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Application of Fuzzy Comprehensive Evaluation in Environmental Quality Assessment

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Abstract. In this paper, the fuzzy comprehensive evaluation method is applied to the comprehensive evaluation of environmental quality, and the air quality in Chengdu, China is comprehensively evaluated and sorted. Improvements were made to the principle of obtaining large and small algorithms and maximum degree of membership. Using the analytic hierarchy process, through the establishment of a fuzzy comprehensive-weighted average model, the evaluation results are more in line with the actual situation, and have a certain practical significance for the control of environmental quality.

Keywords: Fuzzy comprehensive evaluation; AHP; Environmental quality

AMS Mathematics Subject Classification (2010): 62P12

1. Introduction

In 1965, Zadeh [1] put forward the concept of fuzzy sets. After that, fuzzy mathematics was developed as a new branch of mathematics [2,3,4]. Fuzzy clustering analysis, fuzzy pattern recognition, fuzzy comprehensive evaluation [5,6,7,8], fuzzy decision and fuzzy prediction, fuzzy control, fuzzy information processing and other methods constitute a fuzzy system theory.

In the report of the 19th National Congress of China, a new goal “to build a strong, democratic, civilized, harmonious and beautiful socialist modernized country” was proposed, which added the word “beautiful”. At the same time, pollution prevention and control in the three major challenges and battles all show that in the current development process in China, the environment is of great concern to the people of the country, and it is related to the realization of the beautiful goal. With the economic development, the destruction of the environment cannot be avoided. China’s environmental protection has always lagged behind the economic development. Due to the long-term cumulative accumulation of multi-stage, multi-field, and multi-type problems, the environmental carrying capacity has reached or approached the upper limit, and environmental governance is imminent. At the same time, the environmental quality of various places needs to be carried out through reasonable and effective methods. This paper takes the evaluation of environmental quality in Chengdu, China as an example, and uses AHP to establish a fuzzy comprehensive evaluation model [9,10,11,12,13] to construct a comprehensive evaluation system for environmental quality.

2. Establish a comprehensive classification index of environmental quality assessment

The data in this paper comes from the statistical yearbook and the official website. The location chose Chengdu, the largest city in Southwest China.

(1) Establish a set of factors for evaluation, that is, the main indicators that affect the environment.

Table 1: The main air pollutant concentration in chengdu city during 2013-2017

Time	Monthly average concentration of sulfur dioxide (mg / m3)	Monthly average concentration of nitric oxide (mg / m3)	Carbon monoxide 24 hours average 95th percentile (mg / m3)	Ozone maximum 8 hours sliding average 90th percentile (mg / m3)	Inhalable particulate matter (PM10) monthly average concentration (mg / m3)
2013	0.017	0.055	40.8	0.142	0.104
2014	0.019	0.059	48	0.148	0.123
2015	0.014	0.053	48	0.183	0.108
2016	0.014	0.054	43.2	0.168	0.105
2017	0.009	0.046	26.4	0.089	0.06

Note: 1. Chengdu City's ambient air evaluation implements "Ambient Air Quality Standard" "GB3095-2012".
2. The comprehensive index of ambient air quality is a dimensionless index that describes the comprehensive status of urban air quality, taking into account the pollution levels of various pollutants. The greater the composite index of ambient air quality, the heavier the overall pollution level.

(2) Create an evaluation set. According to ambient air standards, the main pollutants are divided into three levels, $V=\{I,II,III\}$.

Table 2: Ambient air quality standard

Unit (mg / m3)	I	II	III
Sulfur dioxide (daily mean)	0.05	0.15	0.8
Nitrogen dioxide (daily average)	0.08	0.12	0.28
Carbon monoxide (hourly average)	5	10	60
Ozone (hourly average)	0.12	0.2	0.4
Inhalable particulate matter (PM10) (daily average)	0.05	0.15	0.35

(1) I, Air Pollution Index (API) (0-50), Excellent Air Quality; II, API (51-100), Good Air Quality; III, API (101-200), the air quality is slightly polluted; IV, API (201-300), air quality is light pollution; V, API (301-400), air quality is moderate pollution; VI, API (401-500), the air quality condition is severe pollution.
(2) This time taking into account the scope of its own monitoring data and international standards, as well as the implementation of the "Ambient Air Quality Standard" "GB3095-2012" of Chengdu City's ambient air assessment, the three grades of I, II, and III are intercepted from the 6 level standards.

