

Computing some Multiplicative Temperature Indices of Certain Networks

V.R.Kulli

Department of Mathematics
Gulbarga University, Kalaburgi (Gulbarga) 585 106, India
E-mail: vrkulli@gmail.com

Received 2 May 2020; accepted 4 June 2020

Abstract. A topological index or a graph index is a numerical parameter mathematically derived from the graph structure. In this paper, we introduce the multiplicative first temperature index, multiplicative total temperature index, multiplicative modified first temperature index, multiplicative temperature inverse degree, multiplicative temperature zeroth order index, multiplicative F -temperature index and multiplicative general first temperature index of a graph. We compute these newly defined indices for oxide networks and honeycomb networks.

Keywords: temperature, multiplicative first temperature index, multiplicative F -temperature index, general multiplicative first temperature index, oxide network, honeycomb network.

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C12

1. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v . We refer [1] for undefined term and notation.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences and Medical Sciences. In Mathematical Chemistry, several topological indices have found some applications, especially in QSPR/QSAR study [2, 3].

The temperature of a vertex u in G is defined as [4]

$$T(u) = \frac{d_G(u)}{n - d_G(u)}$$

where n is the number of vertices of G .

We introduce the following multiplicative temperature indices.

The multiplicative first temperature index of a graph G is defined as

$$T_1H(G) = \prod_{u \in V(G)} T(u)^2.$$

The multiplicative modified first temperature index of a graph G is defined as

V.R.Kulli

$${}^m T_1 H(G) = \prod_{u \in V(G)} \frac{1}{T(u)^2}.$$

The multiplicative total temperature index of a graph G is defined as

$$TTH(G) = \prod_{u \in V(G)} T(u).$$

The multiplicative temperature inverse degree of a graph G is defined as

$$TIDH(G) = \prod_{u \in V(G)} \frac{1}{T(u)}.$$

The multiplicative temperature zeroth order index of a graph G is defined as

$$TZH(G) = \prod_{u \in V(G)} \frac{1}{\sqrt{T(u)}}.$$

The multiplicative F -temperature index of graph G is defined as

$$FTH(G) = \prod_{u \in V(G)} T(u)^3.$$

The general multiplicative temperature index of a graph G is defined as

$$T_1^a H(G) = \prod_{u \in V(G)} T(u)^a,$$

where a is a real number.

Recently, some new temperature indices were studied in [5, 6, 7, 8, 9]. Also recently, some multiplicative indices were studied for example in [10, 11, 12, 13, 14]. In this study, we compute some newly defined multiplicative temperature indices of oxide and honeycomb networks.

2. Oxide networks

An oxide network of dimension n is denoted by OX_n . Oxide networks are of vital importance in the study of silicate networks. A 5-dimensional oxide network is presented in Figure 1.

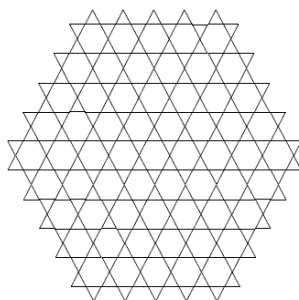


Figure 1: 5-dimensional oxide network

Theorem 1. The multiplicative general first temperature index of OX_n is

$$T_1^a H(OX_n) = 2^{18an^2} \times \left(\frac{1}{9n^2 + 3n - 2} \right)^{6an} \times \left(\frac{1}{9n^2 + 3n - 4} \right)^{3a(3n^2 - n)} \quad (i)$$

Computing some Multiplicative Temperature Indices of Certain Networks

Proof: From Figure 1, we see that the vertices of OX_n are either of degree 2 or 4. By calculation, we find that OX_n has $9n^2 + 3n$ vertices and $18n^2$ edges. We partition the vertex set of OX_n into two sets as follows:

$$\begin{aligned} V_1 &= \{u \in V(OX_n) \mid d(u) = 2\}, & |V_1| &= 6n. \\ V_2 &= \{u \in V(OX_n) \mid d(u) = 4\}, & |V_2| &= 9n^2 - 3n. \end{aligned}$$

Thus we find the vertex partition based on the temperature of the vertices as given in Table 1.

