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On θg*-Closed Sets in Topological Spaces

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Abstract. In this paper, we have introduce a new class of closed sets in topological spaces called θ-generalized star closed set (briefly θg*-closed set) and study some of its properties. Further we introduce the the concept of θg*-continuous functions, θg*-irresolute functions and contra θg*-continuous functions and study the relationship between other existing functions in topological spaces. Also we investigate the composition of the functions between θg*-continuous functions and between continuous and contra θg*-continuous functions and between θg*-continuous functions and θg*-irresolute functions. Moreover, we introduce the application of θg^*-closed sets as three spaces namely, ${}_{\theta}T_{1/2}{}^{*}$ spaces, ${}^{*}{}_{\theta}T_{1/2}$ spaces, ${}_{\theta}T_{1/2}{}^{**}$ spaces in topological spaces and are analyzed.)

Keywords: θg*-closed sets, θg*-continuous functions, θg*-irresolute functions, contra θg*-continuous functions, ${}_{\theta}T_{1/2}{}^{*}$ - space, ${}^{*}{}_{\theta}T_{1/2}$ - space, ${}_{\theta}T_{1/2}{}^{**}$ -space.

AMS Mathematics Subject Classification (2010): 54A05

1. Introduction

The first step of generalized closed sets introduced by Levine [16] in the year of 1970. Velicko [34] defined two subclasses of closed sets namely, δ - closed sets and θ - closed sets in 1968. Levine [16], Mashhour et.al. [21] and Njastad [23] introduced semi-open sets, pre-open sets, α -sets and β -sets respectively. Dontchev, Gnanambal [12] and Palaniappan and Rao [25] are introduced a sets namely gsp -closed sets, gpr -closed sets and rg -closed sets respectively. Veerakumar [33] introduced a new class of sets called g^* -closed sets, which is properly placed in between the class of closed sets and the class of g -closed sets. Arya and Nour [1] are define a set namely, gs -closed sets in 1990. Dontchev and Ganster were introduced semi-generalized closed sets, generalized semi-closed sets, α -generalized closed sets, generalized α -closed sets and respectively.

Dontchev and Maki [8] are introduced θ-generalized closed sets in topological spaces. Sarasak and Rajesh [26] introduced by π -generalized semi-pre closed sets. Park

[24] introduced πgp -closed sets in topological spaces. Dontchev, Noiri [7], Quasi Normal spaces and πg -closed sets are introduced. Aslin, Caksu Guler and Noiri [3] introduced πgs -closed sets in topological spaces.

Balachandran, Sundaram and Maki were introduced generalized continuous functions [4] in the year of 1991. Dontchev [9] introduce a contra continuous functions in 1996. Dontchev and Maki are introduced θg - continuous functions [8] in the year of 1999. Fomin [11] introduced θ-continuous functions in 1943. Veerakumar [30] introduce a new class of sets called g^* - continuous functions in topological spaces.

In this paper, we introduce the new class of sets namely, θg^*-closed sets in topological spaces and study some basic properties. Also, we study the application of ${}_\theta T_{1/2}{}^*$-space, ${}_\theta T_{1/2}{}^{**}$- space and ${}^*_\theta T_{1/2}$ - space. Further, we introduce θg^*-continuous functions and θg^*-irresolute functions and study the relationships of existing functions. Moreover we introduce a new generalization of contra-continuity called contra θg^* -continuous functions.

2. Preliminaries

We recall the following definitions, which are the useful in the sequel.

Definition 2.1. *A subset* A *of a space* (X,τ) *is called*

- a semi-closed set[16] if $int(cl(A)) \subseteq A$.
- a pre-closed set[21] if $cl(int(A)) \subseteq A$.
- a α -closed set[18] if $cl(int(cl(A))) \subseteq A$.
- a semi-pre closed[2] (= β -closed) if $int(cl(int(A))) \subseteq A$.
- a r -closed set[27] if $A = cl(int(A))$.
- a π -closed set[35] if A is the union of regular closed sets.
- a θ-closed set[34] if $A = cl_\theta(A)$,

where $cl_\theta(A) = x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau and\ x \in Uz$.

Definition 2.2. *A subset* A *of a space* (X,τ) *is called*

- a generalized closed [17] (briefly g -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- a semi generalized closed [5] (briefly sg -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.
- a generalized semi closed [33] (briefly gs -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
- a generalized α -closed [20] (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α - open in X.
- a α generalized closed [18] (briefly αg -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

- a regular generalized closed [25] (briefly rg -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- a generalized pre-closed [19] (briefly gp -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- a generalized star closed [33] (briefly g^* -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- a generalized star semi closed [30] (briefly g^*s -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X .
- a generalized pre-regular closed [12] (briefly gpr -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- a weakly generalized closed [22] (briefly wg -closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- a regular weakly generalized closed [22] (briefly rwg -closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .
- a π -generalized closed [7] (briefly πg -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π open in X .
- a π -generalized α closed [15] (briefly $\pi g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π open in X .
- a π -generalized β - closed [26] (briefly $\pi g\beta$ -closed) if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π open in X .
- a π -generalized pre-closed [24] (briefly πgp -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π open in X .
- a π -generalized semi-closed [3] (briefly πgs -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is π open in X .
- a θ-generalized closed [8] (briefly θg -closed) if $cl_{\theta}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- a weakly- closed [28] (briefly w -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- a semi weakly generalized-closed[22] (briefly swg -closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- a $g^{\#}s$ - closed [31] (briefly $g^{\#}s$ -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg -open in X .
- a ψ -closed [31] (briefly ψ -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg -open in X .

