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Hydrological Modeling of Artificial Recharge of Groundwater for Sustainable Water Supply in Dodoma City

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Abstract. Groundwater is an important resource that supports the life of people and the surrounding ecosystem in the world. It is the primary source of safe water to semi-arid areas characterized by limited surface water. In Africa, water scarcity has been hitting major cities and towns. In Tanzania, Dodoma has long experienced shortages of water. Owing to the recent transfer of all significant offices from Dar es Salaam to Dodoma, the City's population has drastically increased. The primary source of water in the City is the Makutupora aquifer. The growing human population has resulted in high demand for water use, which has led to the overexploitation of groundwater aquifer. Therefore, this study was carried out using a Modular finite-difference flow model (MODFLOW) to model artificial recharge products to replenish groundwater in the Makutupora aquifer to ensure water supply sustainability in the City. Before simulation of the artificial recharge was done, groundwater storage was estimated using available borehole data and GIS technique. The results indicated that the total groundwater storage in the Makutupora aquifer was about 24.8 BCM (Billion Cubic Meters). The MODFLOW packages used include well package (WEL), General Head Boundary Package (GHB), Evapotranspiration package (EVT), Drain package (DRN), and Recharge Package (RCH). A total of 21 piezometers were used for model calibration. The statistical calibration was also done to validate the model's calibrated parameters. After simulation of the steady-state reference period, the other four stress periods were simulated, considering the projected population and water demand. The planned injection wells to the model in the first, second, third, and fourth transient state periods resulted in a safe yield of 168,857 m^3/day, 197,760 m^3/day, 360,000 m^3/day, and 600,430 m^3/day, respectively. The recommended artificial recharge source is water from the Kinyasungwe River that flows during rainfall time, generally from November to May. One of the recorded years (2007) indicated a flow of up to 23.646 Million Cubic Meters (MCM). The recommended artificial recharge is possible due to the aquifer's storage capacity of 247.84 Million Cubic Meters (MCM). Other flows in small streams within the well field were recommended in creating artificial recharge structures to add more water to the aquifer and natural recharge. Therefore, information from this study could be used by engineers when constructing artificial engineering structures to replenish the water pumped from the Makutupora aquifer.

Keywords: Groundwater, MODFLOW, Artificial Recharge, Makutupora Basin, Groundwater Model, Semi-arid Regions.

AMS Mathematics Subject Classification (2010): 97M10, 93A30

1. Introduction

The world's water crisis is projected to get worse as a result of both climate change and fast population growth [1, 2]. Various places, especially those found in semi-arid areas where the primary water source is groundwater, always experience water shortage [3]. The problem hits more in cities and towns where demand is more significant than supply due to increased population [4–9]. In Africa, water security is an issue in almost all dimensions like affordability, accessibility, and acceptability, especially to people living in large cities and towns where the population is overgrowing [10].

In Tanzania, Dodoma City has always faced water deficiencies. The City's population has increased quickly and will continue to grow between 2020 and 2051 after the government's relocation of all its significant offices from Dar es Salaam to Dodoma. The relocation was made, favoring Dodoma's central location to ensure that the government services are closer to people [11]. The national census of 1988, 2002, and 2012 indicated that Dodoma City had populations of 203 833, 324 347, and 410 956 people, respectively [12]. But recently, the City's population growth is not following the average expected growth and expansions. Still, it is drastic due to government and International Organization offices' movement along with their employees and families and other service providers from Dar es Salaam to Dodoma City (Table 1). This movement has disrupted and will continue disrupting the population's natural development to support the City's growth [11]. The water demand has also increased and will continue rising as the City grows (ibid) (Table1).

Table 1: Projected City population, water demand, existing water capacity, and deficiency 2025 to 2051

	2015	2025	2026	2036	2051
The population of the City	497,934	689,072	1,069,900	1,713,794	1,972,968
Requirement m^3/day	82,651	137,584	160,730	226,204	418,839
Existing m^3/day	61,500	113,192	113,192	113,192	113,192
Deficit m^3/day	-21,151	-23,992	-47,138	-113,012	-305,247

The groundwater taped from the Makutupora aquifer is the principal primary source for supplying water to the City [13]. As pumping wells draw groundwater from a larger wellfield area, groundwater level declines gradually (ibid). Episodic events and highly seasonal rainfall have been among the factors for sustaining groundwater, but this rarely occurs [14]. The wellfield's ability to continue and maintain the recent intensive pumping to meet the demand for safe water following the City's rapid growth is not evident [15]. Researchers contended that the boreholes could produce 61,500 m^3/day. The aquifer was found to supply between 48,000 m^3/day up to 50,000 m^3/day, and hence it is likely to be overexploited [11], [16–18]. Furthermore, the projections regarding population indicated a more significant water demand than the existing capacity resulting in a deficit (Table 1). Response strategies like mitigation, adaptation, and coping mechanisms are crucial to address water availability in Dodoma City due to increased population [18, 19].

The primary government challenge is finding the ways and methods that can be used to ensure water availability sustainability and have that water cost-effectively. The government plans to bring water from either Farkwa Dam or Lake Tanganyika/Victoria to meet the city needs, but this is cost inhibitive [11]. Internationally, some developed countries like Austria, Greece and the Netherlands have already adopted artificial recharge methods to replenish the aquifers [20, 21]. [22] modeled artificial recharge using MODFLOW, and the results indicated that artificial recharge enhances groundwater recovery effectively. [23] used MODFLOW to model the groundwater flow and assess recharge and water table behavior's potential under varying recharge and pumping rates. The results indicated that the prevailing pumping and recharge rates are not sustainable and that there was a need to implement some artificial recharge techniques. [24] modeled the recovery of storage by MODFLOW to simulate the increase in storage after water scarcity as caused by climate change, urbanization, and an increase in the area's population. The results indicated that making water available to the farms ensures the availability of groundwater in aquifers sustainably.

