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Optimal Solution for Assignment Problem by Average Total Opportunity Cost Method

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Abstract. In this paper, a new method, namely Average Total Opportunity Cost (ATOC) assignment method is proposed to solve the assignment problem followed by a Numerical Example. It minimizes the total cost of certain problems and for few problems it gives a more cost compared to existing Hungarian method. We prove that this method gives the same optimum solution and also some of the variations and special cases in an assignment problem with its applications are discussed in this paper.

Keywords: Assignment Problem, Hungarian assignment Method, ATOC Method.

AMS Mathematics Subject Classification (2010): 90C08

1. Introduction

The assignment problem is one of the most basic applications and it is a special category of linear programming in which our objective is to assign n number of jobs to n number of persons at a minimum cost/ maximum profit. Assignment may be persons to jobs, classes to rooms, operators to machines, drivers to trucks, trucks to delivery routes, or problems to research teams, etc. There are many researchers to solve assignment problem. Goel and Mittal [2], Bazarra et al. [1] and Taha [3] for the history of these methods. Number of methods have been so far presented for an assignment problem some of them are Singh, Dubey, Shrivastava [5], Bertsekas [16] in which, the best known, most used method for solving the assignment problem is the “Hungarian Method”, originally suggested by Kuhn in 1955. Now a days,Ahmed et al., Khandelwal [6], Kotwal [8], Thiruppathiet al [9] developed innovative method to solve the optimal solutions to the balanced assignment problem. Kirtiwant et al [10] introduced a new approach to solve balanced and unbalanced assignment problems. Also Rao and Srinivas [13], developed an effective algorithm for finding the optimal solution of an assignment problem to reduce computational cost.

The Paper is organized as follows: Section 2 deals with Mathematical formulation of Assignment problem. In Section 3, a new method, namely ATOC method is proposed by an Algorithm to find the optimal solution of an assignment problem, followed by a Numerical Example. Variations of the Assignment problem are discussed in section 4. In section 5, special cases of assignment problem and its formulations are explained. The conclusion of the paper is given in Section 6.

2. Mathematical formulation of an assignment problem

Given n workers (rows) and n jobs (column) and effectiveness (in terms of cost, profit, time, etc.,) of each worker for each job the problem lies in allocating each worker to one and only one job so that the given measure of effectiveness is optimized.

		Jobs						
		1	2	.	.	.	n	
	1	C_{11}	C_{12}	.	.	.	C_{1n}	1
Workers	2	C_{21}	C_{22}	.	.	.	C_{2n}	1
	.	.					.	.
	.	.					.	.
	.	.					.	.
	n	C_{n1}	C_{n2}	.	.	.	C_{nn}	1
		1	1	.	.	.	1	

Let X_{ij} be denote the assignment of facility i to job j such that X_{ij} = 0 if the i^{th} worker is not assigned to the j^{th} job and X_{ij} =1if thei^{th} worker is assigned to the j^{th} job and C_{ij} represents the cost of assignment of workers to job j.

Then the Mathematical model of the assignment problem can be stated as the objective function is to

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} C_{ij}$

Subject to the constraints

$\sum_{i=1}^{n} X_{ij} = 1$ and $\sum_{j=1}^{n} X_{ij} = 1$; X_{ij} = 0 or 1.

for all i=1,2,…..n and j=1,2,…..n.

3. ATOC method for solving assignment problem

This section presents a new method (ATOC Method) to make simpler the assignment problem which is different from the prior methods developed.

The various Steps of the algorithm are as follows.

Step 1: Determine the cost table from the given problem.

(i) If the number of sources is equal to the number of destinations, go to step 3.

(ii) If the number of sources is not equal to the number of destinations, go to step2.

Step 2: Add a Dummy source or dummy destination, so that the cost table becomes a square matrix.

The cost entries of dummy source / destinations are always zero Go to Step 3.

Step 3: Obtain the Total Opportunity Cost Table (TOCT).

Step 4: Place the average of total opportunity costs of cells along the row identified as Row Average Total Opportunity Cost (RATOC). Display them along the side of the table against the respective rows. Similarly, calculate the average of total opportunity costs of cells along each column identified as Column Average Total Opportunity Cost (CATOC) and write them below the table against the respective columns.

Step 5: Identify the highest element among the RATOCs and CATOCs. If a tie occurs go to the next step otherwise, Let the highest element be in ith row, then find the smallest cost in that row and let it be c^{ij}. Allocate the cell and cross out the ith row and jth column. Repeat from step 3.

Step 6: (i) If a tie occurs, in the higher element, then that row and column find the minimum cost cell. But, assign to the next minimum cost of the cell and cross out that corresponding row and column. Next, go to step 4.
(ii) If a tie occurs at the minimum cost of the cell, then choose arbitrarily and allocate to it.

Step 7: Repeat the procedure until an optimum solution is attained.

3.1. Numerical example

Consider the following assignment problem. Assign the 4 jobs to 4 machines so as to minimize the total cost.

