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Pentagonal Numbers, Heptagonal Numbers and Pythagorean Triangles

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Abstract. Oblong numbers as figuret numbers, which were first studied by the Pythagoreans are studied in terms of special Pythagorean Triangles. The two consecutive sides and their perimeters of Pythagorean triangles are investigated. In this study, the perimeter of Pythagorean triangles are obtained as addition of pentagonal and heptagonal numbers.

Keywords: Pentagonal numbers, Heptagonal numbers, Pythagorean Triangles, Diophantine equation.

AMS Mathematics Subject Classification (2010): 11D09

1.Introduction

Mathematicians all over the ages have been fascinated by the Pythagorean Theorem and are solving many problems related to it thereby developing mathematics. Integral solutions [1] and [2] and special Pythagorean triangles are generated by [3,4]. [5] Have given perimeter of Pythagorean triangles with their perimeter as addition of pentagonal and heptagonal numbers. Such triangles with two consecutive sides and perimeter as addition of pentagonal and heptagonal numbers are also studied.

2. Method of analysis

The primitive solutions of the Pythagorean Equation,

$X^2+Y^2=Z^2$, is given by [5] (1)

$X=m^2-n^2$, $Y=2mn$, $Z=m^2+n^2$ (2)

for some integers m, n of opposite parity such that m>n>0 and (m, n)=1

2.1. Perimeter is an addition of pentagonal and heptagonal numbers

Definition 1. A natural number P is called addition of pentagonal and heptagonal number if it can be written in the form $\dfrac{(3\alpha^2-\alpha)}{2}+\dfrac{(5\alpha^2-3\alpha)}{2}=2(2\alpha^2-\alpha),\quad \alpha\in N$

If the perimeter of the Pythagorean triangle (X,Y,Z) is addition of pentagonal and heptagonal number α, then

$X + Y + Z = 2(2\alpha^2-\alpha) = P$ (3)

From the equations (2) and (3)

$2m^2 + 2mn = 2(2\alpha^2-\alpha)$, $\alpha \varepsilon N$.

$m(m + n) = \alpha(2\alpha-1)$ (4)

2.2. Hypotenuse and one leg are consecutive

In such cases, $m = n + 1$ (5)

This gives equation (4) as

$(n + 1) (2 n + 1) = \alpha(2\alpha-1)$

Take $\alpha = n + 1$ (6)

Equations (2), (5) & (6) give solution of equations (1) in correspondence with equations (3) and (4)

i.e., $X = 2 n + 1$;

$Y = 2 n (n + 1)$;

$Z = 2 n (n + 1) + 1$;

First ten such special Pythagorean triangles (X, Y, Z) are given in the Table 1 below:

Table 1: Special Pythagorean Triangles

S. No.	**n**	**α**	**P**	**X**	**Y**	**Z**
1	1	2	12	3	4	5
2	2	3	30	5	12	13
3	3	4	56	7	24	25
4	4	5	90	9	40	41
5	5	6	132	11	60	61
6	6	7	182	13	84	85
7	7	8	240	15	112	113
8	8	9	306	17	144	145
9	9	10	380	19	180	181
10	10	11	462	21	220	221

Table 2: Verification of $X^2+Y^2=Z^2$ and $X + Y + Z = 2\,\alpha(2\alpha\text{-}1)$

S.No	X^2	Y^2	X^2+Y^2	Z^2	$X + Y + Z = 2\,\alpha(2\alpha\text{-}1)$
1	9	16	25	25	12=2.2.3
2	25	144	169	169	30=2.3.5
3	49	576	625	625	56=2.4.7
4	81	1600	1681	1681	90=2.5.9
5	121	3600	3721	3721	132=2.6.11
6	169	7056	7225	7225	182=2.7.13
7	225	12544	12769	12769	240=2.8.15
8	289	20736	21025	21025	306=2.9.17
9	361	32400	32761	32761	380=2.10.19
10	441	48400	48841	48841	462=2.11.21

3. Observations and conclusion

1. $(X+Y-Z)^2=(Y+Z-2X+1)$
2. $(X+Z-Y)^2=(Y+Z+2X+1)$
3. $Y+Z=X^2$

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