Definition 2.3. *A function* $f:(X,\tau) \rightarrow (Y,\sigma)$ *from a topological space* X *into a topological space* Y *is called*

- continuous[16] if $f^{-1}(V)$ is a closed in X for every closed set V of Y.
- r-continuous[27] if $f^{-1}(V)$ is a r-closed in X for every closed set V of Y.
- π-continuous[24] if $f^{-1}(V)$ is a π-closed in X for every closed set V of Y.
- πgr-continuous[14] if $f^{-1}(V)$ is a πgr-closed in X for every closed set V of Y.
- πg-continuous[7] if $f^{-1}(V)$ is a πg-closed in X for every closed set V of Y.
- $\pi g\beta$-continuous[19] if $f^{-1}(V)$ is a $\pi g\beta$-closed in X for every closed set V of Y.
- gp-continuous[17] if $f^{-1}(V)$ is a gp-closed in X for every closed set V of Y.
- gs-continuous[33] if $f^{-1}(V)$ is a gs-closed in X for every closed set V of Y
- gpr-continuous[12] if $f^{-1}(V)$ is a gpr-closed in X for every closed set V of Y.
- πgs-continuous[3] if $f^{-1}(V)$ is a πgs-closed in X for every closed set V of Y.

Definition 2.4. *A function $f:(X,\tau)\to(Y,\sigma)$ from a topological space X into a topological space Y is called g^*-irresolute [4] if $f^{-1}(V)$ is a g^*-closed in X for every g^*-closed set V of Y.*

Definition 2.5. *A function $f:(X,\tau)\to(Y,\sigma)$ from a topological space X into a topological space Y is called contra-continuous [9] if $f^{-1}(V)$ is a closed in X for every open set V of Y, contra α - continuous [13] if $f^{-1}(V)$ is a α-closed in X for every open set V of Y.*

Definition 2.6. *A space (X,τ) is called a*

1. T_b-space[6] if every gs-closed set in it is closed.
2. $T_{1/2}$-space[10] if every g-closed set in it is closed.
3. ${}_\alpha T_d$-space[18] if every αg-closed set in it is g-closed.
4. T_d-space[5] if every gs-closed set in it is g-closed.
5. $T_{1/2}{}^*$-space[30] if every g^*-closed set in it is closed.

Lemma 2.7. *If A and B are subsets of a topological space (X,τ), then* $cl_\theta(A\cup B)=cl_\theta(A)\cup cl_\theta(B)$ *and* $cl_\theta(A\cap B)=cl_\theta(A)\cap cl_\theta(B)$.

3. θg^*-closed sets

In this chapter, we introduce and study the notion of θg^*-closed sets in topological spaces and obtain some of its basic properties.

Definition 3.1. *A subset A of a topological space (X,τ) is called θg^*- closed set if*

$cl_\theta(A) \subseteq U$, *whenever* $A \subseteq U$ *and* U *is* g*-open in* (X,τ).

Theorem 3.2. *Every* r*-closed set is* θg^**-closed but not conversely.*

Proof: Suppose that A be a r-closed set in X. Let U be a g-open set such that $A \subseteq U$. Since A is r-closed, then we have $rcl(A) = A \subseteq U$. But, $cl_\theta(A) \subseteq rcl(A) \subseteq U$. Therefore $cl_\theta(A) \subseteq U$. Hence A is a θg^*-closed set.

Example 3.3. *Let* $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$, r*-closed=* $\{X, \phi, \{b,c\}, \{a,c\}\}$ *and* θg^**-closed set=* $\{X, \phi, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$. *Let* $A = \{a\}$. *Then the subset* A *is* θg^**-closed but not a* r*-closed set.*

Example 3.4. *Let* $X = \{a,b,c\}$, $\tau = \{X, \phi, \{b\}, \{a,b\}\}$, θg^**-closed=* $\{X, \phi, \{c\}, \{b,c\}, \{a,c\}\}$, rg*-closed,* πg*-closed,* $\pi g\alpha$*-closed,* πgp*-closed,* πgs*-closed,* $\pi g\beta$*-closed,* $rg\beta$*-closed,* gpr*-closed, and* αgr *closed set=* $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$. *Let* $A = \{a\}$. *Then the subset* A *is* rg*-closed,* πg*-closed,* $\pi g\alpha$*-closed,* πgp*-closed,* πgs*-closed,* $\pi g\beta$*-closed,* $rg\beta$*-closed,* gpr*-closed,* αgr*-closed but not* θg^**-closed.*

Example 3.5. *Let* $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a\}\}$, θg^**-closed=* $\{X, \phi, \{b,c\}\}$, gp *and* gs*-closed=* $\{X, \phi, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$. *Let* $A = \{a,b\}$. *Then the subset* A *is* gp*-closed and* gs*-closed but not* θg^**-closed.*

Remark 3.6. *The following diagram shows that the relationships of* θg^**-closed sets with other known existing sets.*