$T(u) \setminus u \in V(OX_n)$	$\frac{2}{9n^2 + 3n - 2}$	$\frac{4}{9n^2 + 3n - 4}$
Number of vertices	$6n$	$9n^2 - 3n$

Table 1: Vertex partition of OX_n

By definition and by using Table 1, we derive

$$\begin{aligned} T_1^a H(OX_n) &= \prod_{u \in V(OX_n)} T(u)^a \\ &= \left(\frac{2}{9n^2 + 3n - 2} \right)^{6an} \times \left(\frac{4}{9n^2 + 3n - 4} \right)^{a(9n^2 - 3n)} \\ &= 2^{18an^2} \left(\frac{1}{9n^2 + 3n - 2} \right)^{6an} \times \left(\frac{1}{9n^2 + 3n - 4} \right)^{3a(3n^2 - n)} \end{aligned}$$

We obtain the following results by using Theorem 1.

Corollary 1.1. Let OX_n be an oxide network of dimension n . Then

- (1) $T_1 H(OX_n) = 2^{36n^2} \times \left(\frac{1}{9n^2 + 3n - 2} \right)^{12n} \times \left(\frac{1}{9n^2 + 3n - 4} \right)^{6(3n^2 - n)}$.
- (2) ${}^m T_1 H(OX_n) = \frac{1}{2^{36n^2}} \times (9n^2 + 3n - 2)^{12n} \times (9n^2 + 3n - 4)^{6(3n^2 - n)}$.
- (3) $TTH(OX_n) = 2^{18n^2} \times \left(\frac{1}{9n^2 + 3n - 2} \right)^{6n} \times \left(\frac{1}{9n^2 + 3n - 4} \right)^{3(3n^2 - n)}$.
- (4) $TIDH(OX_n) = \frac{1}{2^{18n^2}} \times (9n^2 + 3n - 2)^{6n} \times (9n^2 + 3n - 4)^{3(3n^2 - n)}$.
- (5) $TZH(OX_n) = \frac{1}{2^{9n^2}} \times (9n^2 + 3n - 2)^{3n} \times (9n^2 + 3n - 4)^{\frac{3}{2}(3n^2 - n)}$.
- (6) $FTH(OX_n) = 2^{54n^2} \times \left(\frac{1}{9n^2 + 3n - 2} \right)^{18n} \times \left(\frac{1}{9n^2 + 3n - 4} \right)^{9(3n^2 - n)}$.

Proof: Put $a = 2, -2, 1, -1, -1/2, 3$ in equation (i), we obtain the desired results.

3. Honeycomb networks

Honeycomb network of dimension n is denoted by HC_n , where n is the number of hexagons between central and boundary hexagon. These networks are useful in Computer

Graphics and also in Chemistry. A 4-dimensional honeycomb network is shown in Figure 2.

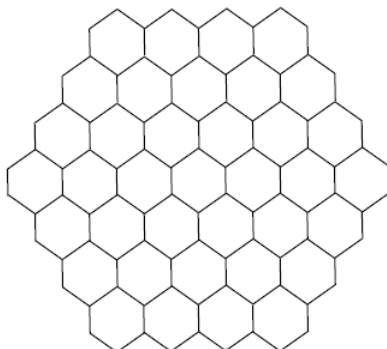


Figure 2: 4-dimensional honeycomb network

Theorem 2. The multiplicative general temperature index of HC_n is

$$T_1^a H(HC_n) = \left(\frac{1}{3n^2 - 1}\right)^{6an} \times \left(\frac{1}{2n^2 - 1}\right)^{6a(n^2 - n)} \quad (\text{ii})$$

Proof: Let HC_n be a honeycomb network of dimension n . Then the vertices of HC_n are either of degree 2 or 3. By calculation, we find that HC_n has $6n^2$ vertices and $9n^2 - 3n$ edges. We partition the vertex set of HC_n into two sets as follows:

$$\begin{aligned} V_1 &= \{u \in V(HC_n) \mid d(u) = 2\}, & |V_1| &= 6n. \\ V_2 &= \{u \in V(HC_n) \mid d(u) = 3\}, & |V_2| &= 6n^2 - 6n. \end{aligned}$$

Thus we obtain the vertex partition based on the temperature of the vertices of HC_n as given in Table 2.