3. Establish membership functions

The method for establishing the membership function of each evaluation parameter for each rating level is as follows:

(1) Level 1 rating

$$u_{i1}(u_i)=\begin{cases} 0 & u_i \ge v_{i2} \\ -\dfrac{u_i-v_{i2}}{v_{i2}-v_{i1}} & v_{i1}<u_i<v_{i2} \\ 1 & u_i \le v_{i1} \end{cases}$$

(2) Grade j rating

$$u_{ij}(u_i)=\begin{cases} 0 & u_i \le v_{ij-1}, u_i \ge v_{ij+1} \\ \dfrac{u_i-v_{ij-1}}{v_{ij}-v_{ij-1}} & v_{ij-1}<u_i<v_{ij} \\ -\dfrac{u_i-v_{ij+1}}{v_{ij+1}-v_{ij}} & v_{ij} \le u_i < v_{ij+1} \end{cases}$$

(3) Grade n rating

$$u_{in}(u_i)=\begin{cases} 0 & u_i \le v_{in} \\ \dfrac{u_i-v_{in-1}}{v_{in}-v_{in-1}} & v_{in-1}<u_i<v_{in} \\ 1 & u_i \ge v_{in} \end{cases}$$

Substituting the monitoring data in Table 1 into the formula for grade J rating, the membership of each grade can be obtained, thus forming a fuzzy relation matrix R.

For example, the degree of membership of SO_2:

Level I is

$$u_{11}(u_1)=\begin{cases} 0 & u_i \ge 0.15 \\ -10(u_1-0.15) & 0.05<u_i<0.15 \\ 1 & u_i \le 0.05 \end{cases}$$

Level II is

$$u_{12}(u_1)=\begin{cases} 0 & u_i \le 0.05, u_i \ge 0.8 \\ -10(u_1-0.8) & 0.05 \le u_i < 0.15 \\ 10(u_1-0.05) & 0.15<u_i<0.8 \end{cases}$$

Level III is

$$u_{13}(u_1)=\begin{cases}0 & u_i \le 0.15\\ (u_1-0.15)/0.65 & 0.15<u_i<0.8\\ 1 & u_i \ge 0.8\end{cases}$$

The fuzzy relation matrix A from 2013 to 2017 can be obtained as follows.

$$R_1=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0.384 & 0.616\\ 0.725 & 0.275 & 0\\ 0.46 & 0 & 0\end{pmatrix}\qquad R_2=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0.24 & 0.76\\ 0.65 & 0.35 & 0\\ 0.27 & 0.73 & 0\end{pmatrix}$$

$$R_3=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0.24 & 0.76\\ 0.65 & 0.35 & 0\\ 0.42 & 0.58 & 0\end{pmatrix}\qquad R_4=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0.336 & 0.664\\ 0.4 & 0.6 & 0\\ 0.45 & 0.55 & 0\end{pmatrix}$$

$$R_5=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0.672 & 0.328\\ 1 & 0 & 0\\ 0.9 & 0.1 & 0\end{pmatrix}$$

4. Create a weight set

Using the normalization formula to determine the weight vector:

$$\omega_i=\frac{C_i/S_i}{\sum_{i=1}^{n}C_i/S_i}(i=1,2,...,n),S_i=\frac{1}{n}\sum_{j=1}^{n}S_{ij}(j=1,2,...,n)$$

where ω_i denotes the normalized weight value of the evaluation parameter i , C_i denotes the monitored concentration of the evaluation parameter i ,and S_i denotes the arithmetic mean of the n level standard of evaluation parameter i . This gives the weight vector $A=(\omega_1,\omega_2,....,\omega_n)$, which is normalized before synthesis, that is

$$\sum_{i=1}^{n}a_i=1,a_i\ge 0,i=1,2,...,n.$$

Substituting the data in Table 1 and Table 2 into the above formula yields sulfur dioxide, nitrogen dioxide, carbon monoxide, ozone, and inhalable particles (PM10). In the five years of 2013, 2014, 2015, 2016 and 2017, the weights are assigned to obtain the fuzzy weight set A of the evaluation parameters.

$A_1=(0.016,0.108,0.512,0.186,0.178)$, $A_2=(0.016,0.101,0.528,0.170,0.185)$,

$A_3=(0.012,0.091,0.527,0.209,0.162)$, $A_4=(0.012,0.100,0.511,0.207,0.170)$,

$A_5=(0.013,0.139,0.510,0.179,0.158)$.

