Journal of Mathematics and Informatics Vol. 13, 2018, 1-12 ISSN: 2349-0632 (P), 2349-0640 (online) Published 1 April 2018 www.researchmathsci.org DOI: http://dx.doi.org/10.22457/jmi.v13a1

Journal of **Mathematics and Informatics**

Application of Gamma Ray Burst in Dark Energy Model

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Received 1 March 2018; accepted 27 March 2018

Abstract. Gamma-ray bursts are the universe's most advantageous resources, and widely found in supernova explosions. In this paper, we use the observed gamma-ray bursts data and other data to a combination of observational constraints on dark energy models. The best fitting value of matter, dark energy density parameter, dark energy state equation parameter and interaction factor and 68% degree of disposition were obtained. And the compliance of the model with the observed data is discussed by information criterion. The conclusions are as follows: (1) The model is fitted with the observed data. (2) Due to the interaction factor, we can see that the model still cannot alleviate the coincidence problem. (3) The obtained parameters are consistent with the cosmological constants model proposed by Einstein in the range of 68%.

Keywords: cosmological parameters; Dark energy; gamma-ray burst data

AMS Mathematics Subject Classification (2010): 28A05

1. Introduction

Gamma Ray Bursts (GRBs) [1-2] is a gamma-ray intensity from the sky in the direction of a sudden increase in a short time, followed by a quick decline of the phenomenon, duration 0.1 to 1000 seconds, the radiation mainly in 0.1-100 MeV energy segment. GRB was found in 1967, for decades, people have to understand its nature is not very clear, but it is almost certain that occur on cosmological scales stellar celestial outbreak in the process.GRB astronomy is one of the most active areas of research, has twice been named in 1997 and 1999 the American magazine "Science" column of ten scientific and technological progress of the Year. Since type Ia supernovae observation team observed accelerating expansion of the universe cite [3], dark energy has become a modern cosmology one of the hottest research. However, even now, we know very little about the nature of dark energy. In order to research the nature of dark energy, we established many types of dark energy

model via obtained information. In many models of dark energy, CDM parametric model dark energy exists coincidence problems[4].We now know the basic structure of the universe according to the standard model of cosmology. There are about 5 percent of the energy content of the universe is made of ordinary baryons and roughly 27 percent consists of a yet-undetected matter component and approximately 68 percent of the dark energy. The first part is below the scope of particle physics and there have a great discovery of the higgs boson in recently [5]. There maining two parts accounted for 95 percent. As previous mentioned the yet-undetected matter component, which is thought to be a massive particle of non-baryonic nature that interacts through weak interaction and gravity only. It is called" cold dark matter". The last part is the energy content, which is the best candidate for dark energy.We can understand the changes in composition of the universe to understand the changes in the universe is accelerating expansion.

It is a huge challenge that accounting for two unknown components [6]. Still regarding the dynamical dark energy, there exists the possibility of interaction between dark energy and dark matter. Meanwhile, it is a possible interaction between dark energy and the other fields. We wanted to find a clue from dark energy section. At least, it is the greatest hope for us by now. In particular, the model of dark matter and dark energy interaction is the best candidate to alleviate the so-called coincidence problem.

Because we know very little about the dark energy and dark matter it is difficult to describe it through the first principles. In recent years, there are a lot of people want to thermodynamic way to describe it, but the effect is not very satisfactory. Broader approach is to use dark energy and dark matter interaction term Q [7],

$$\rho_x + 3H \ (\rho_x + p_x) = -Q, \tag{1}$$

$$\rho_{\rm m} + 3H\rho_{\rm m} = Q, \qquad (2)$$

which preserves the total energy conservation equation $\rho_{tot} + 3H \ (\ \rho_{tot} + p_{tot}) = 0$. If Q is a non-zero function of the scale factor, the interaction makes ρ_m and ρ_x to deviate from the standard scaling.

We will make extensive use of various observation data to limit the dark energy model [8]. In this paper, we mainly use the gamma-ray bursts date limited the dark energy model. In order to highlight the effect of GRBs, we also use other astronomical observations data to jointly limit the dark energy models. Including cosmic microwave background (CMB) observation from the Plank results and Baryon acoustic oscillations (BAO) data. We hope to find the relationship of dark energy and dark matter by observing these restrictions result data. We will use different criteria in the analysis comparing various different results of dark energy model [9]

This paper is organized as follows. We will discuss the information criteria in Section 2. In Section 3 we will give different models and constraining results. We will present

theoretical analysis of the constraint results and discuss it in Section 4. Finally, we summarize the main conclusions in Section 5.

