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The (*a*, *b*)-*KA* *E*-Banhatti Indices of Graphs

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Abstract. In this study, we propose the first and second modified E-Banhatti indices and the first and second (*a*, *b*)-*KA* E-Banhatti indices of a graph. We compute the first and second (*a*, *b*)-*KA* E-Banhatti indices for $HC_5C_7[p, q]$ nanotubes. We also establish some other *E*-Banhatti indices directly as a special case of these indices for some special values of *a* and *b*.

Keywords: first and second modified *E*-Banhatti indices, first and second (*a*, *b*)-*KA* *E*-Banhatti indices, nanotube

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C09, 05C92

1. Introduction

Let *G* be a finite, simple, connected graph. Let *V*(*G*) be the vertex set and *E*(*G*) be the edge set of *G*. The degree $d_G(u)$ of a vertex *u* is the number of vertices adjacent to *u*. For undefined terms and notations, we refer [1].

A graph index is a numerical parameter mathematically derived from the graph structure. The graph indices have their applications in various disciplines of Science and Technology [2].

Kulli [3] defined the Banhatti degree of a vertex *u* of a graph *G* as

$$B(u) = \frac{d_G(e)}{n - d_G(u)},$$

where $|V(G)| = n$ and the vertex *u* and edge *e* are incident in *G*.

Kulli introduced the first and second E-Banhatti indices in [3] and they are defined as

$$EB_1(G) = \sum_{uv \in E(G)} [B(u) + B(v)], \qquad EB_2(G) = \sum_{uv \in E(G)} B(u)B(v).$$

The first and second hyper E-Banhatti indices were proposed by Kulli in [4] and they are defined as

$$HEB_1(G) = \sum_{uv \in E(G)} [B(u) + B(v)]^2, \qquad HEB_2(G) = \sum_{uv \in E(G)} [B(u)B(v)]^2.$$

Kulli [5] proposed the E-Banhatti Nirmala index of a graph G is

$$EBN(G)=\sum_{uv\in E(G)}\sqrt{B(u)+B(v)}.$$

The modified E-Banhatti Nirmala index [5] of a graph G is

$$^{m}EBN(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{B(u)+B(v)}}.$$

The E-Banhatti Sombor index [6] of a graph G is

$$EBS(G)=\sum_{uv\in E(G)}\sqrt{B(u)^2+B(v)^2}.$$

Kulli [6] introduced the modified E-Banhatti Sombor index of a graph G and it is defined as

$$^{m}EBS(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{B(u)^2+B(v)^2}}.$$

Kulli [7] introduced the product connectivity E-Banhatti index and the reciprocal product connectivity E-Banhatti index of a graph G and they are defined as

$$PEB(G)=\frac{1}{\sqrt{B(u)B(v)}},\qquad RPEB(G)=\sum_{uv\in E(G)}\sqrt{B(u)B(v)}.$$

In [8], the FE-Banhatti index of a graph G is defined as

$$FEB(G)=\sum_{uv\in E(G)}\left[B(u)^2+B(v)^2\right].$$

We introduce the first and second modified E-Banhatti indices of a graph *G* and they are defined as

$$^{m}EB_1(G)=\sum_{uv\in E(G)}\frac{1}{B(u)+B(v)},\qquad ^{m}EB_2(G)=\sum_{uv\in E(G)}\frac{1}{B(u)B(v)}.$$

Motivated by the work on E-Banhatti indices, we introduce the first and second (*a*, *b*)-*KA* E-Banhatti indices of a graph and they are defined as

$$KAB^1_{a,b}(G)=\sum_{uv\in E(G)}\left[B(u)^a+B(v)^a\right]^b,\qquad KAB^2_{a,b}(G)=\sum_{uv\in E(G)}\left[B(u)^a B(v)^a\right]^b,$$

where *a* and *b* are real numbers.

Recently, some graph indices were studied in [9, 10, 11, 12].

This paper determines the first and second (*a*, *b*)-*KA* E-Banhatti indices for $HC_5C_7[p, q]$ nanotubes.

