*Journal of Mathematics and Informatics*
*Vol. 22, 2022, 23-27*
*ISSN: 2349-0632 (P), 2349-0640 (online)*
*Published 15 March 2022*
*www.researchmathsci.org*
*DOI: http://dx.doi.org/10.22457/jmi.v22a03205*

Journal of **Mathematics and Informatics**

# Revan Sombor Index

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*Received 2 February 2022; accepted 14 March 2022*

***Abstract.*** In this paper, we introduce the Revan-Sombor index of a graph, which is a combination of the earlier considered Revan- and Sombor-type vertex-degree-based molecular structure descriptors. Some properties of this newly defined topological index are established.

***Keywords:*** Revan index, Sombor index, degree (of graph), topological index

***AMS Mathematics Subject Classification (2010):*** 05C07, 05C09

## 1. Introduction

Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. Let □ and □ denote, respectively, the maximum and minimum degree among the vertices of the graph $G$. The edge connecting the vertices $u$ and $v$ will be denoted by $uv$.

One of the main directions of recent research in chemical graph theory is the study and application of graph-based molecular structural descriptors, usually referred to as "topological indices" [1]. An important group of such descriptors are the vertex-degree-based (VDB) topological indices, whose general form is

$$TI = TI(G) = \sum_{uv \in E(G)} \Phi(d_G(u), d_G(v))$$

where $\Phi(x, y)$ is a pertinently chosen function satisfying the condition $\Phi(x, y) = \Phi(y, x)$. Some of the simplest, oldest, and most detailed studied VDB indices are the first and second Zagreb index

$$M_1 = M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)], \qquad M_2 = M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v)$$

and the so-called "forgotten" topological index

$$F = F(G) = \sum_{uv\in E(G)} \left[ d_G(u)^2 + d_G(v)^2 \right].$$

Another, recently introduced group of VDB indices [2] are the Sombor and reduced Sombor indices

$$SO = SO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}$$

and $$SO_{red} = SO_{red}(G) = \sum_{uv\in E(G)} \sqrt{[d_G(u)-1]^2 + [d_G(v)-1]^2}$$

as well as the reverse Sombor index [3].

$$SO_{rev} = SO_{rev}(G) = \sum_{uv\in E(G)} \sqrt{[\Delta - d_G(u)+1]^2 + [\Delta - d_G(v)+1]^2} \,. \tag{1}$$

Denote by $r_G(u)$ the *Revan vertex degree* of a vertex *u* in *G*, defined as $r_G(u) = \Delta + \delta - d_G(u)$. In 2017 [4], one of the present authors conceived a class of Revan-type indices, defined in analogy to the Zagreb and forgotten indices as

$$R_1(G) = \sum_{uv\in E(G)} \left[ r_G(u) + r_G(v) \right] \,, \quad R_2(G) = \sum_{uv\in E(G)} r_G(u) r_G(v)$$

and

$$FR(G) = \sum_{uv\in E(G)} \left[ r_G(u)^2 + r_G(v)^2 \right]. \tag{2}$$

These indices found numerous applications; for details see the [4,5] and the references cited therein. Directly from their definitions, the following relations with the classical VDB indices can be recognized:

$$R_1(G) = 2(\Delta+\delta)m - M_1(G)$$
$$\mathrm{R}_2(G) = (\Delta+\delta)^2 m - (\Delta+\delta)M_1(G) + M_2(G)$$
$$FR(G) = F(G) - 2(\Delta+\delta)M_1(G) + 2(\Delta+\delta)^2 m.$$

Motivated by the definitions of the Revan and Somber indices, we now introduce the Revan-Sombor index of a graph and defined it as,

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2} = \sum_{uv\in E(G)} \sqrt{[\Delta+\delta-d_G(u)]^2 + [\Delta+\delta-d_G(\mathrm{v})]^2}$$

and establish some of its main properties. It should be noted that if δ = 1, then the Revan-Sombor index coincides with the reverse Sombor index, Eq. (1).