	A	B	C	D
I	10	12	19	11
II	5	10	7	8
III	12	14	13	11
1V	8	15	11	9

The total opportunity cost table is:

	A	B	C	D
I	5	4	21	4
II	0	5	2	3
III	8	7	8	3
1V	3	12	7	2

The allocations with the help of RATOCs and CATOCs are:

	A	B	C	D	RATOC
I	5	4	21	4	(8.5)
II	0	5	2	3	(2.5)
III	8	7	8	3	(6.5)
1V	3	12	7	2	(6)
RATOC	4	7	9.5	3	

By using the algorithm, the highest element is 9.5 which are in the third column. In that corresponding column, choose the minimum cost which is in the cell (II, C) and allocate to it. Cross out the third column and second row. Now the reduced table is as follows:

	B	C	D
II	5	4	4
III	8	7	3
IV	3	12	2

Repeating the steps 4, 5 and 6 the Assignment schedule is (I, B), (II, C), (III, D), (IV, A) and the minimum cost is Rs.38. By using the Hungarian method, we get the same cost.

Problem	Hungarian Method	ATOC Method	Optimum
3.1	**38**	**38**	**38**

4. Variations of the assignment problem

4.1. Unbalanced assignment problem

The Unbalanced assignment problem occurs when the number of persons is less than the number of jobs or the number of jobs is less than the number of persons. However, when the given cost matrix is not a square matrix, the assignment problem is called an unbalanced problem. In such cases a dummy persons(s) or jobs(s) are added in the matrix (with zeros as the cost elements) to make it a square matrix. These cells are the treated the same way as the real cost cells during the solution procedure.

4.1.1. Numerical example

Consider the following assignment problem. Assign 4 jobs to 10 persons with minimum cost

	1	2	3	4
A	11	8	9	8
B	4	5	29	33
C	10	5	29	33
D	1	18	25	31
E	23	22	33	30
F	3	9	13	19
G	6	8	27	32
H	32	30	39	38
I	36	35	31	21
J	15	11	10	28

Using the Algorithm of ATOC Method, the Assignment schedule is (A, 4), (B, 2), (D, 1), (J, 3) and the minimum cost is Rs.24.

By using the Hungarian method we get the optimum cost is 43 and the schedule is (A, 2) (B, 1) (J, 3) (I, 4).

Hence the difference is Rs.19. So, the ATOC method gives the least cost compared to Hungarian method.

4.2. Maximization problem

Sometimes the assignment problem may deal with maximization of the objective function. The maximization problem has to be changed to minimization before the Hungarian method may be applied. This transformation can be done in either of the following two ways:

(i) By subtracting all the elements from the largest element of the matrix.

(ii) By multiplying the matrix elements by -1.

4.2.1. Numerical rxample

Consider the following example for solving maximization problem:

A company is faced with the problem of assigning 4 machines to 4 different jobs (one machine to one job only). The profits are estimated as follows. Solve the problem to maximize the total profits.

	M_1	M_2	M_3	M_4
J_1	140	112	98	154
J_2	90	72	63	99
J_3	110	88	77	121
J_4	80	64	56	88

Reduce the cost matrix by subtracting all the elements from the largest element (i.e. 154) of the matrix, as given in the table below:

	M_1	M_2	M_3	M_4
J_1	14	42	56	0
J_2	64	82	91	55
J_3	44	66	77	33
J_4	74	90	98	66

Using the algorithm of ATOC Method, the optimum assignment is (J_1, M_3), (J_2, M_1), (J_3, M_2), (J_4, M_4) and the minimum total cost is Rs.364.

By using Hungarian method we get the optimum cost as Rs.392 and the schedule is (J_1, M_4), (J_2, M_2),

(J_3, M_1),(J_4, M_3). Here, the difference is Rs.28. and hence, ATOC method gives the less cost compared to Hungarian method.

5. Special cases in assignment problem and its formulation

5.1. Formulation of traveling salesman problem (TSP) as an assignment problem

A traveling salesman has to visit n cities and return to the starting point. He has to start from any one city and visit each city only once. Suppose he starts from the k^{th} city and the last city he visited is m. Let the cost of travel from i^{th} city to j^{th} city be c_{ij}.
Then the objective function is

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} c_{ij}$

Subject to the constraints:
$\sum_{j=1}^{n} x_{ij} = 1$, and $\sum_{i=1}^{n} x_{ij} = 1$, $x_{ij} = 0$ or 1.
for all i=1, 2, 3…..n & j=1, 2,…..n

5.2. Solution procedure

Step 1: Proceed the step 1 to step 4 of the ATOC Method.

Step 2: Since the diagonal routes are not possible, which gives an infinite cost, identify the smallest element among RATOCs and CATOCs. Let the smallest element be in i^{th} row, then find the smallest cost in that row and allocate the cell and cross out the corresponding row and column. If the infinite cost of the row and column are crossed out, then follow the procedure of the ATOC method. If a tie occurs, then follow the step 5 of the ATOC method.

Step 3: The solution thus found is to be in cyclic nature, if not, then include the minimum cost in the table and check whether cyclic assignment is followed. Proceed until the route conditions satisfied.
The procedure is explained through an example below

5.3. Numerical example

Consider the following travel salesman problem so as to minimize the cost per cycle.

	A	B	C	D	E
A	∞	3	6	2	3
B	3	∞	5	2	3
C	6	5	∞	6	4
D	2	2	6	∞	6
E	3	3	4	6	∞

By using the procedure the optimal assignment is (A, D), (B, C), (C, E), (D, B), (E, A) and minimize the total cost is 16.

6. Conclusion

In this paper, we had found an optimal solution for the assignment problem by using Average Total Opportunity Cost Method (ATOC). For any kind of assignment problem, whether its function to be maximized or minimized this method can be employed to solve. It gives minimum cost as compared to existing Hungarian method and also gives the same cost in some problems.

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