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Status Elliptic Sombor and Modified Status Elliptic Sombor Indices of Graphs

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Abstract. In this study, we introduce the status elliptic Sombor and modified status elliptic Sombor indices and their corresponding exponentials of a graph. Furthermore, we compute these indices for wheel graphs and friendship graphs.

Keywords: status elliptic Sombor index, modified status elliptic Sombor index, graphs.

AMS Mathematics Subject Classification (2010): 05C10, 05C69

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1] for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

The elliptic Sombor index was introduced by Gutman et al. in [3] and it is defined as

$$ESO(G) = \sum_{uv \in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}.$$

Recently, some elliptic indices were studied in [4, 5, 6, 7, 8, 9].

The distance $d(u, v)$ between any two vertices u and v is the length of the shortest path connecting u and v. The status $\sigma(u)$ of a vertex u in G is the sum of distances of all other vertices from u in G.

We put forward a new topological index, defined as

$$SES(G) = \sum_{uv \in E(G)} (\sigma(u) + \sigma(v))\sqrt{\sigma(u)^2 + \sigma(v)^2}$$

which we propose to be named as the status elliptic Sombor index.

Considering the status elliptic Sombor index, we introduce the status elliptic Sombor exponential of a graph G and define it as

$$SES(G,x)=\sum_{uv\in E(G)} x^{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}.$$

We define the modified status elliptic Sombor index of a graph G as

$$^{m}SES(G)=\sum_{uv\in E(G)} \frac{1}{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}.$$

Considering the modified status elliptic Sombor index, we introduce the modified status elliptic Sombor exponential of a graph G and defined it as

$$^{m}SES(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}}.$$

Recently, some status indices were studied in [10, 11, 12] and some graph indices were studied in [13, 14].

In this paper, we determine the status elliptic Sombor index, modified status elliptic Sombor index and their corresponding exponentials of wheel graphs and friendship graphs.

2. Results for wheel graphs

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1vertices and 2n edges. A graph W_n is shown in Figure 1.

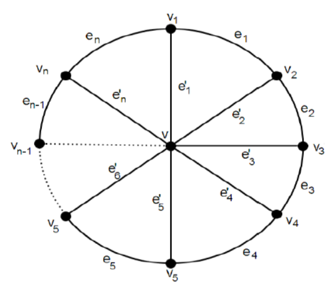

Figure 1: Wheel graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore there are two types of status edges as given in Table 1.

$\sigma(u), \sigma(v) \setminus uv \in E(W_n)$	$(2n-3, 2n-3)$	$(n, 2n-3)$
Number of edges	N	n

Table 1: Status edge partition of W_n

Theorem 2.1. Let $G= W_n$ be the wheel graph. Then

$$SES(G)=2\sqrt{2}n(2n-3)^2+3n(n-1)\sqrt{n^2+(2n-3)^2}.$$

Proof: From definition and by using Table 1, we deduce

$$SES(G) = \sum_{uv \in E(G)} (\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}$$
$$= n(2n-3+2n-3)\sqrt{(2n-3)^2+(2n-3)^2} + n(n+2n-3)\sqrt{n^2+(2n-3)^2}$$
$$= 2\sqrt{2}n(2n-3)^2 + 3n(n-1)\sqrt{n^2+(2n-3)^2}.$$

Theorem 2.2. Let $G = W_n$ be the wheel graph. Then

$$SES(G,x) = nx^{2\sqrt{2}(2n-3)^2} + nx^{3(n-1)\sqrt{n^2+(2n-3)^2}}.$$

Proof: From definition and by using Table 1, we deduce

$$SES(G,x) = \sum_{uv \in E(G)} x^{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}$$
$$= nx^{(2n-3+2n-3)\sqrt{(2n-3)^2+(2n-3)^2}} + nx^{(n+2n-3)\sqrt{n^2+(2n-3)^2}}$$
$$= nx^{2\sqrt{2}(2n-3)^2} + nx^{3(n-1)\sqrt{n^2+(2n-3)^2}}.$$