1.1. Previous studies related to groundwater modelling in makutupora basin

Few previous studies on modelling groundwater flow in Makutupora have been undertaken. [23] used MODFLOW to model groundwater flow under transient state conditions in previous years. However, the picture of the groundwater flow was not clear because the modeled area was too small. [23] used MODFLOW to model flow in the Makutupora catchment and observed that the present geologic structures influence the groundwater and surface water flow. However, the time was not enough to validate the model and determine recharge rates (ibid). Furthermore, a study by [25] on groundwater management using a mathematical model under the MODFLOW code suggested that artificial recharge through infiltration ponds can restore the aquifers. The study did not estimate the recharge rate needed and the aquifer's natural recharge to satisfy the people as the City overgrows.

Additionally, [26] carried out research on Makutupora that assessed the recharge of groundwater using MODFLOW. Her recommendations were to monitor the wellfield by introducing boundaries that reflect the area about time for a better understanding of inflow and outflow rates. [16] contended that research was carried out by a team of scientists from the Sokoine University of Agriculture and the Ministry of Water and

Irrigation both in Tanzania and the University College London (UK). The research involved the compilation of a near-continuous 60-year record of groundwater-level observations. The analysis revealed that recharge depends much on heavy seasonal rainfall associated with El Niño Southern Oscillation (ENSO) that occurs episodically. However, the study did not focus on improving the recharge of the site through artificial recharge. [19] carried a study titled "The Climate Controls and Process of Groundwater Recharge in a Semi-Arid Tropical Environment: Evidence from the Makutupora Basin, Tanzania." The study recommended that the managed artificial recharge replenish the groundwater, but the study did not provide specific information to manage recharge and pumping. Therefore, this study fills in the identified gap by simulating and determining the recharge rate with respect to the estimated increase in water demand that could be brought by the increase in population, considering the figures projected in the Norplan report of 2019 (Table 1).

2. Materials and methods

2.1. Description of the study area

The study was undertaken in the Central Semi-Arid part of Tanzania (Figure 1). The specific site for the study is Makutupora Basin, Dodoma region, Tanzania. The pumping station is located about 30km north of Dodoma City (Figure 2). Its location is 5^0 36′59" and 6^0 14' 50 " S and 35^0 36' 36"' and 36^0 01' 54"'E. It covers an area of approximately 785,937,000 m^2. Dodoma receives an annual rainfall of about 550 mm/year, and it always rains from November to May [27]. The yearly evapotranspiration of the area is approximately 2,000 mm.

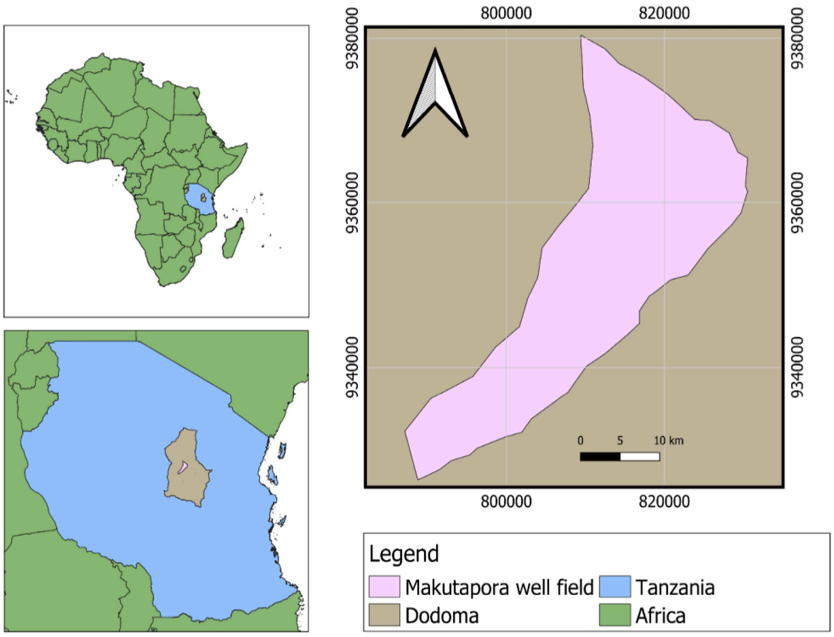

Figure 1: Location Map of the Study area

In conceptualizing the model, the elevation data sets from Earth Explorer were downloaded and visualized in the Geographical Information System (GIS). The National Aeronautics and Space Administration (NASA) website provided the information for creating the study area's digital elevation model. The elevation of the modeled area ranges from 1,058 m.a.s.l to 2,048 m.a.s.l (Figure 2).