5. Fuzzy matrix synthesis

The fuzzy comprehensive evaluation result of the following table 3 is obtained by the synthesis operation of the fuzzy matrix. Take 2013 as an example

$$B=(b_j)_{1\times n}=A\circ R$$

$$b_j=\sum_{i=1}^{m}(\omega_i\wedge r_{ij})=(0.016\quad 0.108\quad 0.512\quad 0.186\quad 0.178)\begin{pmatrix}1&0&0\\1&0&0\\0&0.384&0.616\\0.725&0.27&0\\0.46&0&0\end{pmatrix}=(0.314,0.248,0.315).$$

Note: The rest of the results use the same algorithm.

Table 3: Chengdu's fuzzy comprehensive evaluation results from 2013 to 2017

years	$B=A\circ R$	$(b_j)_{\max}$	Evaluation results
2013	（0.314,0.248,0.315）	0.315	I
2014	（0.28,0.314,0.423）	0.423	IV
2015	（0.215,0.385,0.401）	0.401	III
2016	（0.271,0.389,0.339）	0.389	II
2017	（0.473,0.359,0.167）	0.473	V

The evaluation standard is that the smaller the data is, the better the environmental quality is. Therefore, the five-year comprehensive environmental assessment of Chengdu is: 2013, 2016, 2015, 2014, and 2017.

6. Construction of environmental quality assessment model based on analytic hierarchy process

1) Construction judgment matrix

According to $d_i=c_i/s_{ij}$, get the scale of each year d_i .

$$d_1=(0.051,0.034,1.632,0.592,0.567),\ d_2=(0.057,0.369,1.920,0.617,0.671)$$

$$d_3=(0.042,0.331,1.920,0.763,0.589),\ d_4=(0.042,0.338,1.728,0.700,0.573)$$

$$d_5=(0.027,0.288,1.056,0.371,0.327).$$

The judgment matrix of each year is as follows:

$$R_1=\begin{pmatrix}1&0.148&0.031&0.086&0.090\\6.745&1&0.211&0.581&0.607\\32&4.744&1&2.757&2.878\\11.608&1.721&0.363&1&1.044\\11.118&1.648&0.347&0.958&1\end{pmatrix}\quad R_2=\begin{pmatrix}1&0.154&0.030&0.092&0.085\\6.474&1&0.192&0.598&0.550\\33.684&5.203&1&3.112&2.861\\10.825&1.672&0.321&1&0.920\\11.772&1.818&0.349&1.088&1\end{pmatrix}$$

$$R_3=\begin{pmatrix}1&0.127&0.022&0.055&0.071\\7.881&1&0.172&0.434&0.562\\47.714&5.801&1&2.516&3.260\\18.167&2.305&0.397&1&1.295\\14.024&1.779&0.307&0.772&1\end{pmatrix}\quad R_4=\begin{pmatrix}1&0.124&0.024&0.060&0.073\\8.048&1&0.196&0.483&0.590\\41.143&5.112&1&2.469&3.106\\16.667&2.071&1.405&1&1.222\\13.643&1.695&0.332&0.819&1\end{pmatrix}$$

$$R_5 = \begin{pmatrix} 1 & 0.094 & 0.026 & 0.073 & 0.083 \\ 10.667 & 1 & 1.273 & 0.776 & 0.881 \\ 39.111 & 3.667 & 1 & 2.846 & 3.229 \\ 13.741 & 1.288 & 0.351 & 1 & 1.350 \\ 12.111 & 1.135 & 0.310 & 1.881 & 1 \end{pmatrix}$$

2) Weight set

The square root method is used to find the largest eigenvalue of the judgment matrix, and its eigenvector is normalized to obtain the weight set of the evaluation index. $\lambda_{\max} = 5$, Weight set $A = (0.016, 0.108, 0.512, 0.186, 0.178)$.

3) Consistency test

Consistency indicator CI=0.000, random agreement index RI=1.120. From formula $CR = \frac{CI}{RI}$, we get CR=0.00<0.100 and pass the consistency test.