2. Gamma-ray bursts data and other observations

Recently, as a probe to discover a supernova has become an important function of Gamma-Ray Bursts (GRBs) [10]. Gamma-Ray Bursts are the most violent explosions in the universe, in theory, when the fuel runs out massive stars explode or collapse two adjacent dense star resulting from the merger. Short gamma-ray bursts to the thousandth of a second [11], as long as a few hours, will release huge amounts of energy in a short time. If compared with the sun, the energy which is released in a few minutes is equivalent to the sum of the trillions of sunlight, the emission of a single photon energy is typically several tens of times the typical sunlight. GRBs data is divided into two kinds of high red shift and low red shift. We mainly use high red shift GRBs data in this work.

In this paper, we selected 79 high-red shift GRBs data [12]. The first four columns (100814A, 050318,110213A, 010222) are taken from the work of Wei and Qin and Chen and the last column is using the calibrated Amati relation to get it. These data are named May flower sample, and we always used it to constrain cosmological models. And now, we will briefly introduce Amati relation. Following e.g. [13], we define the Amati relation as

$$\log E_{\rm iso/erg} = \lambda + b \log E_p [300 \, KeV] \tag{3}$$

where "log" means the logarithm to base 10, but λ and b are the constants, their values are not the same when we use different model.

The full information of the Gamma-Ray Bursts data will be found in the table 3 of Wei et al. In this work, we will perform a standard Bayesian analysis to constrain the cosmological parameters by minimizing χ^2

$$\chi^{2}_{\text{GRB}} = \sum \frac{[\text{GRB}^{\text{th}}(i) - \text{GRB}^{\text{obs}}(i)]^{2}}{\sigma(i)^{2}}$$
⁽⁴⁾

where GRBth is the gamma ray burst value in the cosmological model and GRB^{obs} is the measured value with a uncertainty of σ (i)².

We combine the GRB data with the CMB observation from the Planck results and the BAO observation in order to break the degeneracy of model parameters. We also added 580 SN Ia data [14] to limit the dark energy model parameters, and compare the limitations results in other observational data. We use a different method to get CMB and BAO and SNIa data. We obtained by three different aspects of observing to get BAO data and obtained CMB data by Planck measurement and use moduli distance to get the SN Ia data.

3. Models and constraining results

From the viewpoint of the continuity equation £dark energy and dark matter interaction

term must be multiplied by a reciprocal action with time factor, which chosen as the Hubble factor H. About the interaction of the simplest model are [15]

$$Q_1 = 3 \gamma_m H \rho_m \tag{5}$$

and

$$Q_2 = 3 \gamma_x H \rho_x \tag{6}$$

where the constants γ_m and γ_x quantify extent of the interaction about dark matter and dark energy. There is another kind of interaction, but we can only get a parameter about the density $\left(\frac{\rho_x}{\rho_m} = \frac{\rho_{x_n}}{\rho_{x_n}} a^{\xi}\right)$ [16] of dark energy and dark matter from the phenomenon analysis, where ξ is a constant parameter that to quantify the coincidence problem's severity. We find that from the flat FRW universe the corresponding interaction term Q given by Dalal et al. (2001) and Guo et al (2007)

$$Q_{3} = \frac{-(1 - \Omega_{m})(\xi + 3\omega_{x})}{1 - \Omega_{m} + \Omega_{m}(1 + z)^{\xi}} H\rho_{m},$$
(7)

where Ω m is the present value of the dark matter's density parameter. We assume that in spatially flat FRW metric, the dark energy $\omega_x \equiv p/\rho$ by the equation of state (EoS) is a constant in the three phenomenological interaction models. In this paper, we use the matter density parameter to test the constraining power of GRBs, and use the CMB+BAO data as a priori data and combined with other data. We will constrain three interaction dark sectors and each sample the parameters with GRBs+SN+CMB+BAO, SN+CMB+BAO, and GRBs+CMB+BAO in the next work. Now, we will give the best-fit parameters (with 1σ uncertainties) in Table 1.

Table 1: The best-fit values (with the 1σ uncertainties) of the parameters in three IDE models with different data combinations, using BAO+CMB as the priors, all represents GRBs+SN+BAO+CMB, all-GRBs represents SN+BAO+CMB, all-SN means GRBs+BAO+CMB.