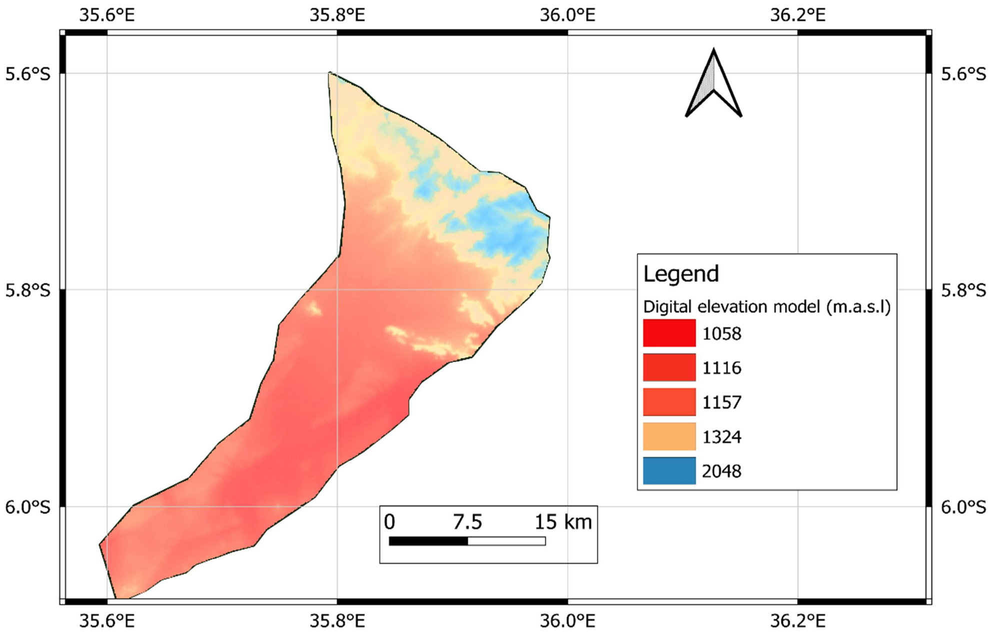

Figure 2: Location Map of the Study Area

2.2. The process of groundwater flow modelling

Several steps were involved in modelling the artificial recharge of groundwater. Various skills related to hydrogeology were used in the whole process of modelling groundwater, involving understanding the equations.

The first step involved using all knowledge and available data obtained from existing literature, a survey conducted in this study, and field data collected in this study to develop a conceptual model. The available data were also used to establish the groundwater storage in the Makutupora aquifer.

The second step involved representing the conceptual model in a mathematical form and construction/setting of the numerical model by applying the analyzed data like recharge rate and evapotranspiration and other boundaries discovered during the conceptualization process.

The third step involved calibration of both steady and transient states that matched the observed and the simulated data. The fourth stage involved the use of the constructed model to predict the flow of groundwater. It also involved the simulation of artificial recharge of groundwater. The modelling process is summarized in Figure 3.

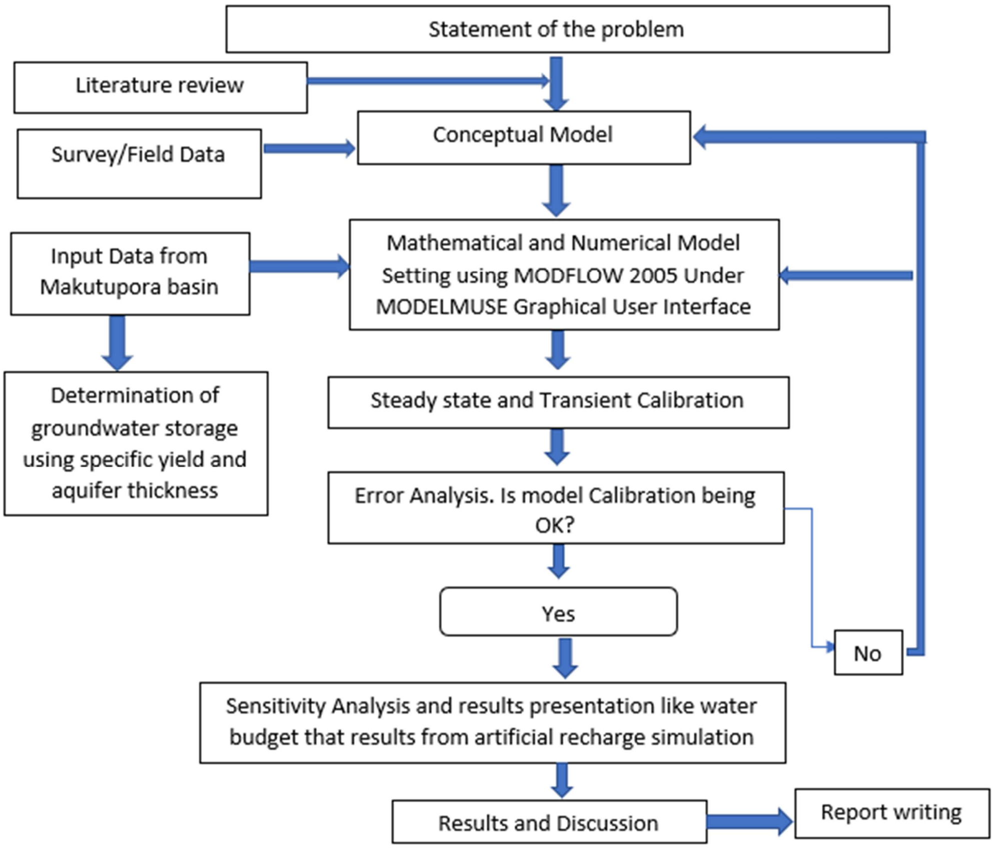

Figure 3: Modelling Protocoll (source: this study, modfied from [26, 28–30])

2.3. Groundwater storage estimation in makutapora aquifer

Estimating groundwater storage in the Makutupora aquifer was done using available borehole data and Geographical Information System (GIS) techniques. The GIS was used to construct the map where the aquifer's surface area and boundary were determined. The available maps were in image form and had no exact coordinate system and hence were imported in QGIS and then georeferenced accordingly. The aquifer was divided into four layers: The layers' determination based on the soil type's dominant concentration. The first and third layer were aquitards, and the second and fourth were the aquifers. The estimated aquifer thickness was adopted from a study by [31]. The specific yield was calculated by averaging the determined values in the study by [32]. Specific yield refers to the amount of water available for groundwater pumping from the material containing the water. The specific yield also depends on the depth of the aquifer. Figure 4 below shows the flowchart of groundwater storage estimation.