Recently, some topological indices were studied in [6,7,8].

## 2. Mathematical properties of the Revan-Sombor index

**Proposition 1.** If *G* is an *r*-regular graph with *n* vertices and $r \geq 1$, then

$$RSO(G) = \frac{nr^2}{\sqrt{2}}.$$

**Proof:** An *r*-regular graph with *n* vertices has $m = nr/2$ edges. In addition,

$$r_G(u) = \Delta + \delta - d_G(u) = \mathrm{r} + \mathrm{r} - \mathrm{r} = \mathrm{r}$$

and therefore

$$RSO(G) = m\sqrt{(r^2 + r^2)} = \frac{nr}{2} \cdot \sqrt{2}r = \frac{nr^2}{\sqrt{2}}. \qquad \blacksquare$$

**Corollary 1.1**. Let $C_n$ be the cycle with $n \geq 3$ vertices. Then $RSO(C_n) = 2\sqrt{2}n$.

**Corollary 1.2**. Let $K_n$ be the complete graph on $n \geq 1$ vertices. Then

$$RSO(K_n) = n(n-1)^2 / \sqrt{2}.$$

It is immediately evident that in the case of regular graphs, the Revan-Sombor and the ordinary Sombor indices coincide. It is worth noting that there are several other graphs for which the equality

$$RSO(G) = SO(G) \tag{3}$$

holds. First of all, Eq (3) is obeyed by complete bipartite graphs $K_{a,b}$ on $a+b$ vertices, $a \geq b \geq 1$. Namely, for such graphs $\Delta = a$, $\delta = b$, and therefore for any edge $uv$, either $r_G(u) = a = d_G(u)$, $r_G(v) = b = d_G(v)$ or $r_G(u) = b = d_G(u)$, $r_G(v) = a = d_G(v)$. Then

$$RSO(K_{a,b}) = SO(K_{a,b}) = ab\sqrt{a^2 + b^2}.$$

Let $G$ be an $r$-regular graph on $n$ vertices and $m$ edges. Let $S(G)$ be its subdivision graph, on $n+m$ vertices and $2m$ edges, obtained by inserting a new vertex on any edge of $G$. Then any edge of $S(G)$ connects a vertex of degree $r$ with a vertex of degree 2. Thus,

$$RSO(S(G)) = SO(S(G)) = 2m\sqrt{r^2 + 2^2}.$$

There are graphs different from $K_{a,b}$ and $S(G)$, for which Eq. (3) holds. Two such examples are depicted in Figure 1. Note that $G(2,3)$ is bipartite whereas $G(3,5)$ is non-bipartite.

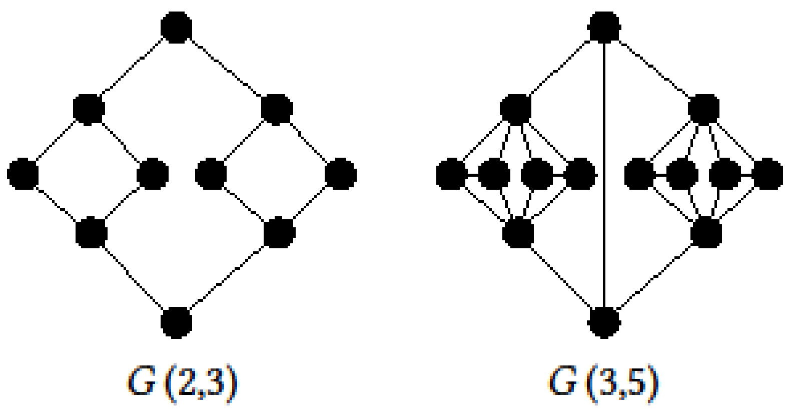


**Figure 1.** Graphs whose Sombor and Revan-Sombor indices coincide.

We can summarize the above observations in the following:

