Journal of Mathematics and Informatics
Vol. 16, 2019, 1-9
ISSN: 2349-0632 (P), 2349-0640 (online)
Published 24 March 2019
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/jmi.134av16a1

Journal of Mathematics and Informatics

Covering-Based Grade Rough Fuzzy Set Models

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Received 28 February 2019; accepted 19 March 2019

Abstract. Based on the discussion of the covering rough set models and the overlapping information between the set and the equivalence classed proposed in this paper, four kinds of covering grade rough fuzzy set models are defined and established by means of the minimum description of neighbor domain, whole neighborhood, rule confidence and membership of the elements. It unifies the results of predecessors. In addition, their properties and relationships are discussed. Finally, an example is given.

Keywords: Rough set, Grade rough set, Covering rough fuzzy set

AMS Mathematics Subject Classification (2010): 28E10, 04A72

1. Introduction

As a data analysis theory for studying uncertain or incomplete knowledge, the rough sets was originally proposed by Polish mathematician Pawlak in 1982 [1]. It has been widely applied in machine learning, knowledge discovery, data processing and so on. As is well known, equivalence relation or partition are the mathematical basis for the theory and play an important role in Pawlak rough set. However, such an equivalence relation is often difficult to obtain in the practical application. Based on this, the extended models of the classical rough set have been studied by many researchers. Grade rough set[2, 3] were proposed under this background. In the grade rough set, it considers the quantitative information which is the overlapping information between the set and the equivalence classed. It reflect absolute quantization information of the approximate space. On the other hand, generalizations of the classical rough set are to weaken the restrictive conditions when the equivalence relations is too strict for the sample classification. In 1983, Zakowski first relaxed the partition of the universe to a covering and established covering generalized rough sets by replacing partitions of a universe with its coverings [4]. Then, many scholars have studied the covering-based rough sets, rough fuzzy sets, fuzzy rough sets and other models and their applications [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. For example, based on the neighbor domain and whole neighborhood of minimum description of elements, Xu et al. [16] and Wei et al. [17] studied the covering rough fuzzy set models. Wang et al. [18] improved the work of the literatures [16, 17] by using the rule confidence, and analyzed the relationship among the above three models. Zhu et al. [19] established a new covering rough fuzzy set model by means of the fuzzy covering-based rough membership, which takes into account the elements and their membership in the minimum description. At the same time, we note that the fuzzy set theory and the rough set model have strong complementarity to each other as mathematical tools to describing and handling uncertain knowledge. Rough set theory is mainly used for knowledge discovery and decision-making in uncertain or incomplete information systems, the basis theorem of the Rough set theory is to approximate the research objects by using the determined upper and lower approximation. Particularly, this descriptive process doesn't need to preprocess the data or providing any priori knowledge. Different from the rough set theory, the fuzzy set theory is mainly used to describe and handle the problem of fuzzy information or concept. In addition, it often needs to rely on the prior knowledge, such as expert systems. As a generalization of the classical rough set theory, Dabois et al. proposed the concept of fuzzy rough sets and rough fuzzy sets in 1990, and their properties have been researched at the same time [20,21,22]. Based on the discussion of the covering rough set models and the overlapping information between the set and the equivalence classed proposed in this paper, four kinds of covering grade rough fuzzy set models are defined and established by means of the minimum description of neighbor domain, whole neighborhood, rule confidence and membership of the elements. It unifies the results of predecessors. In addition, their properties and relationships are discussed. Finally, an example is given.

2. Preliminaries

Let U be a finite and non-empty set called the universe, $C=\{X \mid X \subseteq U\}$ be a family of subsets of U. If no element of C is empty and $\cup_{X\in C} X = U$, then C is called a covering of U. The ordered pair (U,C) is called a covering approximation space. For $x\in U$, $Md(x)=\{K\in C \mid x\in K \wedge (\forall S\in C \wedge x\in S \wedge S\subseteq K \Rightarrow K=S)\}$ is called the minimal description of x [4, 11]; The whole description of x in the approximation space is $Ad_C(x)=\{K\in C \mid x\in K\}$, and denoted by $Ad(x)$; For $x\in U$, $K\in C$, if $K\in Md(x)$, then called K is the neighbourhood of x; And the $\cup\{K \mid K\in Md(x)\}$ is called a whole neighbourhood of x, while denoted by $AN(x)$; Corresponding, the $\cap\{K \mid K\in Md(x)\}$ is called the neighbor domain of x, and denoted by $CN(x)$.