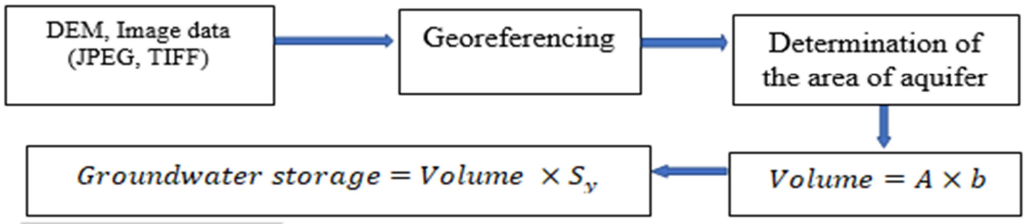

Figure 4: Flowchart showing groundwater storage estimation

A map calculator was used to determine the area of the aquifer. Groundwater calculated using the equation (1) below

$$Groundwater\ storage = A \times b\ \times S_y \quad (1)$$

Table 2: Estimated values of the Specific yield and aquifer thickness for each layer

Layer Number	Specific yield	Aquifer thickness
Layer One	0.0567 = 5.67 x 10^{-2}	55.50m
Layer two	0.2100 = 2.1 x 10^{-1}	62.75m
Layer three	0.0300 = 3 .0 x 10^{-2}	55.50m
Layer four	0.2330 = 2.33x 10^{-1}	62.00m

Source: [31], [32]

2.4. Meteorological data

The meteorological data that were used in the study area include rainfall, temperature, and evapotranspiration data. The data were obtained from the Ministry of Water, and they cover the date January 2017 to December 2019. The driving forces like interception are used to calculate the infiltration rate of the area. The modeled area covers 785.937 km^2. After classification, the vegetation area covers 734.653 km^2 (93%), and the grass area covers 51.284 km^2 (7%). The computed ratio of the grass to other vegetation is 0.07 to 0.93. Then, after determining the area's size and vegetation covers, the interception value was calculated using equation (2).

$$I = RF * (I_g * Area_g + I_{other} * Area_{Other}) \quad (2)$$

where I is the canopy interception (mm d^{-1}), RF is rainfall (mm d^{-1}), I_g and I_{other} are the interception percentage loss for grass and other vegetation covers, respectively. Substituting the values of I_g, $Area_g, I_{other}$, $Area_{Other}$ equation (2) becomes equation (3).

$$I = RF * (0.069 * 0.07 + 0.2 * 0.93) \quad (3)$$

Then the value of interception used to calculate the infiltration rate using equation (4)

$$P_r = P - I \quad (4)$$

where P_r is the infiltration, P is precipitation, and I is the interception

This study has involved the calculation of the weighted interception and infiltration. The minimum, maximum, and average interception were 0 mm/day, 17.270115 mm/day, and 0.302370571 mm/day. The minimum and maximum infiltration were found to be 0 mm/day and 73.229885 mm/day, respectively. The infiltration rate ranges between 0 mm/day and 73 mm/day inclusive. The daily precipitation ranges between 0 mm/day and 90.5 mm/day, inclusive. On average, the value of rainfall was found to be 1.6 mm/day. Lack of data from the Makutupora potential evapotranspiration pan forced us to use other methods such as the Penman-Monteith method of the Food and Agricultural Organization (FAO). The calculated potential evapotranspiration using the Penman-Monteith method ranges between 4.2 mm/day and 11.6 mm/day. Its mean is 8.9 mm/day.

3. Mathematical and numerical groundwater modeling

Numerical modelling of artificial recharge of groundwater has been an essential tool in managing groundwater in various parts of the world [33], [34]. The mathematical formulation and its numerical description used to study groundwater flow in the aquifer are explained in sections 3.1 and 3.2.

3.1. Mathematical description of governing equations

According to [35], the governing equations for a three-dimensional unsteady, transient flow problem with Darcy's law and continuity equations is expressed by equation (10), which is the partial differential equation for physical modelling of the three-dimensional groundwater flow problem. The model is formulated from Darcy's law and continuity equation, as explained below.

Darcy's law states that, for a given type of sand, the rate of flow of water is proportional to the cross-sectional area A and the piezometric head drop or loss $h_2 - h_1$ and it is inversely proportional to the difference in the porous medium's length [36]. Also, the groundwater flows from high to low energy potential [37]. The mathematical expression of Darcy's law is given by equation (5)

$$Q \propto A\frac{h_2-h_1}{l_2-l_1} \rightarrow q = \frac{Q}{A} = -K\frac{h_2-h_1}{l_2-l_1} \tag{5}$$

where $q = (q_{x,}\ q_{y,}\, q_z)$ stands for the specific discharge, K is the hydraulic conductivity, and h is the hydraulic head. A minus sign indicates that the flow is in the direction of decreasing head loss. The medium was assumed to be isotropic and that the discharge rate Q is not dependent on time.

$$q = -\mathrm{k}\frac{dh}{dl} \rightarrow q = -k\ grad\ h \tag{6}$$

where k = $(k_x,\ k_y, k_z)$ stands for the values of hydraulic conductivity along x, y and z axes

The three-dimensional incompressible continuity equation is expressed by equation 7;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

whereby $u, v \; and \; w$ are the components of velocity in the directions of x, y and z. The velocity components u, v and w can be replaced with components of $q = q(q_x, q_y, q_z)$. The transient condition is modelled by adding a storage coefficient. The continuity equation for transient conditions is described by equation (8)

$$\frac{\partial q_x}{\partial x}(b) + \frac{\partial q_y}{\partial y}(b) + \frac{\partial q_z}{\partial z}(b) = N(x, y, z, t) - S\frac{\partial h}{\partial t} \tag{8}$$

where N, is the sink or source, b is the thickness of the aquifer, t is time, and S stands for the storage coefficient. Substituting from equation 8 in Darcy's law for q_x , q_y and q_z result in equation (9) and (10)

$$\left(\frac{\partial}{\partial x}\left(-k_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(-k_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(-k_z\frac{\partial h}{\partial z}\right)\right) = N(x, y, z, t) - S\frac{\partial h}{\partial t} \tag{9}$$

$$\rightarrow \frac{\partial}{\partial x}\left(k_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y b\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial h}{\partial z}\right) = S_s\frac{\partial h}{\partial t} - N(x, y, z, t) \tag{10}$$

where S_s= Specific storage and h = Piezometric head (L),
The equation 9 and 10 solved by finite difference as applied by [38] MHD Arterial Blood Flow and Mass Transfer under the Presence of Stenosis, Body Acceleration and Chemical Reaction: A Case of Magnetic Therapy.