4) Weighted average fuzzy synthesis operator

Combining A and matrix R into a fuzzy comprehensive evaluation matrix B .

$$B = (b_j)_{1\times n} = A \circ R$$

$$b_j = \sum_{i=1}^{m} (\omega_i \wedge r_{ij})$$

$$b_1 = \begin{pmatrix} 0.016 & 0.108 & 0.512 & 0.186 & 0.178 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.384 & 0.616 \\ 0.725 & 0.27 & 0 \\ 0.46 & 0 & 0 \end{pmatrix} = (0.314, 0.248, 0.315),$$

$$b_2 = \begin{pmatrix} 0.016 & 0.101 & 0.528 & 0.170 & 0.185 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.24 & 0.76 \\ 0.65 & 0.35 & 0 \\ 0.27 & 0.73 & 0 \end{pmatrix} = (0.280, 0.314, 0.423),$$

$$b_3 = \begin{pmatrix} 0.012 & 0.091 & 0.527 & 0.209 & 0.162 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.24 & 0.76 \\ 0.65 & 0.35 & 0 \\ 0.42 & 0.58 & 0 \end{pmatrix} = (0.215, 0.385, 0.401),$$

$$b_4 = \begin{pmatrix} 0.012 & 0.10 & 0.511 & 0.207 & 0.170 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.336 & 0.664 \\ 0.4 & 0.6 & 0 \\ 0.45 & 0.55 & 0 \end{pmatrix} = (0.271, 0.389, 0.339),$$

$$b_5 = (0.013 \quad 0.139 \quad 0.510 \quad 0.179 \quad 0.158)\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0.672 & 0.328 \\ 1 & 0 & 0 \\ 0.9 & 0.1 & 0 \end{pmatrix} = (0.473, 0.359, 0.167).$$

Come to the following table 4.

Table 4: Fuzzy comprehensive evaluation results based on analytic hierarchy process

2013	0.314	0.248	0.315
2014	0.28	0.314	0.423
2015	0.215	0.385	0.401
2016	0.271	0.389	0.339
2017	0.473	0.359	0.167

5) Analyze the result of fuzzy comprehensive evaluation

In order to avoid the problem of validity, using the weighted average principle, multiple evaluations can be sorted according to their rank position. With the rank value $B_j\,(j=1,2,\cdots,n)$ as a variable, n and rank values are usually artificially determined, such as $B_j = (1,2,3,4,5)$

Calculate $B_j = \frac{\sum_{i=1}^{n} b_j^k B}{\sum_{j=1}^{n} b_j^k}$ (K is the undetermined coefficient) using the comprehensive evaluation result b_j as a weight.

Using the above methods to analyze the results of air quality assessment in 2013, 2014, 2015, 2016, and 2017 in Chengdu. When K=2, there is

$B_1 = 2.002$, $B_2 = 2.247$, $B_3 = 2.323$, $B_4 = 2.122$, $B_5 = 1.485$, then $B^* = (2.002, 2.247, 2.323, 2.122, 1.485)$.

In 2013, it belonged to Class II, favored Class I, and in 2014 it belonged to Class III and favored Class II. In 2015, it belongs to level III, and in 2016 it belongs to level II. In 2017, it belongs to level I and is biased towards II. Because the smaller the data in the evaluation criteria is, the better the environmental quality is, so the five-year comprehensive environmental assessment of Chengdu is: 2017, 2013, 2016, 2014, 2015.

Comparing the evaluation results obtained by using the largest degree of membership, there are some deviations, but the evaluation results obtained by the latter are more in line with the actual situation (see Table 5 below, from the official website of Chengdu Environmental Protection Bureau).

Table 5: Chengdu environmental air quality comprehensive index

years	2013	2014	2015	2016	2017
composite index	4.05	6.38	6.57	7.17	3.52

The five-year comprehensive assessment of environmental air in Chengdu is: 2017, 2013, 2014, 2015, 2016.

7. Conclusion

This paper establishes a comprehensive evaluation system for air pollutants in Chengdu. It adopts the analytic hierarchy process and fuzzy comprehensive evaluation method to accurately obtain the weight coefficient of each evaluation index. In the past, many fuzzy comprehensive evaluation applications, the principle of maximum degree of membership, often ignore the validity. This time, we use the weighted average principle to evaluate and sort comparatively multiple samples to solve the impact of the validity and the results basically accord with the actual situation. The research method can be applied to other assessments such as wastewater, solid wastes, etc. A scientific evaluation method is a dynamic process. The development of science and technology and economy is getting faster and faster, and people's requirements for a high-quality living environment will become higher and higher. A more comprehensive, systematic, and reasonable environmental assessment system should be established, as well as a more scientific and rigorous evaluation method.

There are still many deficiencies in this article. For example, the evaluation index system that is constructed needs further optimization, and some more targeted indicators can be added. In the future research will be further improved.

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