After more than 30 years of research and development and according to the different between the neighbourhood of the object x and the background of the problem. In general, the covering rough fuzzy set models existed the following four different definitions.

Definition 1. *Let (U,C) be a covering approximation space, the set of all fuzzy sets in the universe U is denoted by $F(U)$. For the fuzzy set $\tilde{A}\in F(U)$, we have four models as follows:*

(**I-type covering-based rough fuzzy set model**)[16] The upper and lower approximate $(\overline{CF}(\tilde{A}),\underline{CF}(\tilde{A}))$ of $\tilde{A}$ in (U,C) is a pair of fuzzy sets, the membership functions of them are defined as follows:

$$\overline{CF}(\tilde{A})(x) = \sup\{\tilde{A}(y) \mid y \in \cup K, K \in Md(x)\} = \sup\{\tilde{A}(y) \mid y \in AN(x)\},$$
$$\underline{CF}(\tilde{A})(x) = \inf\{\tilde{A}(y) \mid y \in \cup K, K \in Md(x)\} = \inf\{\tilde{A}(y) \mid y \in AN(x)\}.$$

(**II-type covering-based rough fuzzy set model**)[17] The upper and lower approximate $(\overline{CS}(\tilde{A}), \underline{CS}(\tilde{A}))$ of $\tilde{A}$ in (U, C) is a pair of fuzzy sets, the membership functions of them are defined as follows:

$$\overline{CS}(\tilde{A})(x) = \sup\{\tilde{A}(y) \mid y \in \cap K, K \in Md(x)\} = \sup\{\tilde{A}(y) \mid y \in CN(x)\},$$

(**III-type covering-based rough fuzzy set model**)[18] The upper and lower approximate $(\overline{CT}(\tilde{A}), \underline{CT}(\tilde{A}))$ of $\tilde{A}$ in (U, C) is a pair of fuzzy sets, the membership functions of them are defined as follows:

$$\overline{CT}(\tilde{A})(x) = \inf_{K \in Md(x)} \{\sup_{y \in K}\{\tilde{A}(y)\}\},$$
$$\underline{CT}(\tilde{A})(x) = \sup_{K \in Md(x)} \{\inf_{y \in K}\{\tilde{A}(y)\}\}.$$

(**IV-type covering-based rough fuzzy set model**)[19] The upper and lower approximate $(\overline{CH}(\tilde{A}), \underline{CH}(\tilde{A}))$ of $\tilde{A}$ in (U, C) is a pair of fuzzy sets, the membership functions of them are defined as follows:

$$\overline{CH}(\tilde{A})(x) = \max\{\tilde{A}(x), \tilde{A}'(x)\}$$
$$\underline{CH}(\tilde{A})(x) = \min\{\tilde{A}(x), \tilde{A}'(x)\},$$

where $\tilde{A}'(x)$ is the fuzzy covering-based rough membership of the object x with respect to $\tilde{A}$, which is defined as follow:

$$\tilde{A}'(x) = \frac{\sum_{y \in (\cup Md(x))} \tilde{A}(y)}{|\cup Md(x)|}.$$

It is known from the commonality of the definition above, that the fuzzy set $\tilde{A}$ is approached by two fuzzy sets in the covering approximation space, that is, upper approximate and lower approximate. Due to the indistinguishability of knowledge C, the degree of membership for any objects in the universe belonging to the fuzzy set $\tilde{A}$ are between the lower approximation and the upper approximation. However, the membership for upper approximation and lower approximation in the I-type covering-based rough fuzzy set model, are defined by the maximum value and the minimum value of the membership in the whole neighborhood of an object, respectively; The membership for upper approximation and lower approximation in the II-type covering-based rough fuzzy set model, are defined by the maximum value and the minimum value of the membership in the neighbor domain of an object, respectively. Obviously, the II-type covering-based rough fuzzy set model reduces the non discernible set of an object; Compared with the I-type and II-type covering-based rough fuzzy set models, III-type covering-based rough fuzzy set model is between the I-type and the II-type covering-based rough fuzzy set models, and the problem of the first two models are corrected which are uncertain separation in the upper approximation and non approximate separation in the lower approximation. I-type and II-type are the extreme states of III-type covering-based rough fuzzy set models respectively, denote as $\underline{CF}(\tilde{A}) \subseteq \underline{CT}(\tilde{A})(x) \subseteq \underline{CS}(\tilde{A})(x) \subseteq \tilde{A} \subseteq \overline{CS}(\tilde{A}) \subseteq \overline{CT}(\tilde{A}) \subseteq \overline{CF}(\tilde{A})$;