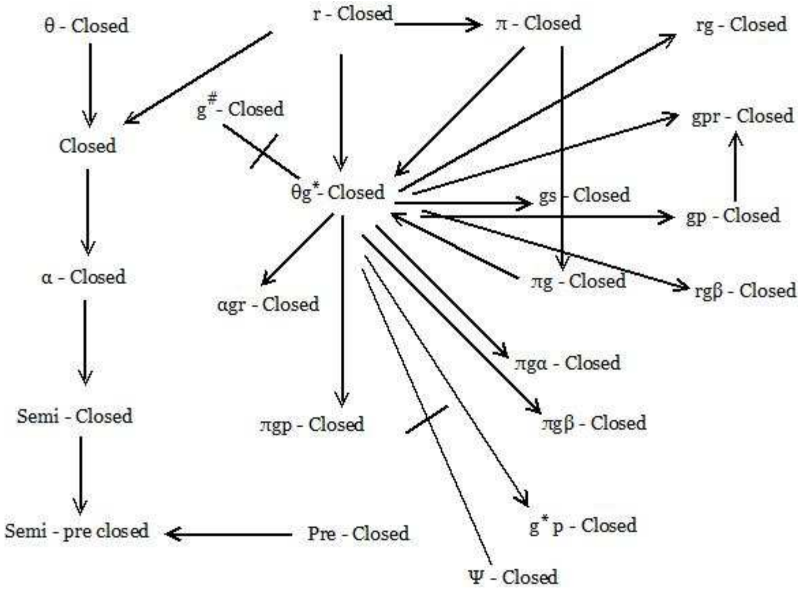

Figure 1:

$A \to B$ represents A implies B but not conversely.

4. Properties of θ-generalized star closed sets

In this section, we discuss the properties of θ-generalized star closed sets.

Theorem 4.1. *The union of two θg^*-closed subsets are θg^*-closed.*

Proof: Let A and B any two θg^*-closed sets in X. Such that $A \subseteq U$ and $B \subseteq U$ where U is g-open in X and so $A \cup B \subseteq U$. Since A and B are θg^*-closed. $A \subseteq cl_\theta(A)$ and $B \subseteq cl_\theta(B)$ and hence $A \cup B \subseteq cl_\theta(A) \cup cl_\theta(B) \subseteq cl_\theta(A \cup B)$. Thus $A \cup B$ is θg^*-closed set in (X, τ).

Example 4.2. *Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ and θg^*-closed = $\{X, \phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}\}$. Let $A = \{a\}$ and $B = \{c\}$, then $A \cup B = \{a,c\}$ is also θg^*-closed set.*

Theorem 4.3. *The intersection of two θg^*-closed subset are θg^*-closed.*

Proof: Let A and B any two θg^*-closed sets in X. Such that $A \subseteq U$ and $B \subseteq U$ where U is g-open in X and so $A \cap B \subseteq U$. Since A and B are θg^*-closed. $A \subseteq cl_\theta(A)$ and $B \subseteq cl_\theta(B)$ and hence $A \cap B \subseteq cl_\theta(A) \cap cl_\theta(B) \subseteq cl_\theta(A \cap B)$. Thus $A \cap B$ is θg^*-closed set in (X, τ).

Example 4.4. *Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{b\}, \{c\}, \{b,c\}, \{a,b\}\}$ and θg^*-closed=*

$\{X,\phi,\{a\},\{c\},\{a,b\},\{a,c\}$. Let $A=\{a,b\}$ and $B=\{a,c\}$, then $A\cap B=\{a\}$ is also θg^*-closed set.

Theorem 4.5. *The intersection of a θg^*-closed set and a θ-closed set is always θg^*-closed.*

Proof: Let A be a θg^*-closed set and let F be θ-closed. Let U be an open set such that $A\cap F\subseteq U$. Set $G=X\setminus F$. Then $A\subseteq U\cup G$. Since G is θ-open, $U\cup G$ is open and since A is θg^*-closed, $cl_\theta(A)\subseteq U\cup G$. Now by Lemma [2.4],

$cl_\theta(A\cap F)\subseteq cl_\theta(A)\cap cl_\theta(F) = cl_\theta(A)\cap F$

$\subseteq (U\cup G)\cap F = (U\cap F)\cup(G\cap F) = (U\cap F)\cup\phi\subseteq U$.

Theorem 4.6. *The intersection of a θg-closed set and a θg^*-closed set is always θg-closed.*

Proof: Let A be a θ generalized-closed set and let F be θg^*-closed. Let U be an open set such that $A\cap F\subseteq U$. Set $G=X\setminus F$. Then $A\subseteq U\cup G$. Since G is θg^*-open, $U\cup G$ is open and since A is θg-closed, $cl_\theta(A)\subseteq U\cup G$. Now by Lemma [2.4], $cl_\theta(A\cap F)\subseteq cl_\theta(A)\cap cl_\theta(F) = cl_\theta(A)\cap F$

$\subseteq (U\cup G)\cap F = (U\cap F)\cup(G\cap F) = (U\cap F)\cup\phi\subseteq U$.

Theorem 4.7. *For any element $x\in X$. The set X is θg^*-closed set or g-open.*

Proof: Suppose $X\setminus\{x\}$ is not g-open, then X is the only g-open set containing $X\setminus\{x\}$. This implies $cl_\theta X\setminus\{x\}\subseteq X$. Hence $X\setminus\{x\}$ is θg^*-closed or g-open in X.

5. Separation axioms of θg^*-closed sets

As applications of θg^*-closed sets, three spaces namely, ${}_\theta T_{1/2}{}^*$ spaces, ${}^*{}_\theta T_{1/2}$ spaces, ${}_\theta T_{1/2}{}^{**}$ spaces are introduced and investigated.

Definition 5.1. *A space (X,τ) is called*

- a ${}_\theta T_{1/2}{}^*$ space if every θg^*-closed set of (X,τ) is a closed set.
- a ${}^*{}_\theta T_{1/2}$ space if every θg^*-closed set of (X,τ) is a g^*-closed set.
- a ${}_\theta T_{1/2}{}^{**}$ space if every θg^*-closed set of (X,τ) is g-closed.