**Proposition 2.** Let $\delta \leq \Delta$ be the minimum and maximum vertex degree of the graph *G*, and let *m* be the number of its edges. If all vertices of *G* are either of degree $\delta$ or of degree $\Delta$, and if all edges of *G* connect a vertex of degree $\delta$ with a vertex of degree $\Delta$, then

$$RSO(G) = SO(G) = m\sqrt{\delta^2 + \Delta^2}\,.$$

**Proof:** If $d_G(u) = \delta$, then $r_G(u) = \Delta + \delta - d_G(u) = \Delta + \delta - \delta = \Delta$. If $d_G(v) = \Delta$, then $r_G(v) = \Delta + \delta - d_G(v) = \Delta + \delta - \Delta = \delta$. Therefore, for any edge *uv*,

$$\sqrt{r_G(u)^2 + r_G(v)^2} = \sqrt{d_G(u)^2 + d_G(v)^2} = \sqrt{\delta^2 + \Delta^2}\,. \quad \blacksquare$$

**Theorem 1.** Let *G* be a connected graph with *m* edges. Then

$$RSO(G) \leq \sqrt{m\,FR(G)}$$

where $FR(G)$ is the Revan-forgotten index, Eq. (2).

**Proof:** Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv\in E(G)} \sqrt{r_G(u)^2 + r_G(v)^2}\right)^2 \leq \sum_{uv\in E(G)} 1 \sum_{uv\in E(G)} \left[r_G(u)^2 + r_G(v)^2\right] = m\,FR(G) \quad \blacksquare$$

**Theorem 2.** Let *G* be a connected graph with *m* edges. Then

$$RSO(G) \leq \sqrt{m\left[2(\Delta+\delta)^2 m + F(G) - 2(\Delta+\delta)M_1(G)\right]}$$

where $M_1(G)$ and $F(G)$ are the first Zagreb and forgotten topological index.

**Proof:** Consider

$$FR(G) = \sum_{uv\in E(G)} \left[r_G(u)^2 + r_G(v)^2\right] = \sum_{uv\in E(G)} \left[\left(\Delta+\delta - d_G(u)\right)^2 + \left(\Delta+\delta - d_G(v)\right)^2\right]$$

$$= 2(\Delta+\delta)^2 m + F(G) - 2(\Delta+\delta)M_1(G).$$

From the above equation and using Theorem 1, we get the desired result. $\blacksquare$

By means of proof techniques analogous to what earlier was used to the Sombor index [9,10], we obtain Theorem 3 and Theorem 4:

**Theorem 3.** Let *G* be a connected graph. Then

$$RSO(G) \geq \frac{1}{\sqrt{2}} R_1(G)\,.$$

Equality holds if and only if *G* is regular.

**Corollary 3.1.** Let *G* be a connected graph. Then

$$RSO(G) \geq \frac{1}{\sqrt{2}}\left[2(\Delta+\delta)m - M_1(G)\right].$$

Equality holds if and only if *G* is regular.

**Theorem 4.** Let *G* be a connected graph. Then

$$RSO(G) \leq \sqrt{2}\left[R_1(G) - R_2(G)\right].$$

**Corollary 4.1.** Let $G$ be a connected graph. Then

$$RSO(G) \leq \sqrt{2}\left[2(\Delta+\delta)m - (\Delta+\delta)^2 m + (\Delta+\delta-1)M_1(G) - M_2(G)\right].$$

Equality holds if and only if $G$ is regular.

## 3. Conclusion

In this study, we have introduced the Revan Sombor index of a graph. Some properties of this newly defined topological index are established.

***Acknowledgement.*** (1) This research is supported by IGTRC No. BNT/IGTRC/ 2022:2203:103 International Graph Theory Research Center, Banhatti 587311, India.
(2) The authors are thankful to the referee for useful comments.

**Conflict of interest.** The authors declare that they have no conflict of interest.

**Authors' Contributions.** All the authors contributed equally to this work.

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