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Computation of Multiplicative (*a, b*)-Status Index of Certain Graphs

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Abstract. The status of a vertex *u* is defined as the sum of the distance between *u* and all other vertices of a graph. In this study, we introduce the multiplicative (*a*, *b*)-status index of a graph. Also we present exact expressions for the multiplicative (*a*, *b*)-status index of wheel graphs and friendship graphs.

Keywords: Status of a vertex, distance, multiplicative (*a*, *b*)-status index, multiplicative *F*-status index, multiplicative symmetric division status index, graph

***AMS Mathematics Subject Classification*:** 05*C*05, 05*C*12, 05*C*35.

1. Introduction

Let $G = (V(G)\ E(G))$ be a finite, simple, connected graph. The degree $d_G(u)$ of a vertex *u* is the number of vertices adjacent to *u*. The distance $d(u, v)$ between any two vertices *u* and *v* is the length of shortest path containing *u* and *v*. The status, denoted by $\sigma(u)$, of a vertex *u* in *G* is the sum of the distances of all other vertices from *u* in G. We refer [1] for any undefined term and notation.

A graph index or a topological index is a numerical parameter mathematically derived from the graph structure. Several graph indices have found some applications in Theoretical Chemistry, especially in *QSPR*/*QSAR* research see [2, 3]. For survey on graph indices, one can refer [4].

In [5], Kulli introduced the multiplicative first status index of a graph, defined as

$$S_1II(G) = \prod_{uv\in E(G)} [\sigma(u) + \sigma(v)].$$

We define the multiplicative *F*-status index of a graph as

$$FSII(G) = \prod_{uv\in E(G)} \left[\sigma(u)^2 + \sigma(v)^2\right].$$

We introduce multiplicative first and second status Gourava indices, defined as

$$SGO_1II(G) = \prod_{uv\in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)].$$

$$SGO_2II(G) = \prod_{uv\in E(G)} \sigma(u)\sigma(v)[\sigma(u) + \sigma(v)].$$

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We propose the multiplicative symmetric division status index of a graph, defined as

$$SDSII(G) = \prod_{uv \in E(G)} \left[\frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)} \right].$$

Motivated by the work on multiplicative graph indices, we introduce the multiplicative (*a*, *b*)-status index of a graph, defined as

$$S_{a,b}II(G) = \prod_{uv \in E(G)} \left[\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a \right]$$

where *a* and *b* are real numbers.

Recently, the hyper Gourava indices were studied in [6]. Recently, some variants of status indices were introduced and studied such as first and second status connectivity indices [7], first and second hyper status indices [8], F_1-status index [9], harmonic status index [10], multiplicative vertex status index [11], (*a*, *b*)-status index [12], status connectivity coincides [13]. Recently, some different multiplicative indices were studied, for example, in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,28, 29,30,31].

In this paper, the multiplicative (*a, b*)-status index of wheel and friendship graphs were computed.

2. Observations

We see the following relationships from the above definitions

a) Multiplicative first status index $S_1II(G) = S_{1,0}II(G)$.

b) Multiplicative *F*-status index $FSII(G) = S_{2,0}II(G)$.

c) Multiplicative second status Gourava index $SGO_2II(G) = S_{2,1}II(G)$.

d) Multiplicative symmetric division status index $SDSII(G) = S_{1,-1}II(G)$.

3. Results for wheel graphs

A wheel graph W_n is the join of K_1 and C_n. A graph W_4 is depicted in Figure 1.

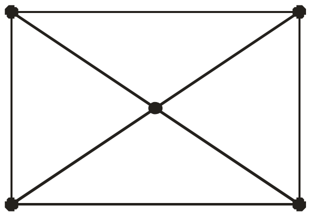

Figure 1: Wheel graph W_4

A wheel graph W_n has n+1 vertices and 2n edges. In W_n, there are two types of edges as given in Table 1.

$d_{W_n}(u), d_{W_n}(v) \setminus uv \in E(W_n)$	(3, 3)	(3, *n*)
Number of edges	*n*	*n*

Table 1: Edge partition of W_n

Thus there are two types of status edges as given Table 2.

$\sigma(u),\sigma(v)\setminus uv\in E(W_n)$	$(2n-3)$ $(2n-3)$	$(n, 2n-3)$
Number of edges	n	n

Table 2: Status edge partition of W_n

Theorem 1. The multiplicative (a, b)-status index of a wheel graph W_n is

$$S_{a,b}II(W_n)=\left[2(2n-3)^{a+b}\right]^n\times\left[n^a(2n-3)^b+n^b(2n-3)^a\right]^n.$$

Proof: From equation and by using Table 2, we derive

$$S_{a,b}II(W_n)=\prod_{uv\in E(W_n)}\left[\sigma(u)^a\sigma(v)^b+\sigma(u)^b\sigma(v)^a\right]$$

$$=\left[(2n-3)^a(2n-3)^b+(2n-3)^b(2n-3)^a\right]^n\times\left[n^a(2n-3)^b+n^b(2n-3)^a\right]^n$$

$$=\left[2(2n-3)^{a+b}\right]^n\times\left[n^a(2n-3)^b+n^b(2n-3)^a\right]^n.$$

We establish the following results from observations and by using Theorem 1.