Theorem 5.2. *Every T_b space is ${}_\theta T_{1/2}{}^*$ space but not conversely.*

Proof: Let (X,τ) be a T_b space. Let A be a θg^*-closed set of (X,τ). Then A is also a gs-closed set. Since (X,τ) is a T_b space, then A is a closed set of (X,τ). Therefore (X,τ) is a ${}_\theta T_{1/2}{}^*$ space.

Example 5.3. *Let* $X = \{a,b,c\}$ *and* $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$. *Then the space* (X,τ) *is not a* T_b *space. Since* $\{a\}$ *is a* gs*-closed set but not a closed set of* (X,τ). *However* (X,τ) *is a* ${}_{\theta}T_{1/2}{}^{*}$ *space.*

Theorem 5.4. *Every* $T_{1/2}$ *space is* $T_{1/2}{}^{*}$ *space but not conversely.*

Proof: Let (X,τ) be a $T_{1/2}$ space. Let A be a g^{*}-closed set of (X,τ). Then A is also a g-closed set. Since (X,τ) is a $T_{1/2}$ space, then A is a closed set of (X,τ). Therefore (X,τ) is a $T_{1/2}{}^{*}$ space.

Example 5.5. *Let* $X = \{a,b,c\}$ *and* $\tau = \{X, \phi, \{a\}\}$. *Then the space* (X,τ) *is not a* $T_{1/2}$ *space. Since* $\{b\}$ *is a* g*-closed set but not a closed set of* (X,τ). *However* (X,τ) *is not a* $T_{1/2}{}^{*}$ *space.*

Theorem 5.6. *Every* $T_{1/2}$ *space is* ${}^{*}T_{1/2}$ *space but not conversely.*

Proof: Let (X,τ) be a $T_{1/2}$ space. Let A be a g-closed set of (X,τ). Then A is also a g-closed set. Since (X,τ) is a $T_{1/2}$ space, then A is a g^{*}-closed set of (X,τ) Therefore (X,τ) is a ${}^{*}T_{1/2}$ space.

Example 5.7. *Let* $X = \{a,b,c\}$ *and* $\tau = \{X, \phi, \{a,c\}\}$. *Then the space* (X,τ) *is not a* $T_{1/2}$ *space. Since* $\{a,b\}$ *is a* g*-closed set but not a closed set of* (X,τ). *However* (X,τ) *is not a* ${}^{*}T_{1/2}$ *space.*

Remark 5.8. *The diagram of Figure 2 shows that the relationship of* ${}_{\theta}T_{1/2}{}^{*}$*-space,* ${}^{*}{}_{\theta}T_{1/2}$*-space, and* ${}_{\theta}T_{1/2}{}^{**}$*-space with other known existing sets.*

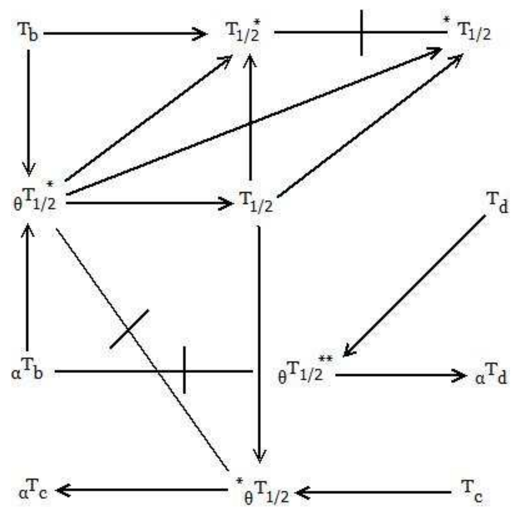

Figure 2:

$A \rightarrow B$ represents A implies B but not conversely.

Example 5.9. *Let* $X = \{a,b,c\}$ *and* $\tau = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$. *Then the space* (X,τ) *is not a* ${}_{\theta}T_{1/2}^{*}$ *space. Since* $\{c\}$ *is a* θg^**-closed set but not a closed set of* (X,τ). *However* (X,τ) *is a* $T_{1/2}$ *space and* $T_{1/2}^{*}$ *space.*

Example 5.10. *Let* $X = \{a,b,c\}$ *and* $\tau = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$. *Then the space* (X,τ) *is not a* ${}^{*}{}_{\theta}T_{1/2}$ *space. Since* $\{c\}$ *is a* θg^**-closed set but not a* g^**-closed set of* (X,τ). *However* (X,τ) *is a* ${}_{\alpha}T_c$ *space.*

Example 5.11. *Let* $X = \{a,b,c\}$ *and* $\tau = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$. *Then the space* (X,τ) *is not a* ${}_{\theta}T_{1/2}{}^{**}$ *space. Since* $\{c\}$ *is a* θg^**-closed set but not a* g*-closed set of* (X,τ). *However* (X,τ) *is a* ${}_{\alpha}T_d$ *space.*

Theorem 5.12. *A space* (X,τ) *is a* $T_{1/2}$ *space if and only if it is* ${}^{*}T_{1/2}$ *and* $T_{1/2}{}^{*}$.

Proof: Necessity: Follows from the Theorems [5.4] and [5.5].

Sufficiency: Suppose (X,τ) is both $T_{1/2}{}^{*}$ and ${}^{*}T_{1/2}$. Let A be a g-closed set of (X,τ). Since (X,τ) is ${}^{*}T_{1/2}$ space, then A is g^*-closed. Since (X,τ) is a $T_{1/2}{}^{*}$ space, then A is a closed set of (X,τ). Thus (X,τ) is a $T_{1/2}$ space.