2. Observations

We observe the following

(1) $EB_1(G)=KAB^1_{1,1}(G)$

(2) $HEB_1(G)=KAB^1_{1,2}(G)$

(3) $EBN(G)=KAB^1_{1,\frac{1}{2}}(G)$

(4) $^{m}EBN(G)=KAB^1_{1,-\frac{1}{2}}(G)$

(5) $EBS(G)=KAB^1_{2,\frac{1}{2}}(G)$

(6) $^{m}EBS(G)=KAB^1_{2,-\frac{1}{2}}(G)$

(7) $FEB(G)=KAB^1_{2,1}(G)$

(8) $^{m}EB_1(G)=KAB^1_{1,-1}(G)$

Furthermore, we also see that

(1) $EB_2(G) = KAB^2_{1,1}(G)$ (2) $HEB_2(G) = KAB^2_{1,2}(G)$

(3) $PEB(G) = KAB^2_{1,-\frac{1}{2}}(G)$ (4) $RPEB(G) = KAB^2_{1,\frac{1}{2}}(G)$

(5) ${}^{m}EB_2(G) = KAB^2_{1,-1}(G)$

3. HC_5C_7 [*p*, *q*] Nanotubes

We consider $HC_5C_7[p, q]$ nanotubes, see Figure 1.

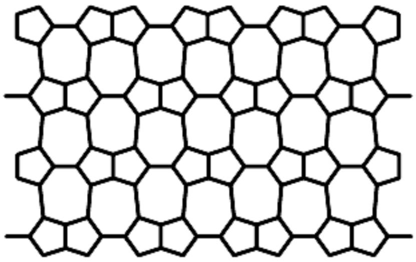

Figure 1: 2-*D* lattice of HC_5C_7 [8, 4] nanotube

The graphs of a nanotube $HC_5C_7[p, q]$ have $4pq$ vertices and $6pq - p$ edges are shown in the above graph. Let $G = HC_5C_7[p, q]$.

In *G*, there are two types of edges as follows:

$E_1 = \{uv \in E(G) \mid d(u)=2, d(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv \in E(G) \mid d(u)= d(v) = 3\}$, $|E_2| = 6pq - 5p$.

Therefore, in *G*, we obtain that $\{B(u), B(v): uv$ □ $E(NHPX[m, n])\}$ has two Banhatti edge set partitions.

$$BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{4pq-2}, B(v) = \frac{3}{4pq-3}\}, \qquad |BE_1| = 4p.$$

$$BE_2 = \{uv \in E(G) \mid B(u) = B(v) = \frac{4}{4pq-3}\}, \qquad |BE_2| = 6pq-5p.$$

We calculate the first (*a*, *b*)-*KA* E-Banhatti index of a nanotube $HC_5C_7[p,q]$ as follows:

Theorem 1. Let $G = HC_5C_7[p,q]$ be a nanotube. Then

$$KAB^1_{a,b}(G) = 4p\left[\left(\frac{3}{4pq-2}\right)^a + \left(\frac{3}{4pq-3}\right)^a\right]^b + 2^b(6pq-5p)\left(\frac{4}{4pq-3}\right)^{ab}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of *G*, we obtain

$$KAB^1_{a,b}(G) = \sum_{uv \in E(G)} \left[B(u)^a + B(v)^a\right]^b$$

$$=4p\left[\left(\frac{3}{4pq-2}\right)^a+\left(\frac{3}{4pq-3}\right)^a\right]^b+(6pq-5p)\left[\left(\frac{4}{4pq-3}\right)^a+\left(\frac{4}{4pq-3}\right)^a\right]^b.$$

By solving the above equation, we get the desired result.

We obtain the following results by using Theorem 1.

Corollary 1.1. Let $G= HC_5C_7[p,q]$ be a nanotube. Then

(i) [3] $EB_1(G)=KAB^1_{1,1}(G)=\frac{12p(8pq-5)}{(4pq-2)(4pq-3)}+\frac{8(6pq-5p)}{(4pq-3)}.$

(ii) $HEB_1(G)=KAB^1_{1,2}(G)=\frac{36p(8pq-5)^2}{(4pq-2)^2(4pq-3)^2}+\frac{64(6pq-5p)}{(4pq-3)^2}.$

(iii) $EBN(G)=KAB^1_{1,\frac{1}{2}}(G)=\frac{4\sqrt{3}p\sqrt{(8pq-5)}}{\sqrt{(4pq-2)(4pq-3)}}+\frac{2\sqrt{2}(6pq-5p)}{\sqrt{(4pq-3)}}.$