3.2. Numerical equations

Numerical modelling of equation (10) is done by the compact finite difference approximation as expressed by equation (11):

$$\Delta_x\left(T_x\Delta_x h^{t+\Delta t}\right) + \Delta_y\left(T_y\Delta_y h^{t+\Delta t}\right) + \Delta_z\left(T_z\Delta_z h^{t+\Delta t}\right) = \frac{V_b S_s}{\Delta_t}\left(h^{t+\Delta t} - h^t\right) - V_b N \tag{11}$$

where V_b is the volume of grid block at the location (i, j, k), T is the transmissibility, Δt is the time level increment, t is the current time level, $t + \Delta t$ is the next time step. Figure 5 represents the hypothetical aquifer in three dimensions. The block centred finite difference approach is used to simulate groundwater flow.

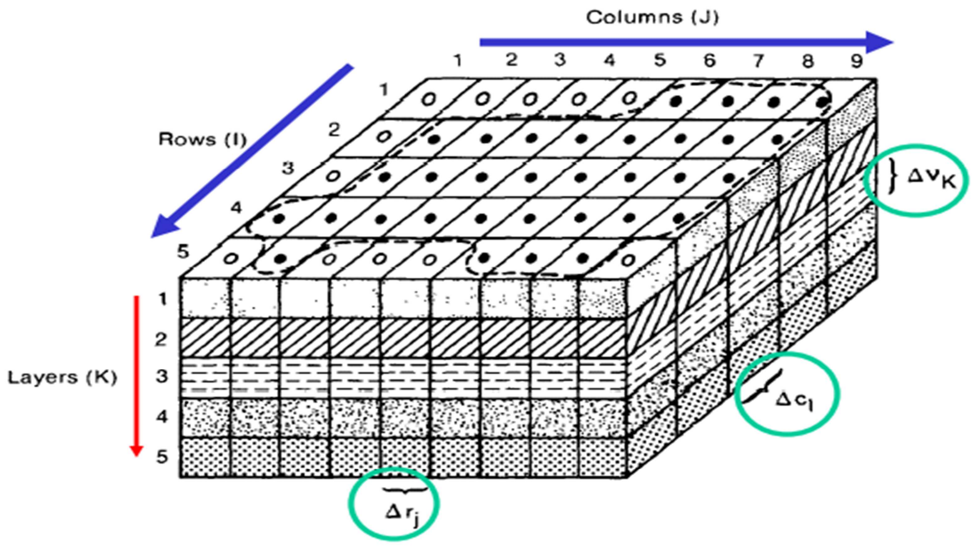

Figure 5: A discretized hypothetical aquifer system in three dimensions (Haurbaugh, 2005).

Equation (10) was discretized using the finite difference quotients of equation (11) form a numerical discrete equation (12):

$$\frac{1}{(\Delta x)_{i,j,k}}\left[Kx_{\left(i+\frac{1}{2},j,k\right)}\frac{h_{i+1,j,k}^{n+1}-h_{i,j,k}^{n+1}}{(\Delta x)_{i+\frac{1}{2},j,k}}-Kx_{\left(i-\frac{1}{2},j,k\right)}\frac{h_{i,j,k}^{n+1}-h_{i-1,j,k}^{n+1}}{(\Delta x)_{i-\frac{1}{2},j,k}}\right]$$
$$+\frac{1}{(\Delta y)_{i,j,k}}\left[Ky_{\left(i,j+\frac{1}{2},k\right)}\frac{h_{i,j+1,k}^{n+1}-h_{i,j,k}^{n+1}}{(\Delta y)_{i,j+\frac{1}{2},k}}-Ky_{\left(i,j-\frac{1}{2},k\right)}\frac{h_{i,j,k}^{n+1}-h_{i,j-1,k}^{n+1}}{(\Delta y)_{i,j-\frac{1}{2},k}}\right]$$
$$+\frac{1}{(\Delta z)_{i,j,k}}\left[Kz_{\left(i,j,k+\frac{1}{2}\right)}\frac{h_{i,j,k+1}^{n+1}-h_{i,j,k}^{n+1}}{(\Delta z)_{i,j,k+\frac{1}{2}}}-Kz_{\left(i,j,k-\frac{1}{2}\right)}\frac{h_{i,j,k}^{n+1}-h_{i,j,k-1}^{n+1}}{(\Delta z)_{i,j,k-\frac{1}{2}}}\right]$$
$$=S_s\frac{h_{i,j,k}^{n+1}-h_{i,j,k}^{n}}{\Delta t}-N_{(i,j,k)} \tag{12}$$

This study used MODFLOW-2005 code for the numerical model and the ModelMuse Graphical User Interface (GUI) to implement the numerical model and do simulations for the groundwater flow in the Makutupora aquifer.

3.2. MODFLOW-2005 model construction and discretization

In this study, numerical modelling of the groundwater flow, including defining boundary conditions, was done through several MODFLOW-2005 packages. ModelMuse (the Graphical user interface (GUI) software) was used for inputting data independently in the MODFLOW-2005 numerical model, generating and editing the numerical model grid, defining and redefining discretization, executing the numerical model, doing simulations and displaying the results of the MODFLOW-2005 numerical model.

The site covering 785,937,000 m^2 was discretized in the grid of 500mx500m comprising 115 rows and 55 columns. Four layers were defined in the model regarding the available bore log and lithological profile data having the length elevation of 40, 75, 105, and 150. The model was run under transient conditions except for the first steady-state stress period. The digital elevation model (DEM) data were downloaded from NASA (ASTER GDEM 2). The Modelmuse GUI was used to define the model grids for the active and inactive cells. Figure 6 shows the discretized model.