6. θg^*-continuous functions and θg^*-irresolute functions

This section is devoted to introduce θg^*-continuous functions and θg^*-irresolute functions and discussed the relationships between the other known existing functions.

Definition 6.1. *A function* $f:(X,\tau) \rightarrow (Y,\sigma)$ *is called* θg^**-continuous if* $f^{-1}(V)$ *is a* θg^**-closed set of* (X,τ) *for every closed set* V *of* (Y,σ).

Theorem 6.2. *For a function* $f:(X,\tau) \rightarrow (Y,\sigma)$, *every continuous function is* θg^**-continuous but not coversely.*

Proof: Let f be a continuous function and let V be a closed set in (Y,σ), then $f^{-1}(V)$ is closed set in (X,τ). Since every closed set is θg^*-closed set, $f^{-1}(V)$ is θg^*- closed set in (X,τ). Therefore f is θg^*-continuous.

Example 6.3. *Let* $X = Y = \{a,b,c\}$ *with* $\tau = \{\phi, \{a\}, \{a,c\}, X\}$ *and* $\sigma = \{\phi, \{c\}, Y\}$. *Let the function* $f:(X,\tau) \rightarrow (Y,\sigma)$ *be defined by* $f(a) = b, f(b) = a, f(c) = c$. *Then* f *is* θg^**-continuous but not continuous. Since for the closed set* $\{a,b\}$ *in* Y,

$f^{-1}(\{a,b\}) = \{a,b\}$ *is* θg^**-closed but not closed set in* (X,τ).

Theorem 6.4. *For a function* $f:(X,\tau)\to(Y,\sigma)$*, the following hold.*
Every θg^**-continuous function is* rg*-continuous,* gpr*-continuous,* gs*-continuous,* gp*-continuous,* πg*-continuous,* πgs*-continuous,* $\pi g\beta$*-continuous.*

Proof: Let f be a θg^*-continuous function and let V be a closed set in (Y,σ), then $f^{-1}(V)$ is θg^*-closed set in (X,τ). Since every θg^*- closed set is rg-closed set (gpr -closed, gs-closed, gp-closed, πg-closed, πgs-closed, $\pi g\beta$-closed), $f^{-1}(V)$ is rg-closed (gpr-closed, gs-closed, gp-closed, πg-closed, πgs-closed, $\pi g\beta$ -closed) set in (X,τ). Therefore f is rg-continuous (gpr-continuous, gs -continuous, gp-continuous, πg-continuous, πgs-continuous, $\pi g\beta$-continuous).

Example 6.5.

1. Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{c\},Y\}$. Let the function $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=b$, $f(b)=a$, $f(c)=c$. Then f is rg-continuous but not θg^*-continuous. Since for the closed set $\{a,b\}$ in Y, $f^{-1}(\{a,b\}) = \{a,b\}$ is rg-closed but not θg^*-closed set in (X,τ).

2. Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi,\{a\},\{c\},\{a,c\},X\}$ and $\sigma = \{\phi,\{b\},Y\}$. Let the function $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=c$, $f(b)=b$, $f(c)=a$. Then f is gpr-continuous but not θg^*-continuous. Since for the closed set $\{a,c\}$ in Y, $f^{-1}(\{a,c\}) = \{a,c\}$ is gpr-closed but not θg^*-closed set in (X,τ).

3. Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi,\{a\},\{a,c\},X\}$ and $\sigma = \{\phi,Y,\{b\},\{a,b\}\}$. Let the function $f:(X,\tau)\to(Y,\sigma)$ be an identity function, then f is gs-continuous but not θg^*-continuous. Since for the closed sets $\{a,c\}$ and $\{c\}$ in Y, $f^{-1}(\{a,c\}) = \{a,c\}$ and $f^{-1}(\{c\}) = \{c\}$ is gs-closed but not θg^*-closed set in (X,τ).

4. Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,Y,\{a,c\}\}$. Let the function $f:(X,\tau)\to(Y,\sigma)$ be an identity function, then f is gp-continuous but not θg^*-continuous. Since for the closed set $\{b\}$ in Y, $f^{-1}(\{b\}) = \{b\}$ is gp-closed but not θg^*-closed set in (X,τ).

5. Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{ ,\{b\},\{c\},\{b,c\},\{a,c\},Y\}$. Let the function $f:(X,\tau)\to(Y,\sigma)$ be an identity function, then f is πg-continuous but not θg^*-continuous. Since for the closed set $\{\{a,b\},\{a,c\},\{a\},\{b\}\}$ in Y, $f^{-1}(\{a,b\}) = \{a,b\}$, $f^{-1}(\{a,c\} = \{a,c\}$, $f^{-1}(\{a\}) = \{a\}$ and $f^{-1}(\{b\}) = \{b\}$ is πg-closed but not θg^*-closed set in (X,τ).

6. Let $X = Y = \{a,b,c\}$ with $\tau = \{ ,\{b\},\{c\},\{a,b\},\{b,c\},X\}$ and $\sigma = \{ ,\{c\},\{a,c\},\{b,c\},Y\}$. Let the function $f:(X,\tau) \to (Y,\sigma)$ be defined by f(a)= c, f(b)= b, f(c)= a. Then f is π gs- continuous but not θg^* - continuous. Since for the closed set {a,b}, {b}, {a} in Y, f^{-1}({a,b}) = {b,c}, f^{-1}({b}) = {b} and f^{-1}({a}) = {c}, which is π gs- closed but not θg^* - closed set in (X, τ).