(iv) ${}^mEBN(G)=KAB^1_{1,-\frac{1}{2}}(G)=\frac{4p\sqrt{(4pq-2)(4pq-3)}}{\sqrt{3}\sqrt{(8pq-5)}}+\frac{(6pq-5p)\sqrt{(4pq-3)}}{2\sqrt{2}}$

(v) $EBS(G)=KAB^1_{2,\frac{1}{2}}(G)=\frac{12p\sqrt{(4pq-2)^2+(4pq-3)^2}}{(4pq-2)(4pq-3)}+\frac{4\sqrt{2}(6pq-5p)}{(4pq-3)}.$

(vi) ${}^mEBS(G)=KAB^1_{2,-\frac{1}{2}}(G)=\frac{4p(4pq-2)(4pq-3)}{3\sqrt{(4pq-2)^2+(4pq-3)^2}}+\frac{(6pq-5p)(4pq-3)}{4\sqrt{2}}$

(vii) $FEB(G)=KAB^1_{2,1}(G)=\frac{36p\left[(4pq-2)^2+(4pq-3)^2\right]}{(4pq-2)^2(4pq-3)^2}+\frac{32(6pq-5p)}{(4pq-3)^2}.$

(viii) ${}^mEB_1(G)=KAB^1_{1,-1}(G)=\frac{4p(4pq-2)(4pq-3)}{3(8pq-5)}+\frac{(6pq-5p)(4pq-3)}{8}.$

We calculate the second (*a*, *b*)-*KA* E-Banhatti index of a nanotube $HC_5C_7[p,q]$ as follows:

Theorem 2. Let $G= HC_5C_7[p,q]$ be a nanotube. Then

$$KAB^2_{a,b}(G)==4p\left(\frac{9^a}{(4pq-2)(4pq-3)}\right)^b+(6pq-5p)\left(\frac{4}{4pq-3}\right)^{2ab}.$$

Proof: From the definition and by cardinalities of the Banhatti edge partition of *G*, we obtain

$$KAB^2_{a,b}(G)=\sum_{uv\in E(G)}\left[B(u)^a\times B(v)^a\right]^b$$

$$= 4p\left[\left(\frac{3}{4pq-2}\right)^a \times \left(\frac{3}{4pq-3}\right)^a\right]^b + (6pq-5p)\left[\left(\frac{4}{4pq-3}\right)^a \times \left(\frac{4}{4pq-3}\right)^a\right]^b.$$

By solving the above equation, we get the desired result.

We obtain the following results by using Theorem 1.

Corollary 2.1. Let $G = HC_5C_7[p,q]$ be a nanotube. Then

(i) [3] $EB_2(G) = KAB_{1,1}^2(G) = \dfrac{36p}{(4pq-2)(4pq-3)} + \dfrac{16(6pq-5p)}{(4pq-3)^2}.$

(ii) $HEB_2(G) = KAB_{1,2}^2(G) = \dfrac{324p}{(4pq-2)^2(4pq-3)^2} + \dfrac{256(6pq-5p)}{(4pq-3)^4}.$

(iii) $PEB(G) = KAB_{1,-\frac{1}{2}}^2(G) = \dfrac{4p\sqrt{(4pq-2)(4pq-3)}}{3} + \dfrac{(6pq-5p)(4pq-3)}{4}.$

(iv) $^mRPEB(G) = KAB_{1,\frac{1}{2}}^2(G) = \dfrac{12p}{\sqrt{(4pq-2)(4pq-3)}} + \dfrac{4(6pq-5p)}{(4pq-3)}.$

(v) $^mEB_2(G) = KAB_{1,-1}^2(G) = \dfrac{4p(4pq-2)(4pq-3)}{9} + \dfrac{(6pq-5p)(4pq-3)^2}{16}.$

4. Conclusion

In this study, we have defined the first and second modified E-Banhatti indices and the first and second (*a*, *b*)-*KA* E-Banhatti indices of a graph. The first and second (*a*, *b*)-*KA* *E*-Banhatti indices and some other *E*-Banhatti indices for particular values of *a* and *b* for $HC_5C_7[p, q]$ nanotubes have been determined.

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