectively.



(a) (b)

Figure 3: Binary image and its thinning

(3) Straight line fitting of binary image

If the straight line slope of the thinned binary image is directly arctangent calculated, the straight line direction obtained will have a certain deviation from the actual blur direction. Therefore, line fitting is performed on the refined image to accurately calculate the direction of bright white lines. If the coordinate point of the thin line is $(x_i, y_i), i = 1, 2, ..., n$ and the fitted line equation is $y = kx + b$, then the slope k and intercept b of the fitted line can be calculated according to equation (17).

$$\begin{cases} b = \bar{y} - k\bar{x} \\ k = \frac{L_{yy} - L_{xx} + \sqrt{(L_{xx} - L_{xy})^2 + 4L_{xy}^2}}{2L_{xy}} \end{cases} \tag{17}$$

Among them, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $L_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$, $L_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2$, $L_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$.

(4) Estimating motion blur angles

By performing the arctangent operation on the slope k of the fitted line obtained in step (3), the angle $\theta = \arctan k$ between it and the axis x of the image coordinate system can be obtained. The direction of the fitted line is the blur direction of the blurred image, and the angle between the blur direction of the blurred image and the axis x of the image is also $\theta = \arctan k$.

(5) Estimate the length of motion blur

By performing first-order differentiation on a blurred image and performing autocorrelation operations, a discriminative curve can be obtained, where symmetrical correlation peaks appear with negative peaks. The distance between the two peaks is the length of the motion blur . Mark the coordinates of the two negative correlation peaks as x_1 and x_2 , and use Data Cursor in Matlab to obtain the values between the two negative peaks. Then the fuzzy scale L can be derived from $L = x_1 - x_2$.

After estimating the blur parameters using the above 5 steps, construct a PSF according to formula (11), and then restore the blurred image using the Wiener filter deconvolution algorithm.

4. Experimental results and analysis

4.1. Experimental study on the calculation accuracy of fuzzy direction recognition method

To test the effectiveness of the method proposed in this article in estimating motion blur angles, two test images, Livingroom and Cameraman, were selected for this section of the experiment. The setting range of blur angle is from 0° to 180°, and 10 motion blur parameters are randomly generated by the computer to test the algorithm in the article. Table 1 shows the test results.

Table 1: Test results of motion blur angle estimation algorithm

Fuzzy parameter (L/θ)	livingroom		cameraman	
	Experimental result	Absolute error	Experimental result	Absolute error
12/23.91	24.12	0.21	23.62	0.29
41/68.82	68.7	0.12	68.8	0.02
16/10.63	10.68	0.05	10.53	0.1
8/126.71	126.45	0.26	126.7	0.18
26/134.93	134.9	0.03	135.08	0.15
10/125	124.92	0.08	125.11	0.11
31/54.42	54.2	0.22	54.14	0.28
25/129.33	129.68	0.35	129.65	0.33
44/60.20	60.24	0.04	60.47	0.27
11/106.31	106.01	0.3	106.23	0.07
Mean error	0.166		0.18	

According to the experimental data in Table 3.1, it can be seen that the algorithm proposed in this article has achieved good results in 20 tests. The maximum error values for fuzzy angle measurement in Livingroom and Cameraman images are 0.35 and 0.33, respectively, and the average error value does not exceed 0.18. Compared to the average error of 0.395 in reference [17] and 1.0 in reference [18], the algorithm proposed in this paper has a certain degree of robustness and meets the accuracy requirements of image restoration algorithms for motion blur angle estimation.

4.2. Image restoration experiment

To verify the effectiveness of the PSF estimation method proposed in this article, two standard images (livingroom, cameraman) and two captured real images (flower, landscape) were used as test images in this section. Linear motion blur with a blur length of 20 and a blur angle of 30 were added, respectively. The classic linear image restoration method, Wiener filtering, was used to restore the image. Figure 4 shows the restoration results of blurred images. From top to bottom, there are four images: livingroom, camera, flower, and landscape. From left to right, there are original images, blurred images, and restored images.

From the experimental results, it can be seen that although the restored image still contains some noise, it is visually closer to the original image. It shows that the improvement of motion blur angle accuracy in point spread function has played a certain role in optimizing the effect of image restoration.

(a) Original image (b) Blurred image (c) Restore image

Figure 4: Restoration result of motion blurred image

5. Conclusion

The restoration of motion blurred images is mainly affected by the blur angle and blur length in PSF estimation. Traditional PSF estimation methods are affected by the bright cross in the center of the spectrum, which reduces the accuracy of blur angle estimation and thus affects the effectiveness of image restoration. Therefore, this article proposes a new motion blur parameter estimation scheme and conducts blur angle accuracy measurement experiments using optical images as an example[19]. The experimental results show that the PSF estimation algorithm in this paper improves the estimation accuracy of motion blur angles, thereby achieving the quality of image deconvolution restoration, which meets the practical engineering requirements of image restoration problems.

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Conflicts of Interest. The authors declare no conflicts of interest.

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