The γ_d IDE Model					
	$\Omega_{_{\mathrm{m}}}$	ω _x	γ_{d}		
all	$0.296^{\text{+0.014}}_{\text{-0.014}}(1\sigma)$	$-1.065^{\scriptscriptstyle +0.014}_{\scriptscriptstyle -0.067}(1\sigma)$	$-0.006^{+0.012}_{-0.012}(1\sigma)$		
all-GRB	$0.296_{\text{-0.014}}^{\text{+0.014}}(1\sigma)$	$-1.066^{+0.066}_{-0.067}(1\sigma)$	$-0.006^{+0.0001}_{-0.0120}(1\sigma)$		

all-SN	$0.239^{+0.035}_{-0.035}(1\sigma)$	1.772 ^{+0.449} _{-0.467} (1σ)	$-0.036^{+0.011}_{-0.041}(1\sigma)$		
priors	$0.242^{\scriptscriptstyle +0.043}_{\scriptscriptstyle -0.036}(1\sigma)$	$-1.828^{+0.576}_{-0.514}(1\sigma)$	$-0.040^{+0.021}_{-0.019}(1\sigma)$		
The γ_m IDE Model					
	$\Omega_{_{\mathrm{m}}}$	ω _x	$\gamma_{\rm d}$		
all	$0.295^{\text{+0.014}}_{\text{-0.014}}(1\sigma)$	$-1.060^{+0.061}_{-0.061}(1\sigma)$	$-0.002^{+0.004}_{-0.004}(1\sigma)$		
all-GRB	$0.295^{\text{+0.015}}_{\text{-0.015}}(1\sigma)$	$-1.059^{\scriptscriptstyle +0.061}_{\scriptscriptstyle -0.060}(1\sigma)$	$-0.002^{+0.004}_{-0.004}(1\sigma)$		
all-SN	$3.223^{+0.235}_{-0.254}(1\sigma)$	$-1.341^{+0.311}_{-0.289}(1\sigma)$	$-0.005^{+0.005}_{-0.005}(1\sigma)$		
priors	$0.251^{\tiny +0.036}_{\tiny -0.035}(1\sigma)$	$-1.556^{+0.388}_{-0.372}(1\sigma)$	$-0.005^{+0.004}_{-0.005}(1\sigma)$		

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The ξ IDE Model					
	$\Omega_{_{\mathrm{m}}}$	$\omega_{\rm x}$	${\cal Y}_{ m d}$		
all	$0.295_{-0.014}^{+0.014}(1\sigma)$	$-1.062^{+0.065}_{-0.065}(1\sigma)$	$3.217^{+0.253}_{-0.253}(1\sigma)$		
all-GRB	$0.295_{-0.014}^{+0.014}(1\sigma)$	$-1.064^{+0.066}_{-0.065}(1\sigma)$	$3.223^{+0.253}_{-0.254}(1\sigma)$		
all-SN	$0.269_{-0.031}^{+0.032}(1\sigma)$	$-1.468^{+0.336}_{-0.323}(1\sigma)$	$4.561^{_{+1.072}}_{_{-1.111}}(1\sigma)$		
priors	$0.307^{\scriptscriptstyle +0.011}_{\scriptscriptstyle -0.011}(1\sigma)$	$-1.002^{+0.094}_{-0.094}(1\sigma)$	$3.028^{+0.365}_{-0.367}(1\sigma)$		

A. The γ_{d} IDE model We take the most simple interaction model form $Q_1 = 3\gamma_m H \rho_m$, we can obtain the Hubble parameter [17].

$$E^{2}(z) = \frac{\omega_{x}\Omega_{m}}{\gamma_{d} + \omega_{x}} (1 + z)^{3(1-\gamma_{d})} + (1 - \frac{\omega_{x}\Omega_{m}}{\gamma_{d} + \omega_{x}})(1 + z)^{3(1+\omega_{x})}$$
(8)

where $\Omega_{\rm m} = 8\pi G \rho_{\rm m_0} / (3H_0^2)$ is the present fractional energy density of dark matter. In this work, we take $H_0, \Omega_{\rm m}, \omega_{\rm x}$ and $\gamma_{\rm m}$ as the free parameters. Firstly, we use minimizing the three-dimensional χ^2 function to determine the Hubble constant H_0 . We will statistical analyze the remaining parameters $\Omega_{\rm m}$ and $\omega_{\rm x}$ and $\gamma_{\rm m}$. We will display the results in Fig.1.