$T(u) \setminus u \in V(HC_n)$	$\frac{2}{6n^2 - 2}$	$\frac{3}{6n^2 - 3}$
Number of vertices	$6n$	$6n^2 - 6n$

Table 2: Vertex partition of HC_n

By definition and by using Table 2, we obtain

$$\begin{aligned} T_1^a H(HC_n) &= \prod_{u \in V(HC_n)} T(u)^a = \left(\frac{2}{6n^2 - 2}\right)^{a6n} \times \left(\frac{3}{6n^2 - 3}\right)^{a(6n^2 - 6n)} \\ &= \left(\frac{1}{3n^2 - 1}\right)^{6an} \times \left(\frac{1}{2n^2 - 1}\right)^{6a(n^2 - n)} \end{aligned}$$

From Theorem 2, we establish the following results.

Corollary 2.1. Let HC_n be a honeycomb network of dimension n . Then

- (1) $T_1 H(HC_n) = \left(\frac{1}{3n^2 - 1}\right)^{12n} \times \left(\frac{1}{2n^2 - 1}\right)^{12(n^2 - n)}$.
- (2) ${}^m T_1 H(HC_n) = (3n^2 - 1)^{12n} \times (2n^2 - 1)^{12(n^2 - n)}$.

Computing some Multiplicative Temperature Indices of Certain Networks

$$(3) \quad TTII(HC_n) = \left(\frac{1}{3n^2-1}\right)^{6n} \times \left(\frac{1}{2n^2-1}\right)^{6(n^2-n)}.$$

$$(4) \quad TIDII(HC_n) = (3n^2-1)^{6n} \times (2n^2-1)^{6(n^2-n)}.$$

$$(5) \quad TZII(HC_n) = (3n^2-1)^{3n} \times (2n^2-1)^{3(n^2-n)}.$$

$$(6) \quad FTII(HC_n) = \left(\frac{1}{3n^2-1}\right)^{18n} \times \left(\frac{1}{2n^2-1}\right)^{18(n^2-n)}.$$

Proof: Put $a = 2, -2, 1, -1, -1/2, 3$ in equation (ii), we get the desired results.

4. Conclusion

In this paper, we have computed the multiplicative first temperature index, multiplicative modified first temperature index, multiplicative total temperature index, multiplicative temperature inverse degree, multiplicative temperature zeroth order index, multiplicative F -temperature index, multiplicative general temperature index of oxide and honeycomb networks.

REFERENCES

1. V.R.Kulli, *Multiplicative Connectivity Indices of Nanostructures*, LAP LEMBERT Academic Publishing, (2018).
2. R.Todeschini and V.Consonni, *Handbook of Molecular Descriptors for Chemo-informatics*, Wiley-VCH, Weinheim, (2009).
3. V.R.Kulli, *Collegiate Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
4. S.Fajtlowicz, On conjectures of Graffiti-II, *Congr. Numer* 60 (1987) 187-197.
5. V.R.Kulli, Computation of some temperature indices of $HC_5C_5[p,q]$ nanotubes, *Annals of Pure and Applied Mathematics*, 20(2) (2019) 69-74.
6. V.R.Kulli, The (a, b) -temperature index of H-Naphtalenic nanotubes, *Annals of Pure and Applied Mathematics*, 20(2) (2019) 85-90.
7. V.R.Kulli, Some multiplicative temperature indices of $HC_5C_7[p,q]$ nanotubes, *International Journal of Fuzzy Mathematical Archive*, 17(2) (2019) 91-98.
8. V.R.Kulli, Multiplicative (a, b) -temperature index of H-Naphtalenic nanotubes, submitted.
9. T.K.Atework, P.N.Kisori and S.Dickson, Atom bond connectivity temperature index, *Journal of Mathematical Nanoscience*, 8(2) (2018) 67-75.
10. V.R.Kulli, Multiplicative Gourava indices of armchair and zigzag polyhex nanotubes, *Journal of Mathematics and Informatics*, 17 (2019) 107-112.
11. V.R.Kulli, Multiplicative connectivity KV indices of dendrimers, *Journal of Mathematics and Informatics*, 15 (2019) 1-7.
12. V.R.Kulli, Computation of multiplicative (a, b) -status index of certain graphs, *Journal of Mathematics and Informatics*, 18 (2020) 50-55.
13. V.R.Kulli, Computation of multiplicative status Gourava indices of graphs, *International Journal of Mathematical Archive*, 11(5) (2020) 23-28.
14. V.R.Kulli, Computation of multiplicative status indices of graphs, *International Journal of Mathematical Archive*, 11(4) (2020) 1-6.