Computation of Multiplicative $(a, b)$-Status Index of Certain Graphs

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

Received 12 December 2019; accepted 25 January 2020

Abstract. The status of a vertex $u$ is defined as the sum of the distance between $u$ and all other vertices of a graph. In this study, we introduce the multiplicative $(a, b)$-status index of a graph. Also we present exact expressions for the multiplicative $(a, b)$-status index of wheel graphs and friendship graphs.

Keywords: Status of a vertex, distance, multiplicative $(a, b)$-status index, multiplicative $F$-status index, multiplicative symmetric division status index, graph

AMS Mathematics Subject Classification: 05C05, 05C12, 05C35.

1. Introduction

Let $G = (V(G), E(G))$ be a finite, simple, connected graph. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. The distance $d(u, v)$ between any two vertices $u$ and $v$ is the length of shortest path containing $u$ and $v$. The status, denoted by $\sigma(u)$, of a vertex $u$ in $G$ is the sum of the distances of all other vertices from $u$ in $G$. We refer [1] for any undefined term and notation.

A graph index or a topological index is a numerical parameter mathematically derived from the graph structure. Several graph indices have found some applications in Theoretical Chemistry, especially in QSPR/QSAR research see [2, 3]. For survey on graph indices, one can refer [4].

In [5], Kulli introduced the multiplicative first status index of a graph, defined as

$$S,II(G) = \prod_{u \in E(G)} [\sigma(u) + \sigma(v)].$$

We define the multiplicative $F$-status index of a graph as

$$FSII(G) = \prod_{u \in E(G)} [\sigma(u)^2 + \sigma(v)^2].$$

We introduce multiplicative first and second status Gourava indices, defined as

$$SGO,II(G) = \prod_{u \in E(G)} [\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)].$$

$$SGO,II(G) = \prod_{u \in E(G)} [\sigma(u)\sigma(v)[\sigma(u) + \sigma(v)].$$
Computation of Multiplicative \((a, b)\)-Status Index of certain Graphs

We propose the multiplicative symmetric division status index of a graph, defined as

\[
SDS\text{II}(G) = \prod_{uv \in E(G)} \left[ \frac{\sigma(u)}{\sigma(v)} + \frac{\sigma(v)}{\sigma(u)} \right].
\]

Motivated by the work on multiplicative graph indices, we introduce the multiplicative \((a, b)\)-status index of a graph, defined as

\[
S_{a,b}\text{II}(G) = \prod_{uv \in E(G)} \left[ \sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a \right]
\]

where \(a\) and \(b\) are real numbers.

Recently, the hyper Gourava indices were studied in [6]. Recently, some variants of status indices were introduced and studied such as first and second status connectivity indices [7], first and second hyper status indices [8], \(F_1\)-status index [9], harmonic status index [10], multiplicative vertex status index [11], \((a, b)\)-status index [12], status connectivity coincides [13]. Recently, some different multiplicative indices were studied, for example, in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,28, 29,30,31].

In this paper, the multiplicative \((a, b)\)-status index of wheel and friendship graphs were computed.

2. Observations
We see the following relationships from the above definitions

a) Multiplicative first status index \(S_{1,0}\text{II}(G) = S_{1,0}\text{II}(G)\).

b) Multiplicative \(F\)-status index \(FS\text{II}(G) = S_{2,0}\text{II}(G)\).

c) Multiplicative second status Gourava index \(SG_{0,2}\text{II}(G) = S_{2,1}\text{II}(G)\).

d) Multiplicative symmetric division status index \(SDS\text{II}(G) = S_{1,-1}\text{II}(G)\).

3. Results for wheel graphs
A wheel graph \(W_n\) is the join of \(K_1\) and \(C_n\). A graph \(W_4\) is depicted in Figure 1.

![Figure 1: Wheel graph \(W_4\)](image)

A wheel graph \(W_n\) has \(n+1\) vertices and \(2n\) edges. In \(W_n\), there are two types of edges as given in Table 1.

<table>
<thead>
<tr>
<th>(d_{w_1}(u), d_{w_2}(v) \setminus uv \in E(W_n))</th>
<th>((3, 3))</th>
<th>((3, n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>(n)</td>
<td>(n)</td>
</tr>
</tbody>
</table>

*Table 1: Edge partition of \(W_n\)*
Thus there are two types of status edges as given Table 2.

\[
\begin{array}{ccc}
\sigma(u), \sigma(v) \backslash uv \in E(W_n) & (2n - 3) & (2n - 3) \\
\text{Number of edges} & n & n \\
\end{array}
\]

Table 2: Status edge partition of \( W_n \)

**Theorem 1.** The multiplicative \((a, b)\)-status index of a wheel graph \( W_n \) is

\[
S_{a,b}I(W_n) = \left[2(2n-3)^{a+b}\right]^n \times \left[n^a(2n-3)^b + n^b(2n-3)^a\right]^n.
\]

**Proof:** From equation and by using Table 2, we derive

\[
S_{a,b}I(W_n) = \prod_{uv \in E(W_n)} \left[\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a\right]
\]

\[
= \left[(2n-3)^a(2n-3)^b + (2n-3)^b(2n-3)^a\right]^n \times \left[n^a(2n-3)^b + n^b(2n-3)^a\right]^n
\]

\[
= \left[2(2n-3)^{a+b}\right]^n \times \left[n^a(2n-3)^b + n^b(2n-3)^a\right]^n.
\]

We establish the following results from observations and by using Theorem 1.