7. Let X=Y= {a,b,c} with τ = { ϕ,{a},{a,b},X} and $\sigma = \{ \phi,\{a\}, \{b,c\},Y \}$. Let the function f:(X, τ) $\to$ (Y,σ) be defined by f(a)= b, f(b)= c, f(c)= a. Then f is $\pi g\beta$- continuous but not θg^* - continuous. Since for the closed set {a} and {b,c} in Y, f^{-1}({a}) = {c} and f^{-1}({b,c}) = {a,b} which is $\pi g\beta$ - closed but not θg^* - closed set in (X, τ).

Remark 6.6. *The following diagram shows the relationship of θg^* - continuous with other known exixting sets.*

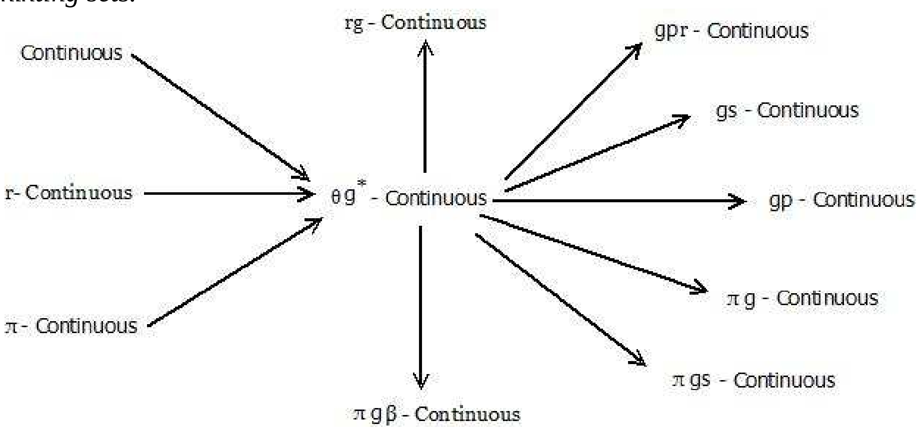

Figure 3:

$A \to B$ represents A implies B but not conversely.

Definition 6.7. *A function f: (X,τ) $\to$ (Y,σ) is called θg^* - irresolute if f^{-1}(V) is a θg^*-closed set of (X, τ) for every θg^*-closed set of (Y,σ).*

Theorem 6.8. *For a function $f:(X,\tau) \to (Y,\sigma)$, every θg^* - irresolute function is θg^* - continuous but not conversely.*

Proof: Let V be a closed set in (Y,σ). Since every closed set is θg^*-closed set. Therefore V is θg^*-closed set of Y. Since f is θg^* - irresolute, then f^{-1}(V) is θg^* -closed set in X. Thus f is θg^*-continuous.

Example 6.9. *Let X=Y= {a,b,c} with τ = { ϕ,{c},X}, θg^* = { ϕ,{a,b},X},*

σ = { ϕ,{a},{b,c},X} and θg^* = { ϕ,{a},{b,c},Y}. Define a function f(a)= a, f(b)= b, and f(c)= c then f^{-1}({a})= {a}, f^{-1}({b,c})= {b,c} which is not θg^*- irresolute. Since it is θg^*- closed set of Y but the inverse is not a θg^*- closed set of X. But it is θg^*- continuous.

Theorem 6.10. *Let a function $f:(X,\tau)\to(Y,\sigma)$ be a θg^*- continuous function. If (X, τ) is ${}_\theta T_{1/2}{}^*$- space, then f is continuous function.*

Proof: Let V be a closed set in (Y,σ). Since f is θg^*-continuous, f^{-1}(V) is θg^*- closed in (X, τ). Since (X, τ) is ${}_\theta T_{1/2}{}^*$, f^{-1}(V) is closed in (X, τ). Therefore f is continuous.

Remark 6.11. *The composition of two θg^*- continuous functions need not be θg^*- continuous as shown in the following example.*

Example 6.12. *Let X=Y=Z= {a,b,c} with τ = { ϕ,{a},{a,b},X}, σ = { ϕ,{b},{b,c},Y} and η = { ϕ,{c},{a,b},Z}. Define $f:(X,\tau)\to(Y,\sigma)$ by f(a)= a, f(b)= b, f(c)= c. Define g:(Y, σ) $\to$ (Z, η) by g(a)=b, g(b)= a, g(c)= c.*
Then θg^*C(X, τ)= { ϕ,X,{c},{b,c},{a,c}} and θg^*C(Y, σ)={ ϕ,Y,{a},{a,b},{a,c}}.
Here {a,b} is a closed set in (Z, η). But $(g\,o\,f)^{-1}$({a,b})= {a,b} is not a θg^*- closed set in (X, τ). Therefore g o f is not θg^*- continuous.

7. Contra θg^*-continuous functions

In this section, we introduce a new class of continuous function called contra θg^*-continuous functions and studied the composition between θg^*-continuous functions and θg^*-irresolute functions.

Definition 7.1. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be contra θg^*- continuous if f^{-1}(V) is θg^*- closed set in X for every open set V in Y.*

Theorem 7.2. *For the function $f:(X,\tau)\to(Y,\sigma)$, the following hold.*

[a] Every contra r-continuous function is contra θg^*-continuous.

[b] Every contra θg^*-continuous function is contra rg-continuous (contra gpr-continuous, contra gs-continuous, contra gp-continuous, contra π g-continuous, contra πgs-continuous, contra π gβ-continuous).

Proof: [a] Suppose we take V be an open set in Y. Since f is contra r-continuous, then f^{-1}(V) is r-closed in X. Since every r- closed set is θg^*- closed, f^{-1}(V) is θg^*- closed in X. Thus we have f is contra θg^*- continuous.