Theorem 2.3. Let $G = W_n$ be the wheel graph. Then

$$^{m}SES(G) = \frac{n}{2\sqrt{2}(2n-3)^2} + \frac{n}{3(n-1)\sqrt{n^2+(2n-3)^2}}.$$

Proof: From definition and by using Table 1, we obtain

$$^{m}SES(G) = \sum_{uv \in E(G)} \frac{1}{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}$$
$$= \frac{n}{(2n-3+2n-3)\sqrt{(2n-3)^2+(2n-3)^2}} + \frac{n}{(n+2n-3)\sqrt{n^2+(2n-3)^2}}$$
$$= \frac{n}{2\sqrt{2}(2n-3)^2} + \frac{n}{3(n-1)\sqrt{n^2+(2n-3)^2}}.$$

Theorem 2.4. Let $G = W_n$ be the wheel graph. Then

$$^{m}SES(G,x) = nx^{\frac{1}{2\sqrt{2}(2n-3)^2}} + nx^{\frac{1}{3(n-1)\sqrt{n^2+(2n-3)^2}}}.$$

Proof: From the definition and by using Table 1, we get

$$^{m}SES(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}}$$
$$= nx^{\frac{1}{(2n-3+2n-3)\sqrt{(2n-3)^2+(2n-3)^2}}} + 2nx^{\frac{1}{(n+2n-3)\sqrt{n^2+(2n-3)^2}}}$$
$$= nx^{\frac{1}{2\sqrt{2}(2n-3)^2}} + nx^{\frac{1}{3(n-1)\sqrt{n^2+(2n-3)^2}}}.$$

3. Results for friendship graphs

A friendship graph F_n, $n \geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is presented in Figure 2.

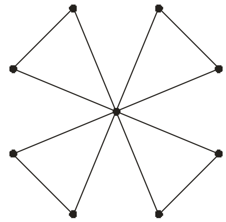

Figure 2: Friendship graph F_4

Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}, \qquad |E_1| = n.$$

$$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}, \qquad |E_2| = 2n.$$

Therefore, in F_n, there are two types of status edges as given in Table 2.

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	$(4n-2, 4n-2)$	$(2n, 4n-2)$
Number of edges	n	$2n$

Table 2: Status edge partition of F_n

Theorem 3.1. Let $G= F_n$ be the friendship graph. Then

$$SES(G) = 8\sqrt{2}n(2n-1)^2 + 4n(3n-1)\sqrt{4n^2 + (4n-2)^2}.$$

Proof: From definition and by using Table 2, we deduce

$$SES(G) = \sum_{uv \in E(G)} (\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2 + \sigma(v)^2}$$

$$= n(4n-2+4n-2)\sqrt{(4n-2)^2 + (4n-2)^2} + 2n(2n+4n-2)\sqrt{(2n)^2 + (4n-2)^2}$$

$$= 8\sqrt{2}n(2n-1)^2 + 4n(3n-1)\sqrt{4n^2 + (4n-2)^2}.$$

Theorem 3.2. Let $G= F_n$ be the friendship graph. Then

$$SES(G,x) = nx^{8\sqrt{2}(2n-1)^2} + 2nx^{2(3n-1)\sqrt{4n^2+(4n-2)^2}}.$$

Proof: From definition and by using Table 2, we deduce

$$SES(G,x) = \sum_{uv \in E(G)} x^{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}$$

$$= nx^{(4n-2+4n-2)\sqrt{(4n-2)^2+(4n-2)^2}} + 2nx^{(2n+4n-2)\sqrt{(2n)^2+(4n-2)^2}}$$

$$= nx^{8\sqrt{2}(2n-1)^2} + 2nx^{2(3n-1)\sqrt{4n^2+(4n-2)^2}}.$$

Theorem 3.3. Let $G= F_n$ be the friendship graph. Then

$$^{m}SES(G) = \frac{n}{8\sqrt{2}(2n-1)^2} + \frac{2n}{2(3n-1)\sqrt{4n^2+(4n-2)^2}}.$$