IV-Type covering-based rough fuzzy set model establishes the relationship between the coverage C and the membership of element in the fuzzy set $\tilde{A}$, which is using the fuzzy covering-based rough membership. It not only reflects the relationship between the elements and their minimum description, but also fuses the degree of membership of the elements in a given fuzzy set and its minimum description. At the same time, we have $\underline{CF}\tilde{A}) \subseteq \underline{CH}(\tilde{A}) \subseteq \tilde{A} \subseteq \overline{CH}(\tilde{A}) \subseteq \overline{CF}(\tilde{A})$.

3. Covering-based grade rough fuzzy set models

Definition 2. *Let (U,R) be a Pawlak approximation space, X be a nonempty subsets of U, k be a non-negative integer. Then the degree k upper and lower approximations of X with respect to the Pawlak approximation space (U,R) are defined as follows:*

$$\overline{R}_k X = \{x \in U \,\|\, [x]_R \cap X | > k\},$$
$$\underline{R}_k X = \{x \in U \,\|\, [x]_R | - |[x]_R \cap X | \le k\}.$$

Definition 3. *Let (U,C) be a covering approximation space, the set of all fuzzy sets in the universe U is denoted by $F(U)$. For the fuzzy set $\tilde{A} \in F(U)$, we have four models as follows:*

(**I-type covering-based grade rough fuzzy set model**) The upper and lower approximate $(\overline{CF}_k(\tilde{A}), \underline{CF}_k(\tilde{A}))$ of $\tilde{A}$ on U is a pair of fuzzy sets, the membership function of them are defined as follows:

$$\overline{CF}_k \tilde{A}(x) = \bigvee_{y \in (\cup Md(x))} \{\tilde{A}(y) \,|\, \sum_{y \in (\cup Md(x))} \tilde{A}(y) > k\},$$
$$\underline{CF}_k \tilde{A}(x) = \bigwedge_{y \in (\cup Md(x))} \{\tilde{A}(y) \,\|\, \cup Md(x)| - \sum_{y \in (\cup Md(x))} \tilde{A}(y) \le k\}.$$

(**II-type covering-based grade rough fuzzy set model**) The upper and lower approximate $(\overline{CS}_k(\tilde{A}), \underline{CS}_k(\tilde{A}))$ of $\tilde{A}$ on U is a pair of fuzzy sets, the membership function of them are defined as follows:

$$\overline{CS}_k \tilde{A}(x) = \bigvee_{y \in (\cap Md(x))} \{\tilde{A}(y) \,|\, \sum_{y \in (\cap Md(x))} \tilde{A}(y) > k\},$$
$$\underline{CS}_k \tilde{A}(x) = \bigwedge_{y \in (\cap Md(x))} \{\tilde{A}(y) \,\|\, \cap Md(x)| - \sum_{y \in (\cap Md(x))} \tilde{A}(y) \le k\}.$$

(**III-type covering-based grade rough fuzzy set model**) The upper and lower approximate $(\overline{CT}_k(\tilde{A}), \underline{CT}_k(\tilde{A}))$ of $\tilde{A}$ on U is a pair of fuzzy sets, the membership function of them are defined as follows:

$$\overline{CT}_k\tilde{A}(x) = \bigwedge_{K\in Md(x)} \{\tilde{A}(y) \mid \sum_{y\in K}\sup \tilde{A}(y) > k\},$$

$$\underline{CT}_k\tilde{A}(x) = \bigvee_{K\in Md(x)} \{\tilde{A}(y) \| K| - \sum_{y\in K}\inf \tilde{A}(y) \le k\}.$$

(**IV-type covering-based grade rough fuzzy set model**) For the different of neighbor domain and whole neighborhood of x, the upper and lower approximate of $\tilde{A}$ on (U,C) have a couple of fuzzy sets of the following two forms. In this discussion, we used $(\overline{CH}_k(\tilde{A}),\underline{CH}_k(\tilde{A}))$ to represent this pair of fuzzy sets for simplicity, the membership function of them are defined as follows:

(1) $$\overline{CH}_k^{\cup}\tilde{A}(x) = \bigvee_{y\in(\cup Md(x))} \{\tilde{A}(y) \mid \sum_{y\in(\cup Md(x))} \min(\tilde{A}(y),\tilde{A}'(y)) > k\},$$

$$\underline{CH}_k^{\cup}\tilde{A}(x) = \bigwedge_{y\in(\cup Md(x))} \{\tilde{A}(y) \| \cup Md(x)| - \sum_{y\in(\cup Md(x))} \max(\tilde{A}(y),\tilde{A}'(y)) \le k\}.$$

(2) $$\overline{CH}_k^{\cap}\tilde{A}(x) = \bigvee_{y\in(\cap Md(x))} \{\tilde{A}(y) \mid \sum_{y\in(\cap Md(x))} \min(\tilde{A}(y),\tilde{A}''(y)) > k\},$$

$$\underline{CH}_k^{\cap}\tilde{A}(x) = \bigwedge_{y\in(\cap Md(x))} \{\tilde{A}(y) \| \cap Md(x)| - \sum_{y\in(\cap Md(x))} \max(\tilde{A}(y),\tilde{A}''(y)) \le k\}.$$

where $\tilde{A}'(x)$ and $\tilde{A}''(x)$ are the fuzzy covering-based rough membership of the object x with respect to $\tilde{A}$, respectively.

In the covering approximation space, the fuzzy set $\tilde{A}$ is approached by the degree k upper and lower approximation sets. If the upper approximation and the lower approximation are equal, then $\tilde{A}$ is said to be exact with respect to the covering approximation space (U,C), otherwise, $\tilde{A}$ is said to be rough. For convenience of description, we using $(\overline{C}_k(\tilde{A}),\underline{C}_k(\tilde{A}))$ to denote $(\overline{CF}_k(\tilde{A}),\underline{CF}_k(\tilde{A}))$;

or $(\overline{CS}_k(\tilde{A}),\underline{CS}_k(\tilde{A}))$; or $(\overline{CT}_k(\tilde{A}),\underline{CT}_k(\tilde{A}))$; or $(\overline{CH}_k(\tilde{A}),\underline{CH}_k(\tilde{A}))$ respectively.

Theorem 1. *Let* (U,C) *be a covering approximation space,* $\tilde{A},\tilde{B}\in F(U)$, k,l *be two non-negative integers. Then the covering-based grade rough fuzzy set models have the following properties:*

(1) $\overline{C}_k(\emptyset) = \emptyset$, $\underline{C}_k(U) = U$;

(2) $\tilde{A}\subseteq\tilde{B} \Rightarrow \underline{C}_k(\tilde{A})\subseteq\underline{C}_k(\tilde{B})$, $\overline{C}_k(\tilde{A})\subseteq\overline{C}_k(\tilde{B})$;

(3) $\underline{C}_k(\tilde{A}\cap\tilde{B})\subseteq\underline{C}_k(\tilde{A})\cap\underline{C}_k(\tilde{B})$, $\overline{C}_k(\tilde{A}\cap\tilde{B})\subseteq\overline{C}_k(\tilde{A})\cap\overline{C}_k(\tilde{B})$;

(4) $\underline{C}_k(\tilde{A}\cup\tilde{B})\supseteq\underline{C}_k(\tilde{A})\cup\underline{C}_k(\tilde{B})$, $\overline{C}_k(\tilde{A}\cup\tilde{B})\supseteq\overline{C}_k(\tilde{A})\cup\overline{C}_k(\tilde{B})$;

(5) $k\ge l \Rightarrow \underline{C}_l(\tilde{A})\subseteq\underline{C}_k(\tilde{A})$, $\overline{C}_k(\tilde{A})\subseteq\overline{C}_l(\tilde{A})$;

(6) $\underline{C}_k(\tilde{A}) = \sim \overline{C}_k(\sim \tilde{A}), \overline{C}_k(\tilde{A}) = \sim \underline{C}_k(\sim \tilde{A})$.

Proof: Taking the I-type covering-based grade rough fuzzy set model as example: the properties (1)-(5) are obvious, we will only prove (6).