[b] Suppose we take V be an open set in Y. Since f is contra θg^*- continuous, then f^{-1}(V) is θg^*- closed in X. Since every θg^*- closed set is rg- closed (gpr-closed,gs-closed,gp-closed, π g-closed, π gs-closed, π g β -closed), f^{-1} (V) is rg-closed (gpr-closed,gs- closed,gp-closed, π g-closed, π gs-closed, π g β -closed) in X. Thus we have f is contra rg-continuous (contra gpr-continuous, contra gs-continuous, contra gp-continuous, contra π g-continuous, contra π gs-continuous, contra π gβ-continuous).

Example 7.3.
[a] Let X= {a,b,c} =Y with τ = {X, ϕ ,{a},{b},{a,b}} and σ = {Y, ϕ ,{b},{c},{b,c}}. Let the function $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is contra θg^* -continuous but not in contra r-continuous. Since for the open set {c} in Y, f^{-1}({c})={c} is θg^* - closed but not a r- closed set in (X, τ).

[b] Let X= {a,b,c} =Y with τ = {X, ϕ ,{c},{b,c}} and σ = {Y, ϕ ,{b,c}}. Let the function $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is contra rg- continuous but not in contra θg^*-continuous. Since for the open set {b,c} in Y, f^{-1}({b,c})={b,c} is rg- closed but not θg^* - closed set in (X, τ).

[c] Let X= {a,b,c} =Y with τ = {X, ϕ ,{a},{a,b}} and σ = {Y, ϕ ,{b}, {a,b}} . Let the function $f:(X,\tau)\to(Y,\sigma)$. Define a set f(a)=b, f(b)=a, f(c)=c. Then $f^{-1}(\{b\})$ ={a} and f^{-1} ({a,b})={a,b} which is contra gpr - continuous but not in contra θg^* -continuous. However f is contra gpr- continuous.

[d] Let X= {a,b,c} =Y with τ = {X, ϕ ,{a}} and σ = {Y, ϕ ,{a}, {a,b}}. Let the function $f:(X,\tau)\to(Y,\sigma)$. Defined by the set f(a)=b, f(b)=a, f(c)=c. Then f^{-1} ({a})={b}, f^{-1} ({a,b})={a,b}, which is contra gs - continuous but not in contra θg^* -continuous. However f is contra gs- continuous.

[e] Let X= {a,b,c} =Y with τ = {X, ϕ ,{c},{b,c}} and σ = {Y, ϕ ,{b}}. Let the function $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is contra gp - continuous but not in contra θg^* - continuous. Since for the open set {b} in Y, f^{-1} ({b})={b} is gp - closed but not θg^* - closed set in (X, τ).

[f] Let X= {a,b,c} =Y with τ = {X, ϕ ,{c},{b,c}} and σ = {Y, ϕ ,{b}, {b,c}}. Let the function $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is contra π g - continuous but not in contra θg^* -continuous. Since for the open sets {b} and {b,c} in Y, f^{-1}

({b})={b} and f^{-1}({b,c})={b,c} which is π g- closed but not θg^* - closed set in (X,τ).

[g] Let X= {a,b,c} =Y with τ = {X, ϕ ,{b},{c},{b,c}} and σ = {Y, ϕ ,{b}}. Let the function $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is contra π gs - continuous but not in contra θg^* -continuous. Since for the open set {b} in Y, f^{-1}({b})={b} is π gs-closed but not θg^* - closed set in (X, τ).

[h] Let X= {a,b,c} =Y with τ = {X, ϕ ,{a},{b},{a,b},{a,c}} and σ = {Y, ϕ ,{a}}. Let the function $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is contra π g β - continuous but not in contra θg^* -continuous. Since for the open sets {a} in Y, f^{-1} ({a})={a} is π g β - closed but not θg^* - closed set in (X, τ).

Theorem 7.4. *Let $f:(X,\tau)\to(Y,\sigma)$ be a contra θg^* - continuous function and g:(Y, σ) $\to$ (Z, η) be a continuous function then gof: (X, τ) $\to$ (Z, η) is contra θg^* - continuous.*

Proof: Let V be any open set in Z. Since g:(Y, σ) $\to$ (Z, η) be a continuous, g^{-1} (V) is open in Y. Since $f:(X,\tau)\to(Y,\sigma)$ be a contra θg^* - continuous, $f^{-1}(g^{-1}(V))$ is a θg^* - closed set in X. Hence $(gof)^{-1}(V)=f^{-1}(g^{-1}(V))$ is a θg^* - closed set in X. Therefore gof: (X, τ) $\to$ (Z, η) is contra θg^* - continuous.

Theorem 7.5. *Let $f:(X,\tau)\to(Y,\sigma)$ be a θg^* -irresolute and g:(Y, σ) $\to$ (Z, η) be a contra θg^* - continuous function then gof: (X, τ) $\to$ (Z, η) is contra θg^* - continuous.*

Proof: Now we take V be any open set in Z. Since g:(Y, σ) $\to$ (Z, η) be a contra θg^* -continuous, g^{-1} (V) is θg^* - closed in Y. Since $f:(X,\tau)\to(Y,\sigma)$ be a θg^* -irresolute, $f^{-1}(g^{-1}(V))$ is a θg^* - open set in X. Therefore gof: (X, τ) $\to$ (Z, η) is contra θg^* - continuous.