Figure 1: The 2-D regions with the 2 σ contours of parameters ω_x and γ_d , Ω_m and γ_d , Ω_m and ω_x in the γ_d IDE model. The BAO+CMB priors are shown in black line, the red dot represents the fits from GRBs+BAO+CMB+SN, the black line represent those from SN+BAO+CMB, and the blue line represent those from GRBs+BAO+CMB.

We will get the best fit value of the parameters what when we take joint restrictions of GRBs+BAO+CMB+SN, and they are $\Omega_{\rm m} = 0.296^{+0.014}_{-0.014}$, $\omega_{\rm x} = -1.065^{+0.014}_{-0.067}$, and $\gamma_{\rm d} = -0.006^{+0.012}_{-0.012}$. We found that they are within the margin of error analysis data in the Table 1. And we are also shows the restriction results of the model parameters ($\omega_{\rm x}$ and $\gamma_{\rm m}$, $\omega_{\rm x}$ and $\Omega_{\rm m}$, $\gamma_{\rm m}$ and $\Omega_{\rm m}$) in Fig. 1. We found that although the best-fit value is slightly larger than zero, but it shows that you can turn the dark matter to dark energy is, which for alleviate the coincidence problem is beneficial.

In order to highlight the effect of GRBs data, we used two different way of limitation in this paper, and we take that are SN+CMB+BAO without GRBs and GRBs+CMB+BAO without SN in Fig. 1. The analysis revealed that after joining the GRBs data, display better graphics tend to center. However, we find that the GRBs data on the parameter constraints'

the non-negligible effect in the other two following models.

B. The γ_m IDE model

Above us the simplest form, this time we take other dark energy density $Q_2 = 3\gamma_x H\rho_x$, we obtain the Hubble parameter



Figure 2: The same as Figure 1, but for the γ_m IDE model.

Though minimizing the total χ^2_{total} , our statistical analysis gives the matter density implied $\Omega_m = 0.295^{+0.015}_{-0.014}$. The dark energy parameters' best fit obtained are $\omega_x = -1.040^{+0.065}_{-0.065}$ and $\gamma_m = -0.002^{+0.004}_{-0.004}$. And we are also shows the restriction results of the model parameters (ω_x and γ_m , ω_x and Ω_m , γ_m and Ω_m) in Fig. 2. We found that although the best-fit value is slightly larger than zero, but it shows that you can turn the dark matter to the dark energy. In order to highlight the effect of GRBs data, we used two different way of limitation in this paper, and we take that are SN+CMB+BAO without GRBs and GRBs+CMB+BAO without SN in Fig. 2. The analysis revealed that after joining the GRBs data, display better graphics tend to center. The 79 GRBs data we use the calculated results are within the range of error in 1σ .

C. The ξIDE model

In the ξ IDE model, we give the interaction between dark matter and dark energy by

 $Q_{3} = \frac{-(1 - \Omega_{m})(\xi + 3\omega_{x})}{1 - \Omega_{m} + \Omega_{m}(1 + z)^{\xi}} H \rho_{m}, \text{ and the corresponding Hubble parameter now has}$

the form



Figure 3: The same as Figure 1, but for the *ξIDE* model

We will give the constrained results from the joint analysis in Fig. 1 and we can take the best fit is $\Omega_m = 0.295 \frac{+0.014}{-0.014}$, $\omega_x = -1.062 \frac{+0.065}{-0.065}$ and $\xi = 3.217 \frac{+0.253}{-0.253}$. We found that after the addition of GRBs data, it has played a very important role in limiting the results. We find another parameter $\gamma = -(\xi + 3\omega_x)$ in the other two models, and try to test the energy transfer about dark energy and dark matter. It is clear that the interaction's negative value is still favored in the framework of complicated ξ IDE model. We found that GRBs data plays a very important role in limiting the dark energy model by analyzing the data in Table 1. The analysis revealed that after joining the GRBs data, display better graphics tend to center.

Similarly, we found that the Λ CDM model is still supported within the range of error in 1 σ , and also played a very good effect limit. However, join GRBs data only alleviate the coincidence problem does not solve it. We also need other astronomical observation data to limit the dark energy model, and the model parameters come in different ways, to eventually solve the coincidence problem. To this end, we need to keep looking for a various of probe on interactions of dark energy and dark matter.

4. Analysis

We will use the information criteria (IC) to compare the three interacting DE models in this section and check the consistency of GRBs data and the dark energy model by best-fit parameters. We also verify the use of different models of dark energy and matter density parameter to alleviate the effect of coincidence problem.