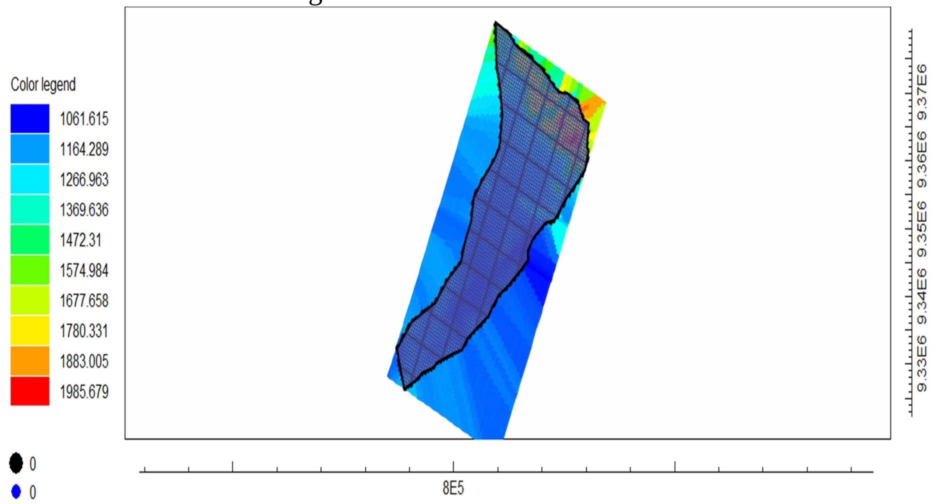

Figure 6: Model grid in Modelmuse GUI with elevation data

3.3. Boundary conditions

The proper assignment of the boundary conditions is vital when constructing a groundwater flow model, especially when establishing the area's hydrological processes. In the case of this study, the following MODFLOW-2005 packages were used to study the area.

- Recharge package (RCH): The recharge package was used as the inflow from the precipitation. The infiltration coefficient used is 68% of the rainfall. It was used as the infiltration in the reference period (which had 1.1 mm/day of rain).

- WEL package (WEL): The WEL package was used to assign the pumping rate and artificial recharging rate to the aquifer. The simulation involved a plan of 50 pumping wells and 50 recharging wells. General head boundary package (GHB): The GHB always defines the general groundwater flow condition. It allows water to move either out or into the system, depending on the elevation. The area's groundwater flow in the system is influenced by faults, especially Kitope and Mlemu Fault. The estimated values of hydraulic conductance used in the simulation were 1.119375E-8 m^2/day, 1.50807233796296E-14 m^2/day, and 1.17994097E-8 m^2/day for both Kitope 1, Kitope 2, and Mlemu Fault, respectively. The boundary head was set as the model top for all faults.

- Evapotranspiration package (EVT). The evapotranspiration package is used to implement the outflow of water from evaporation and transpiration. In the reference period, the evapotranspiration rate of 8mm/day.
- HOB package: The HOB package was used to introduce a hydraulic head in the model, was later used for model calibration.
- Drain (DRN). The Kinyasungwe River that flows within the Makutupora area was assigned as a drain because it drains the water from the upper part of the study area. The stream flows seasonally, and the assigned hydraulic conductivity was 0.1331m ^{2}/day, and the elevation for this was set as Model Top.

3.4. Time discretization

The stress periods were defined based on the projected population rise and the projected water demand for Dodoma City. The number of stress periods was five. The first stress period was the steady-state, and the remainder were in a transient state. The stress periods were in groups of time steps 1, 5, 10, 1, and 15 years, from 2019 to 2051 (Table 3).

Table 3: Flow Conditions and Time Discretization

Stress period	Time Step (Years)	Duration (Years)	State	Calendar period
1	1	1	Steady	2016
2	5	6	Transient	2017-2022
3	1	7	Transient	2023-2024
4	10	17	Transient	2025-2035
5	15	32	Transient	2036-2051

4. Results and discussions

4.1. Estimated groundwater storage in makutupora aquifer

The aquifer has an area of 785 937 000 m^2. The estimated values of specific yield for the aquifer ranges from 3 .0 x 10^{-2} to 0.2330 = 2.33x 10^{-1}. The values of the specific yield and aquifer thickness for each layer is indicated in Table 2. The total groundwater storage was estimated to be 247.84 MCM. The description of the estimation of groundwater storage layer-wise is indicated in Table 4.

Table 4: Estimated groundwater storage capacity of the Makutupora aquifer

Layer Number	Groundwater storage	Groundwater storage in Million Cubic Meters (MCM)
Layer one	24042419.87m^3	24.04
Layer two	100678134.600m^3	100.68
Layer three	12720857.08m^3	12.72
Layer four	110369670.50m^3	110.40
Total	247811082.1m^3	247.84

4.2. Calibrated parameters for the calibrated model

The calibration involved adjusting the recharge rate and hydraulic conductivities and systematically to match the observed and simulated head. Table 5 shows the calibrated hydraulic conductivities, recharge rate, evapotranspiration rate, and drain conductance. The hydraulic conductivities of the area differ from one layer to another and influences the area's recharge.

Table 5: Hydraulic Parameters

Parameter	Value	Description
HKL1	1E-7 m/day	Hydraulic conductivity of the first layer
HKL2	0.0001 m/day	Hydraulic conductivity of the second layer
HKL3	0.0001 m/day	Hydraulic conductivity of the third layer
HKL4	1E-6 m/day	Hydraulic conductivity of the fourth layer
GHB	0.1331 m^3/day	The general head boundary conductance
RCH	1.2991979E-9 m^3/day	Recharge rate of groundwater
EVT	5.12E-7 m^3/day	Evapotranspiration rate of groundwater