(6) $\sim \overline{CF}_k(\sim \tilde{A}) = 1 - \bigvee_{y\in(\cup Md(x))} \{1-\tilde{A}(y) \mid \sum_{y\in(\cup Md(x))} 1-\tilde{A}(y) > k\}$

$$= \bigwedge_{y\in(\cup Md(x))} \{1-\tilde{A}(y) \mid \sum_{y\in(\cup Md(x))} 1-\tilde{A}(y) \le k\}$$

$$= \bigwedge_{y\in(\cup Md(x))} \{\tilde{A}(y) \| \cup Md(x)| - \sum_{y\in(\cup Md(x))} \tilde{A}(y) \le k\}$$

$$= \underline{CF}_k \tilde{A}$$

It follows that $\underline{C}_k(\tilde{A}) = \sim \overline{C}_k(\sim \tilde{A})$.

Similarly, $\overline{C}_k(\tilde{A}) = \sim \underline{C}_k(\sim \tilde{A})$ can be proved easily.

Theorem 2. *Let (U, C) be a covering approximation space. I-type and II-type covering-based grade rough fuzzy set models are equivalent, if and only if, for any $x \in U$, C is an unary coverage.*

Theorem 3. *Let (U, C) be a covering approximation space. If $\tilde{A} \in F(U)$, k is an non-negative integer, then*

$$\underline{CF}_k(\tilde{A}) \subseteq \underline{CH}_k^{\cup}(\tilde{A}), \overline{CH}_k^{\cup}(\tilde{A}) \subseteq \overline{CF}_k(\tilde{A}),$$

$$\underline{CS}_k(\tilde{A}) \subseteq \underline{CH}_k^{\cap}(\tilde{A}), \overline{CH}_k^{\cap}(\tilde{A}) \subseteq \overline{CS}_k(\tilde{A}).$$

Proof: Suppose $m = \inf\{\tilde{A}(y) \mid y \in \cup Md(x)\}$, $M = \sup\{\tilde{A}(y) \mid y \in \cup Md(x)\}$. For any $x \in U$, $y \in (\cup Md(x))$, we have $m \le \tilde{A}(x) \le M$. In addition, by the fuzzy covering-based rough membership, we can obtain $m \le \tilde{A}'(x) \le M$.

(1) If $\tilde{A}'(x) \le \tilde{A}(x)$, then we have

$$\overline{CH}_k^{\cup} \tilde{A}(x) = \bigvee_{y\in(\cup Md(x))} \{\tilde{A}(y) \mid \sum_{y\in(\cup Md(x))} \min(\tilde{A}(y), \tilde{A}'(y)) > k\}$$

$$= \bigvee_{y\in(\cup Md(x))} \{\tilde{A}(y) \mid \sum_{y\in(\cup Md(x))} \min \tilde{A}'(y) > k\}$$

$$\le \bigvee_{y\in(\cup Md(x))} \{\tilde{A}(y) \mid \sum_{y\in(\cup Md(x))} \tilde{A}(y) > k\} = \overline{CF}_k \tilde{A}(x)$$

Hence $\overline{CH}_k^{\cup}(\tilde{A}) \subseteq \overline{CF}_k(\tilde{A})$.

(2) If $\tilde{A}(x) \le \tilde{A}'(x)$, then we can obtain

$$\underline{CH}_k^{\cup} \tilde{A}(x) = \bigwedge_{y\in(\cup Md(x))} \{\tilde{A}(y) \| \cup Md(x)| - \sum_{y\in(\cup Md(x))} \max(\tilde{A}(y), \tilde{A}'(y)) \le k\}$$

$$= \bigwedge_{y\in(\cup Md(x))} \{\tilde{A}(y) \| \cup Md(x)| - \sum_{y\in(\cup Md(x))} \max \tilde{A}'(y) \le k\}$$

$$\ge \bigwedge_{y\in(\cup Md(x))} \{\tilde{A}(y) \| \cup Md(x)| - \sum_{y\in(\cup Md(x))} \tilde{A}(y) \le k\} = \underline{CF}_k \tilde{A}(x)$$

It follows that $\underline{CF}_k(\tilde{A}) \subseteq \underline{CH}_k^{\cup}(\tilde{A})$.

In summary, we have

$$\underline{CF}_k(\tilde{A}) \subseteq \underline{CH}_k^{\cup}(\tilde{A}), \overline{CH}_k^{\cup}(\tilde{A}) \subseteq \overline{CF}_k(\tilde{A}).$$

Similarly, $\underline{CS}_k(\tilde{A}) \subseteq \underline{CH}_k^{\cap}(\tilde{A}), \overline{CH}_k^{\cap}(\tilde{A}) \subseteq \overline{CS}_k(\tilde{A})$ can be proved easily.