Corollary 1.1. Let W_n be a wheel graph with n +1 vertices and $2n$ edges. Then

(1) $S_1II(W_n)=2^n(2n-3)^n(3n-3)^n.$

(2) $FSII(W_n)=2^n(2n-3)^{2n}\left(5n^2-12n+9\right)^n.$

(3) $SGO_2II(W_n)=2^n(2n-3)^{3n}\left(2n^3-n^2-3n\right)^n.$

(4) $SDSII(W_n)=2^n\left(\frac{5n^2-12n+9}{n(2n-3)}\right)^n.$

Theorem 2. The multiplicative first status Gourava index of a wheel graph W_n is

$$SGO_1II(W_n)=\left(4n^2-8n+3\right)^n\times\left(2n^2-3\right)^n.$$

Proof: From definition and by using Table 2, we derive

$$SGO_1II(W_n)=\prod_{uv\in E(W_n)}\left[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)\right]$$

$$=\left[(2n-3)+(2n-3)+(2n-3)(2n-3)\right]^n\times\left[n+2n-3+n(2n-3)\right]^n$$

$$=\left(4n^2-8n+3\right)^n\times\left(2n^2-3\right)^n.$$

4. Result for friendship graphs

A friendship graph F_n , $n\geq 2$, is a graph that can be constructed by joining n copies of C_3 with a common vertex. A graph F_4 is shown in Figure 2.

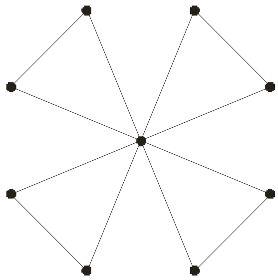

Figure 2: Friendship graph F_4

If F_n is a friendship graph, then F_n has $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as given in Table 3.

$d_{F_n}(u), d_{F_n}(v) \setminus uv \in E(F_n)$	(2, 2)	(2, 2*n*)
Number of edges	*N*	2*n*

Table 3: Edge partition of F_n

Thus F_n has two types of status edges as given in Table 4

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	(4*n* – 2) (4*n* – 2)	(2*n*, 4*n* – 2)
Number of edges	*n*	2*n*

Table 4: Status edge partition of F_n

Theorem 3. The multiplicative (*a*, *b*)-status index of a friendship graph F_n is

$$S_{a,b}II(F_n) = \left[2(4n-2)^{a+b}\right]^n \times \left[(2n)^a(2n-2)^b + (2n)^b(4n-2)^a\right]^{2n}.$$

Proof: From equation and by using Table 4 , we deduce

$$S_{a,b}II(F_n) = \prod_{uv \in E(F_n)} \left[\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a\right]$$

$$= \left[(4n-2)^a(4n-2)^b + (4n-2)^b(4n-2)^a\right]^n \times \left[(2n)^a(4n-2)^b + (2n)^b(4n-2)^a\right]^{2n}$$

$$= \left[2(4n-2)^{a+b}\right]^n \times \left[(2n)^a(2n-2)^b + (2n)^b(2n-3)^a\right]^{2n}.$$

From observations and by using Theorem 3, we obtain the following results.

Corollary 3.1. Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(1) $S_1II(F_n) = (8n-4)^n(6n-2)^{2n}$.

(2) $FSII(F_n) = \left[2(4n-2)^2\right]^n(20n^2-16n+4)^{2n}$.

(3) $SGO_2II(F_n) = 2^n(4n-2)^{3n}(48n^3-40n^2+8n)^{2n}$.

(4) $SDSII(F_n) = 2^n \left(\frac{5n^2 - 4n + 1}{2n^2 - n} \right)^{2n}$.

Theorem 4. The multiplicative second status Gourava index of a friendship graph F_n is

$$SGO_1II(F_n) = (16n^2 - 8n)^n (8n^2 + 2n - 2)^{2n}.$$

Proof: from definition and by using Table 4, we obtain

$$SGO_1II(F_n) = \prod_{uv \in E(F_n)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)]$$

$$= [4n - 2 + 4n - 2 + (4n-2)(2n-2)]^n \times [2n + 4n - 2 + 2n(4n-2)]^{2n}$$

$$= (16n^2 - 8n)^n \times (8n^2 + 2n - 2)^{2n}.$$

5. Conclusion

In this paper, the expressions for the multiplicative (*a*, *b*)-status index , multiplicative *F*-status index, multiplicative first and second status Gourava indices of wheel graphs and friendship graphs have been computed.

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