Proof: From definition and by using Table 2, we obtain

$$
\begin{aligned}
{}^{m}SES(G) &= \sum_{uv\in E(G)} \frac{1}{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}} \\
&= \frac{n}{(4n-2+4n-2)\sqrt{(4n-2)^2+(4n-2)^2}} \\
&\quad + \frac{2n}{(2n+4n-2)\sqrt{(2n)^2+(4n-2)^2}} \\
&= \frac{n}{8\sqrt{2}(2n-1)^2} + \frac{2n}{2(3n-1)\sqrt{4n^2+(4n-2)^2}}.
\end{aligned}
$$

Theorem 3.4. Let $G= F_n$ be the friendship graph. Then

$$
{}^{m}SES(G,x) = nx^{\frac{1}{8\sqrt{2}(2n-1)^2}} + 2nx^{\frac{1}{2(3n-1)\sqrt{4n^2+(4n-2)^2}}}.
$$

Proof: From definition and by using Table 2, we get

$$
\begin{aligned}
{}^{m}SES(G,x) &= \sum_{uv\in E(G)} x^{\frac{1}{(\sigma(u)+\sigma(v))\sqrt{\sigma(u)^2+\sigma(v)^2}}} \\
&= nx^{\frac{1}{(4n-2+4n-2)\sqrt{(4n-2)^2+(4n-2)^2}}} + 2nx^{\frac{1}{(2n+4n-2)\sqrt{(2n)^2+(4n-2)^2}}} \\
&= nx^{\frac{1}{8\sqrt{2}(2n-1)^2}} + 2nx^{\frac{1}{2(3n-1)\sqrt{4n^2+(4n-2)^2}}}.
\end{aligned}
$$

4. Conclusion

This paper computes the status elliptic Sombor index, modified status elliptic Sombor index, and their corresponding exponentials for certain graphs.

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Conflicts of interest. The author declares no conflicts of interest.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2019) 66-91.
3. I.Gutman, B.Furtula and M.S.Oz, Geometric approach to vertex degree based topological indices-Elliptic Sombor index theory and application, *International Journal of Quantum Chemistry*, 124(2) (2024) e27151.

4. V.R.Kulli, Modified elliptic Sombor index and its exponential of a graph, *International Journal of Mathematics and Computer Research*, 12(1) (2024) 3949-3954.
5. V.R.Kulli, Multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices, *Annals of Pure and Applied Mathematics*, 29(1) (2024) 19-23.
6. V.R.Kulli, Reverse elliptic Sombor and modified reverse elliptic Sombor indices, *International Journal of Mathematical Archive*, 15(1) (2024) 1-7.
7. V.R.Kulli, Elliptic Revan index and its exponential of certain networks, *International Journal of Mathematics and Computer Research*, 12(2) (2024) 4055-4061.
8. V.R.Kulli, Modified elliptic Revan index of certain two families of nanotubes, *Annals of Pure and Applied Mathematics*, 29(2) (2024) 103-107.
9. V.R.Kulli, Elliptic delta and modified elliptic delta indices of certain networks, *International Journal of Mathematics and its Applications*, 12(2) (2024) 105-116.
10. H.S. Ramane and A.S. Yalnaik, Status connectivity indices graphs and its applications to the boiling point of benzenoid hydrocarbons, *Journal of Applied Mathematics and Computing*, 55 (2017) 607-627.
11. H.S.Ramane, B.Basavagoud and A.S.Yalnaik, Harmonic status index of graphs, *Bulletin of Mathematical Sciences and Applications*, 17 (2016) 24-32.
12. V.R.Kulli, Computation of multiplicative (a, b)-status index of certain graphs, *Journal of Mathematics and Informatics* 18(2020) 45-50. DOI: http://dx.doi.org/10.22457/jmi.v18a4165.
13. V.R.Kulli and I.Gutman, Revan Sombor index, *Journal of Mathematics and Informatics*,22(2022)23-27. DOI: http://dx.doi.org/10.22457/jmi.v22a03205.
14. V.R.Kulli, The (a, b)-KA E-Banhatti indices of graphs, *Journal of Mathematics and Informatics*, 23 (2022) 55-60. DOI: http://dx.doi.org/10.22457/jmi.v23a06214.