8. Conclusion

In this paper, a new class of sets called θg^* - closed sets has been introduced and some of its properties has been studied. Based on this sets, some of the functions called θg^* - continuous functions, θg^* - irresolute functions and contra θg^* -continuous functions are also introduced in the topological spaces and some of its properties has been studied. Further, the application of θg^* -closed sets has been introduced interms of spaces namely, ${}_{\theta}T_{1/2}{}^*$ - spaces and investigated its properties.

REFERENCES

1. S.P.Arya and T.M.Nour, Characterization of s-normal spaces, *Indian J. Pure Appl. Math.*, 21(8) (1990) 717-719.
2. D.Andrijevic, Semi pre-open sets, *Mat. Vesnik*, 38(1) (1986) 24-32.
3. G.Aslim and A.Caksu Guler, Noiri On π gs- closed sets in topological spaces, *Acta. Math. Hunger.*, 112(4) (2006) 275-283.
4. K.Balachandran, P.Sundaram and H.Maki, On generalized continuous maps in topological spaces, *Mem. Fac. Kochi Univ. Ser. A, Math.*, 12 (1991) 5-13.
5. P.Bhathacharyya and B.K.Lahiri, Semi generalized closed sets in topological spaces, *Indian J.Pure Apple.Math.*, 29 (1987) 376-382.
6. R.Devi, H.Maki and K.Balachanran, Semi generalized homeomorphisms and generalized semi homeomorphisms, *Indian J. Pure. Appl. Math.*, 26(3) (1995) 271 -284.
7. J.Dontchev and T.Noiri, Quasi normal spaces and πg -closed sets, *Acta Math. Hungar.*, 89(3) (2000) 211-219.
8. J.Dontchev and H.Maki, On θ-generalized closed sets, *Int.J. Math. Math. Sci.* 22 (1999) 239-249.
9. J.Dontchev, Contra-continuous functions and strongly s- closed spaces, *International Journal of Mathematical Sciences*, 19(2) (1996) 303-310.
10. W.Dunham, $T_{1/2}$ space, *Kyungpook Math.J.*, 17 (1977) 161-169.
11. S.V.Fomin, Extensions of topological spaces, *Ann. of Maths.*, 44 (1943) 471-480.
12. Y.Gnanambal, On generalized pr-closed sets in topological spaces, *Indian. J. Pure Appl. Math.*, 28 (1997) 351-360.
13. S.Jafari and T.Noiri, Contra-α -continuous functions between topological space, *Iranian International Journal of Science*, 2(2) (2001) 153-167.
14. V.Jeyanthi and C.Janaki, π gr- closed sets in topological spaces, *Asian Journal of Current Engg. Math.*, 1 (2012) 241-246.
15. C.Janaki, Studies on $\pi g\alpha$ - closed sets in topological spaces, Ph.D Thesis, Bharathiar University, Cbe (2009).
16. N.Levine, Semi-open and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70 (1963) 36-41.
17. N.Levine, Generalized closed sets in topological spaces, *Rend. Circ. Mat. Patermo*, 2 (19) (1970) 89-96.
18. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α -closed sets and α -generalized closed sets, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math.,* 15 (1994) 51-63.
19. H.Maki, J.Umehara and T.Noiri, Every topological space is pre-$T_{1/2}$, *Mem. Fac.Sci. Kochi Univ. Ser. A. Math.*, 17 (1996) 33-42.
20. H.Maki, R.Devi and K.Balachandran, Generalized α -closed sets in topology, *Bull Fukuoka Univ. Ed.*, 42 (1993) 13-21.
21. A.S.Mashhour, M.E.Abd, El.Monsef and S.N.E.Deebl, On pre continuous and weak pre continuous mappings, *Proc. Math. and Phys. Soc. Egypt*, 53 (1982) 47-53.
22. N.Nagaveni, Studies on generalized homeomorphisms in topological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 1999.

23. O.Njastad, Some classes of nearly open sets, *Pacific J. Math*., 15 (1965) 961-970.
24. J.H.Park, On π gp- closed sets in topological spaces, *Indian J. Pure Appl. Math*, 2004.
25. N.Palaniappan and K.C.Rao, Regular generalized-closed sets, *Kyung-pook Math. J*., 33 (1993) 211-219.
26. M.S.Sarasak and N.Rajesh, π -Generalized semi-preclosed sets, International *Mathematical Forum,* 5 (2010) 473-478.
27. M.Stone, Application of the theory of boolean rings to general topology, *Trans. Amer. Math. Soc.*, 41 (1937) 374-481.
28. M.Sheikjohn, w- closed sets in topology, *Acta Ciencia Indica*, 4 (2000) 389-392.
29. M.K.R.S.Veerakumar, Semi-closed sets and semi-pre-closed sets, *Rend. Intit. Mat. Univ. Trieste*, XXXII (2000) 25-41.
30. M.K.R.S.Veerakumar, g^*-semi-closed sets in topology, *Acta Ciencia India*, 29M(1) (2003) 81-90.
31. M.K.R.S.Veerakumar, $g^\#$ -semi-closed sets in topology, *Indian J. Math.*, 44(1) (2002) 73-87.
32. M.K.R.S.Veerakumar, Between closed sets and g- closed sets, *Mem. Fac.Sci. Kochi Univ. (Math.)* 21 (2000) 1-19.
33. M.K.R.S.Veerakumar, gs- closed sets in topological spaces, *Kochi University Series A*, 21 (2000)1-19.
34. N.V.Velicko, H-closed topological spaces, *Trans. Aner. Math. Soc.*, 78 (1968) 103-118.
35. Zaitsev, On certain classes of topological spaces and their bicomactifications, *Donkl. Akad. Nauk. SSSR.*, 178 (1968) 778-779.