4. Illustrative example

Suppose the universe $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $C = \{K_1, K_2, K_3, K_4, K_5\}$ constructs a covering of U, where the

$K_1 = \{x_1, x_3, x_4\}, K_2 = \{x_1, x_3\}, K_3 = \{x_2, x_6\}, K_4 = \{x_3, x_4, x_5\}, K_5 = \{x_2, x_5\}$,

$\tilde{A} = \{\frac{0.85}{x_1}, \frac{0.3}{x_2}, \frac{0.9}{x_3}, \frac{0.8}{x_4}, \frac{0.5}{x_5}, \frac{0}{x_6}\}$ is a fuzzy set.

Firstly, according to definition of the coverage, the minimal description of every object in U can be computed as follows:

$$Md(x_1) = \{\{x_1, x_3, x_4\}, \{x_1, x_3\}\};$$
$$Md(x_2) = \{\{x_2, x_6\}, \{x_2, x_5\}\};$$
$$Md(x_3) = \{\{x_1, x_3, x_4\}, \{x_1, x_3\}, \{x_3, x_4, x_5\}\};$$
$$Md(x_4) = \{\{x_1, x_3, x_4\}, \{x_3, x_4, x_5\}\};$$
$$Md(x_5) = \{\{x_3, x_4, x_5\}, \{x_2, x_5\}\};$$
$$Md(x_6) = \{\{x_2, x_6\}\}$$

Then, due to the definition of the fuzzy covering-based rough membership of the object x with respect to $\tilde{A}$ we have

$$\tilde{A}' = \{\frac{0.85}{x_1}, \frac{0.27}{x_2}, \frac{0.76}{x_3}, \frac{0.76}{x_4}, \frac{0.63}{x_5}, \frac{0.15}{x_6}\}, \quad \tilde{A}'' = \{\frac{0.88}{x_1}, \frac{0.3}{x_2}, \frac{0.9}{x_3}, \frac{0.85}{x_4}, \frac{0.5}{x_5}, \frac{0.15}{x_6}\}.$$

Let $k = 1$, then by the *Definition 3*, we can obtain the upper and lower approximation, negative region, boundary region of covering-based grade rough fuzzy set models with respect to the fuzzy set $\tilde{A}$ as follows:

$$\underline{CF}_k \tilde{A} = \{\frac{0.8}{x_1}, \frac{0}{x_2}, \frac{0.5}{x_3}, \frac{0.5}{x_4}, \frac{0}{x_5}, \frac{0}{x_6}\}, \ BnCF_k \tilde{A} = \{\frac{0.2}{x_1}, \frac{0}{x_2}, \frac{0.5}{x_3}, \frac{0.5}{x_4}, \frac{0.9}{x_5}, \frac{0}{x_6}\},$$

$$\overline{CF}_k \tilde{A} = \{\frac{0.9}{x_1}, \frac{0}{x_2}, \frac{0.9}{x_3}, \frac{0.9}{x_4}, \frac{0.9}{x_5}, \frac{0}{x_6}\}, \ NegCF_k \tilde{A} = \{\frac{0.1}{x_1}, \frac{1}{x_2}, \frac{0.1}{x_3}, \frac{0.1}{x_4}, \frac{0.1}{x_5}, \frac{1}{x_6}\}.$$

$$\underline{CS}_k\tilde{A}=\{\frac{0.85}{x_1},\frac{0.3}{x_2},\frac{0.9}{x_3},\frac{0.8}{x_4},\frac{0.5}{x_5},\frac{0}{x_6}\},\quad BnCS_k\tilde{A}=\{\frac{0.15}{x_1},\frac{0}{x_2},\frac{0}{x_3},\frac{0.2}{x_4},\frac{0}{x_5},\frac{0}{x_6}\},$$

$$\overline{CS}_k\tilde{A}=\{\frac{0.9}{x_1},\frac{0}{x_2},\frac{0}{x_3},\frac{0.9}{x_4},\frac{0}{x_5},\frac{0}{x_6}\},\quad NegCS_k\tilde{A}=\{\frac{0.1}{x_1},\frac{1}{x_2},\frac{1}{x_3},\frac{0.1}{x_4},\frac{1}{x_5},\frac{1}{x_6}\}.$$