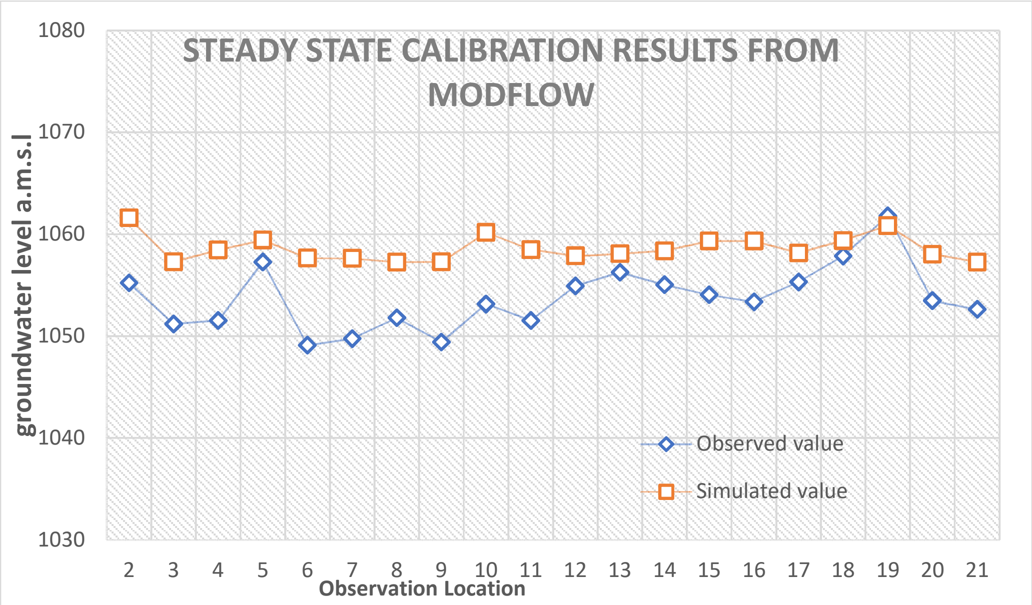

Figure 7: Steady-State Calibrated values from the MODFLOW.

Various statistical tests like Chi-square and the coefficient of determination were used to validate the calibrated results. Equation (5) was used to compute the value of Root Mean Squared Error (RMSE).

$$RMSE = \left(\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{n}\right)^{1/2} \qquad (11)$$

The RMSE from the statistical test was 5.51. The value coincides with the value produced by the MODFLOW 2005 and Modelmuse graphical user interface, which is 5.38 with a slight deviation. Using equation (6), we computed the value of Chi-square, where the. Chi-square results were good as they range between 0 and 1.

$$\chi 2 = \sum \frac{(O_i - E_i)^2}{E_i} \qquad (12)$$

where $\chi 2$ = Chi-squared, E_i =Expected value O_i = Observed value, and n is the total number of observed values.

The chi-squared test value is 0.57, which is the same as 57%, indicating that the two values are not much different. The statistical findings suggest that the model works as long as the variance between the actual and predicted values does not vary greatly.

4.3. Scenarios simulated

4.3.1. First scenario

The simulated steady-state flow was the initial point with the hydraulic conductivity's initial calibrated parameters, recharge, evapotranspiration, and drain conductance values. The model was first set by considering the effect of natural recharge that usually occurs through infiltration after precipitation. After that, a pumping rate of 0.1 m^3 /day for about ten years indicated a decline in groundwater level. Hence, other strategies to replenish the aquifers from the extra water that flows out of the system during the rainy season are vital. The plan to fill the aquifer was simulated after introducing the artificial injecting artificial wells that could artificially increase the groundwater storage. This model attempts to simulate artificial wells' introduction to replenish the overexploited groundwater in Makutupora, given the surge increase in Dodoma City population.

4.3.2. Second scenario

Then, a second scenario was done by which a total of 50 injecting wells were assigned to add the water to the ground by the artificial aquifer recharge methods. In this scenario, it was assumed that no pumping was undertaken in the area (Table 6). The second scenario enabled the increase in the apparent amount of water volume that could have been recharged to the aquifers. The restored water could specify the optimum amount of water to be pumped from the wells for a sustainable water supply. The insertion of 50 wells in the area strategically increased the cumulative volume in recharge. After the simulation, the daily recharge increased from 168,857 m^3/day to 600,430 m^3/day. Table 6 shows the results of the implementation of artificial recharge. We simulated this following the cumulative recharged volume and the rate of the daily recharge.

Table 6: Volume Results After Artificial Recharge

No	Duration (Years)	State	Number of active reservoirs	Cumulative volume recharged m^3/year	Daily Recharge m^3/day
1	1	Steady	0	0	0
2	6	Transient	6	61 801 828	168 857
3	7	Transient	10	505 474 560	197 760
4	17	Transient	20	2 236 151 296	360 000
5	32	Transient	50	7 013 803 008	600 430

One of the significant identified sources where the water could be drawn for recharge is from the Little Kinyasungwe river flow. The river flows seasonally, especially during the rainy period. The data available from the Ministry of Water in Tanzania indicated a flow of 23.646 Million Cubic Meters (MCM) in 2007. The river discharges its water to the Hombolo dam that has 184 Million Cubic Meters (MCM). The dam is important as it provides water for various uses like fishing, irrigation and domestic water use. Due to the expanded surface area of water created by dams, enormous water volumes are lost to evaporation, much more than would have been lost in the dam's absence. Therefore, a portion of the water flowing in the Kinyasungwe river is recommended for the aquifers' artificial recharge. Also, implementers need to put a lot of efforts into artificially recharge the aquifers in the years that El Niño Southern Oscillation (ENSO) occurs as it contributes a lot of flow due to a huge amount of rainfall.

4.3.3. Third scenario

The third scenario involved the simulation of the pumping rate that can be done after artificial recharge. The results indicated that the recharged water could increase the aquifer yield to the extent that the groundwater's abstraction about the projected population meets the demand sustainably. For example, in the year 2051, the estimated existing water would be 113,192 m^3/day. The projected amount of water demanded was expected to be 418,839 m^3/day. The projected demand indicated a deficiency of 305,247 m^3/day. In the second scenario, we see from Table 5 that the injection of artificial wells has increased the recharge rate to 600,430 m^3/day. So, the simulated artificial recharge has given the estimated value of 600,430 m^3/day. This study carried out pumping rate simulations against recharge rate after artificial recharge, as illustrated in Figure 5. The pumping rate assignment indicates that the recharge rate is higher than the pumping rate's speed. For example, in the last stress period, the recharge rate was 0.6 million cubic meters per day while pumping was 0.42 million cubic meters per day.