In this paper, we mainly used different information criteria (IC) for discussion of our results, including the Akaike Information Criteria (AIC) [18] and Bayesian Information Criteria (BIC) and Kullback Information Criterion (KIC). By these criteria, we can get a variety of data to limit the results good or bad. We can use these results to elect those that best meet our requirements of model parameters. There are a lot of studies have been through these criterion get the results what they want.

The BIC is given by

$$BIC = -2\ln\zeta_{max} + k\ln N, \qquad (11)$$

the AIC is defined as

$$AIC = -2\ln\zeta_{\max} + 2k, \qquad (12)$$

and the KIC is defined as

$$KIC = -2\ln\zeta_{max} + 3k \tag{13}$$

where ζ_{max} is the maximum likelihood, k is the number of parameters, and N is the number of data points. Note that for Gaussian errors, $\chi^2_{min} = -2\ln \zeta_{max}$. we obtain χ^2_{min} and calculate their corresponding AIC, BIC and KIC values shown in Table 2.

analysis.					
IC	$\gamma_{ m m}$ IDE model	γ_d IDE model	ξ IDE model		
$\chi^2_{ m min}$	597.82	597.84	597.82		
BIC	623.84	623.86	623.84		
AIC	605.82	605.84	605.82		
KIC	609.82	609.84	609.82		

 Table 2: The information criteria (IC) values for the three models considered in this

In the one-on-one model comparison, model M_α with characterizing IC_α has the likelihood

$$P(M_{\alpha}) = \frac{\exp(-IC_{\alpha})/2}{\exp(-IC_{1}/2) + \exp(-IC_{2}/2)}$$
(14)

of being the correct choice, and the difference $\Delta IC = IC_2 - IC_1$ determines the extent to which M_1 is favored over M_2 . We compare these two sets of data $\gamma_m IDE$ and $\gamma_d IDE$ in χ^2_{min} £they only differ 0.02. Accordingly, we can calculate the probability of their respective $P(M_1) \approx 50\% - 51\%$ and $P(M_2) \approx 49\% - 50\%$. Similarly, we can compare the data of γ_m IDE and ξIDE model, we find that their values are the same. Then the probability of their respective is $P(M_1) \approx 50\% = P(M_2) \approx 50\%$. We find the results by comparing other sets of data is the same.

Through the above analysis, we know that limiting effect γ_d IDE model is the best

play, with the best-fit parameters from GRBs+BAO+CAB+SN. We can clearly see the difference between the theoretical and observed values. Through our analysis, we can clearly understand that our data can be used as cosmological probes to check the results obtained from other tests above.

5. Conclusion

In this article, we use the 79 GRBs data and CMB data and BAO and supernova data for three different dark energy interaction model for joint restrictions, which allow transfer between the dark energy and the dark matter. It also offers the possibility of alleviated the coincidence problem. With these models we can find interaction term Q can change the direction to dark matter density ($\propto \gamma_m \rho_m$), it also can change the direction to the dark energy density ($\propto \gamma_x \rho_x$) and the cosmological scaling factor ($\frac{\rho_x}{\rho_m} \propto a^{\xi}$) in power-law function

Firstly, we found that the interaction term Q value is very small, almost zero. We have found that the dark energy interaction parameters are within the error range through analysis of the data in the Table 1. Similarly, we also found that the addition GRBs data makes the display more visible, and are within the range of error in 1 σ . We compare the γ_d IDE model and the γ_m IDE model and the ξ IDE model are found their biggest difference is that the value of the γ_d and γ_m are negative, while in ξ is positive. We can also be found from the table one, with the addition of GRBs data later, restrictions on the model parameters Ω_m and w x also played a very good effect. Let's look at the graphic by COSMOMC program and after the processing by MATYLAB program. Through observation, we found that after adding the GRBs data, graphics have central tendency.

Secondly, we also offer three different types of Information Criteria (IC) results in the Table 2. We compared with two models of γ_m IDE and γ_d IDE, and we found that the γ_m IDE model provides better fits to observational data. Similarly, we also found that the ξ IDE

model played the same role with γ_m . According to compare A and B and C of the data in the Table 2, we find the same result. However, we are here not yet completely solved the problem coincidence, we are just proposed a possible in the framework of three interacting dark energy models.

Finally, we will get more GRBs data in the future. We will also get more astronomical observational data through a variety of means of observation. This time, our study raises the importance of the application of GRBs data limitations dark energy model. We also hope that in the future there can be more astronomical observations data apply to our study of dark energy model, including high red shift SNeIa from SDSS-II and SNLS collaborations and low red shift GRBs data and weak lensing survey combined CMB measurements.

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