$$\underline{CT}_k\tilde{A}=\{\frac{0.85}{x_1},\frac{0}{x_2},\frac{0.85}{x_3},\frac{0.8}{x_4},\frac{0}{x_5},\frac{0}{x_6}\},\quad BnCT_k\tilde{A}=\{\frac{0.15}{x_1},\frac{0}{x_2},\frac{0.15}{x_3},\frac{0.2}{x_4},\frac{0.5}{x_5},\frac{0}{x_6}\},$$

$$\overline{CT}_k\tilde{A}=\{\frac{0.9}{x_1},\frac{0}{x_2},\frac{0.9}{x_3},\frac{0.9}{x_4},\frac{0.5}{x_5},\frac{0}{x_6}\},\quad NegCT_k\tilde{A}=\{\frac{0.1}{x_1},\frac{1}{x_2},\frac{0.1}{x_3},\frac{0.1}{x_4},\frac{0.5}{x_5},\frac{1}{x_6}\}.$$

$$\underline{CH}_k^{\cup}\tilde{A}=\{\frac{0.8}{x_1},\frac{0}{x_2},\frac{0.63}{x_3},\frac{0.63}{x_4},\frac{0}{x_5},\frac{0}{x_6}\},\quad BnCH_k^{\cup}\tilde{A}=\{\frac{0.2}{x_1},\frac{0}{x_2},\frac{0.37}{x_3},\frac{0.37}{x_4},\frac{0.76}{x_5},\frac{0}{x_6}\},$$

$$\overline{CH}_k^{\cup}\tilde{A}=\{\frac{0.85}{x_1},\frac{0}{x_2},\frac{0.85}{x_3},\frac{0.85}{x_4},\frac{0.76}{x_5},\frac{0}{x_6}\},\quad NegCH_k^{\cup}\tilde{A}=\{\frac{0.15}{x_1},\frac{1}{x_2},\frac{0.15}{x_3},\frac{0.15}{x_4},\frac{0.24}{x_5},\frac{1}{x_6}\}.$$

$$\underline{CH}_k^{\cap}\tilde{A}=\{\frac{0.88}{x_1},\frac{0.3}{x_2},\frac{0.9}{x_3},\frac{0.85}{x_4},\frac{0.5}{x_5},\frac{0}{x_6}\},\quad \overline{CH}_k^{\cap}\tilde{A}=\{\frac{0.9}{x_1},\frac{0}{x_2},\frac{0}{x_3},\frac{0.9}{x_4},\frac{0}{x_5},\frac{0}{x_6}\},$$

$$BnCH_k^{\cap}\tilde{A}=\{\frac{0.12}{x_1},\frac{0}{x_2},\frac{0}{x_3},\frac{0.15}{x_4},\frac{0}{x_5},\frac{0}{x_6}\},\quad NegCH_k^{\cap}\tilde{A}=\{\frac{0.1}{x_1},\frac{1}{x_2},\frac{1}{x_3},\frac{0.1}{x_4},\frac{1}{x_5},\frac{1}{x_6}\}.$$

The approximate classification quality and rough degree of covering-based grade rough fuzzy set models with respect to the fuzzy set $\tilde{A}$ are $(30.0\%,50.0\%)$, $(55.8\%,86.1\%)$, $(41.7\%,21.9\%)$, $((34.3\%,37.8\%)^{\cup},(56.7\%,88.9\%)^{\cap})$, respectively.

By the Example, we have that $Md(x_6)=\{\{x_2,x_6\}\}$ is an unary coverage and $|Md(x_6)|=1, |\cup Md(x_6)|=|\cap Md(x_6)|=2,$ then

$$P(\cup Md(x_6),\tilde{A})=P(\cap Md(x_6),\tilde{A})\neq P(K,\tilde{A}),$$

so the relative error classification rate of object x_6 is equal in the I-type and II-type covering-based grade rough fuzzy set models. At the same time, it is obvious that $\underline{CF}_k(\tilde{A})\subseteq \underline{CH}_k^{\cup}(\tilde{A}),\overline{CH}_k^{\cup}(\tilde{A})\subseteq \overline{CF}_k(\tilde{A}),\ \underline{CS}_k(\tilde{A})\subseteq \underline{CH}_k^{\cap}(\tilde{A}),\overline{CH}_k^{\cap}(\tilde{A})\subseteq \overline{CS}_k(\tilde{A})$ are also hold.

Acknowledgements. This research was supported by the National Natural Science Fund of China (61763044).

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