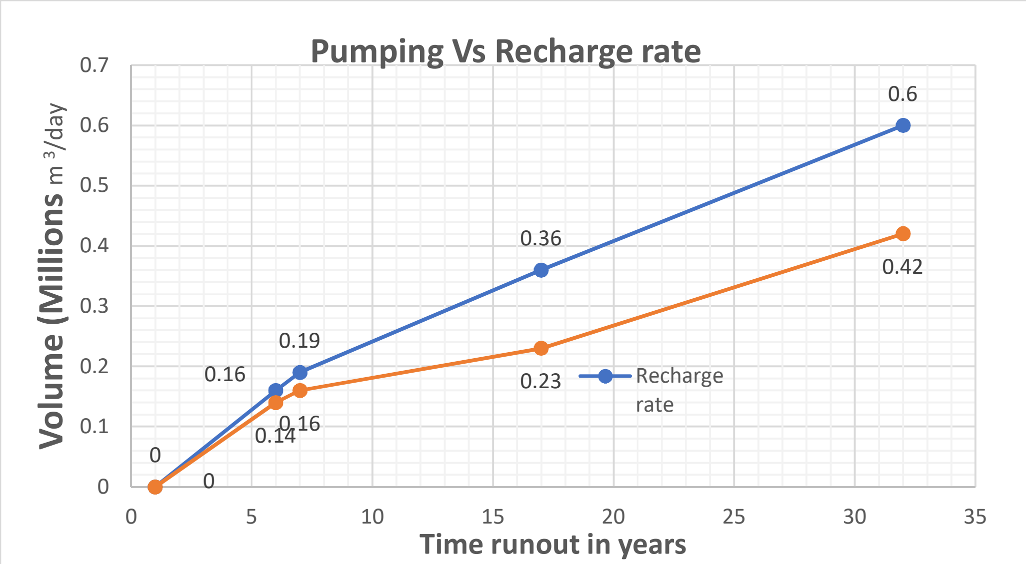

Figure 8: Recharge rate Versus the Pumping rate

Table 7 indicates the projected population as the City grows, the existing water, and the water demand. The deficiency is also shown. The values enabled the water simulation to be pumped from the aquifer after artificial recharge be carried out. The simulation results showed water surplus, which suggests that artificial wells are essential to replenish the aquifer depleted after increasing the population and other factors like climate change and variability. For example, Table 7 shows that in 2025, the projected water demand is 137,584 m^3/day, while the simulated amount is 168,857 m^3/day, resulting in a surplus of 31,273 m^3/day.

Table 7: City Population concerning Water Demand, Existing Water, Deficiency, Simulated and Surplus

	City Population	**Water demand** m^3/day	**Existing water** m^3/day	**Deficiency** m^3/day	**Simulated** m^3/day	**Surplus** m^3/day
2025	689 072	137 584	113 192	-23 992	168 857	31 273
2026	1 069 900	160 730	113 192	-47 138	197 760	37 030
2036	1 713 794	226 204	113 192	-113 012	360 000	133 796
2051	1 972 968	418 839	113 192	-305 247	600 430	181 591

4.4. Water budget of the groundwater at the end of the last stress period

Table 8 indicates the water budget at the end of the last stress period. The results show that artificial recharge mechanisms enhance aquifer sustainability. About 99.9% of the inflow is estimated to come from artificial wells. The aquifer's effective planning could support water production of up to 69.75% of the outflow from the planned 50 pumping wells. The rate of natural recharge alone is not enough to help keep the replenishment of the aquifer. The aquifer's sustainability is disturbed as the rate of natural recharge is meagre.

Table 8: Water Budget after Pumping the Water From 50 Pumping Wells

	Input		**Output**	
	m^3/day	**%**	m^3/day	**%**
Storage	16.0882	0.0027	177,418	29.5
Wells	600,000	99.9	418,819	69.75
Drains	0	0	4,178.2241	0.0796
Head Dependent Bounds	413.5103	0.07	0.2726	0.0000454
Recharge	1.0094	0.00017	0	0
Evapotranspiration	0	0	6.272	0.00104
Total	600,430.5625	100	600,422.1250	100

5. Conclusion

The relocation of government offices from Dar es Salaam to Dodoma has led to a surge in population, resulting in overexploitation of water in the Makutupora aquifer. The ability of the well to support the population's sustainability through water supply is likely to be overtaken by the abstraction. The sustainability of groundwater to meet the demand of the Cities that overgrows should be studied. Therefore, this study simulated the power and ability of artificial recharge and natural recharge to increase the aquifers' water for suitable water supply in Dodoma City and mitigate the shortage. Based on the climate data, the recharge rate and evapotranspiration rate were computed.

The groundwater flow model which was used is MODFLOW-2005 with the support of Modelmuse Graphical User Interface. The packages used for the model are RCH, WEL, EVT, DRN, HOB, and GHB packages. Both the steady-state and transient flow models were calibrated by systematically adjusting the parameters. The steady-state model was used as a reference for the transient, and it had a Root Mean Square Residue of 5.3.

The groundwater flow simulation results of artificial recharge indicate that artificial recharge mechanisms could enhance the Makutupora aquifer sustainability. The developed model might help the decision-makers to manage the groundwater flow in the Makutupora aquifer. However, data from the area (Makutupora Dodoma) were minimal, especially the evapotranspiration data. Furthermore, the areas stratigraphic units' complexity was a barrier to estimating the hydraulic conductivities. The results indicated a need for about 90% of the flow to replenish the aquifers.

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