**Corollary 1.1.** Let \( W_n \) be a wheel with \( n + 1 \) vertices and \( 2n \) edges. Then

1. \( S_1I(W_n) = 2^n(2n-3)^n(3n-3)^n \).
2. \( FSII(W_n) = 2^n(2n-3)^2^n(5n^2 - 12n + 9)^n \).
3. \( SGO_1I(W_n) = 2^n(2n-3)^3^n(2n^3 - n^2 - 3n)^n \).
4. \( SDSII(W_n) = 2^n\left(\frac{5n^2 - 12n + 9}{n(2n-3)}\right)^n \).

**Theorem 2.** The multiplicative first status Gourava index of a wheel graph \( W_n \) is

\[
SGO_1I(W_n) = (4n^2 - 8n + 3)^n \times (2n^2 - 3)^n.
\]

**Proof:** From definition and by using Table 2, we derive

\[
SGO_1I(W_n) = \prod_{uv \in E(W_n)} \left[\sigma(u) + \sigma(v) + \sigma(u)\sigma(v)\right]
\]

\[
= \left[(2n-3) + (2n-3) + (2n-3)(2n-3)\right]^n \times \left[n + 2n - 3 + n(2n-3)\right]^n
\]

\[
= (4n^2 - 8n + 3)^n \times (2n^2 - 3)^n.
\]

**4. Result for friendship graphs**

A friendship graph \( F_n \), \( n \geq 2 \), is a graph that can be constructed by joining \( n \) copies of \( C_3 \) with a common vertex. A graph \( F_4 \) is shown in Figure 2.
Computation of Multiplicative \((a, b)\)-Status Index of certain Graphs

If \( F_n \) is a friendship graph, then \( F_n \) has \( 2n+1 \) vertices and \( 3n \) edges. By calculation, we obtain that there are two types of edges as given in Table 3.

<table>
<thead>
<tr>
<th>( d_{F_n}(u), d_{F_n}(v) \setminus uv \in E(F_n) )</th>
<th>((2, 2))</th>
<th>((2, 2n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>( N )</td>
<td>( 2n )</td>
</tr>
</tbody>
</table>

**Table 3: Edge partition of \( F_n \)**

Thus \( F_n \) has two types of status edges as given in Table 4.

<table>
<thead>
<tr>
<th>( \sigma(u), \sigma(v) \setminus uv \in E(F_n) )</th>
<th>((4n - 2)) ((4n - 2))</th>
<th>((2n, 4n - 2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>( n )</td>
<td>( 2n )</td>
</tr>
</tbody>
</table>

**Table 4: Status edge partition of \( F_n \)**

**Theorem 3.** The multiplicative \((a, b)\)-status index of a friendship graph \( F_n \) is

\[
S_{a,b}II(F_n) = \left[ 2(4n - 2)^a (4n - 2)^b \right]^{2n} \times \left[ (2n)^a (2n - 2)^b + (2n)^b (4n - 2)^a \right]^{2n}.
\]

**Proof:** From equation and by using Table 4, we deduce

\[
S_{a,b}II(F_n) = \prod_{\omega \in \Omega(F_n)} \left[ \sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a \right]^{2n}.
\]

\[
= \left[ (2(4n - 2)^a (4n - 2)^b + (4n - 2)^b (4n - 2)^a) \right]^{2n} \times \left[ (2n)^a (4n - 2)^b + (2n)^b (4n - 2)^a \right]^{2n}.
\]

From observations and by using Theorem 3, we obtain the following results.

**Corollary 3.1.** Let \( F_n \) be a friendship graph with \( 2n+1 \) vertices and \( 3n \) edges. Then

1. \( S_{a,b}II(F_n) = (8n - 4)^a (6n - 2)^{2n} \).
2. \( FSII(F_n) = \left[ 2(4n - 2)^b \right]^{2n} (20n^2 - 16n + 4)^{2n} \).
3. \( SGO_{2}II(F_n) = 2^n (4n - 2)^{3n} (48n^3 - 40n^2 + 8n)^{2n} \).
Theorem 4. The multiplicative second status Gourava index of a friendship graph $F_n$ is

$$SGO_{II}(F_n) = (16n^2 - 8n)^n (8n^2 + 2n - 2)^{2n}.$$  

Proof: from definition and by using Table 4, we obtain

$$SGO_{II}(F_n) = \prod_{\sigma \in \mathcal{E}(F_n)} \left[ \sigma(u) + \sigma(v) + \sigma(u)\sigma(v) \right]$$

$$= [4n^2 - 2n - 2 + (4n - 2)(2n - 2)]^n \times [2n + 4n - 2 + 2n(4n - 2)]^{2n}$$

$$= (16n^2 - 8n)^n \times (8n^2 + 2n - 2)^{2n}.$$  

5. Conclusion

In this paper, the expressions for the multiplicative $(a, b)$-status index, multiplicative $F$-status index, multiplicative first and second status Gourava indices of wheel graphs and friendship graphs have been computed.

REFERENCES

12. V.R.Kulli, The $(a, b)$-status index of graphs, submitted.
Computation of Multiplicative ($a$, $b$)-Status Index of certain Graphs


