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Alpha set of Double Bi-Topological Space

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Abstract. The concept of intuitionistic topology or double topology is an important topic that addresses many of life's uncertainties. The aim of this paper is to define the concept of alpha set and the concept of neighborhood set on a double bi-topological space and to study the most important properties of these concepts.

Keywords: Double set; Đɳɱ$\alpha - open$ set; Đɳɱα-neighborhood point

AMS Mathematics Subject Classification (2010): 11N45

1. Introduction

The first to introduce the topic of intuitionistic set or double set is the mathematician Coker [4]. In 1998, Coker studied the concept of topology and the concept of neighborhood [6] around the same set. After that a lot of studies about this set were mentioned and we mention them. In 2000, an introduction to intuitionistic topological spaces [5] was studied. In 2014, generalized preregular closed sets were defined by the double set [8]. In the same year, the double set was used to define the concept of semi open [11]. In 2016, notes were made about semi open double set [7], study of the concept of fixed point around double set [9], and definition of intuitionistic β-open sets [1]. In 2017, a topic titled common fixed points of occasionally weakly compatible in intuitionistic fuzzy metric space was launched [10]. In 2019, the topic generalized closed set in intuitionistic fuzzy topology was studied. [2]. In 2020, the topic of some properties of double minimal space [3] was studied.In this paper we will know alpha set on double bi-topological space and study its most important properties.

2. Preliminaries

In this section, we remember some basic concepts related to double sets.

Definition 2.1. [4] Double set (ĐS for short) $\ddot{A}$ of a non-void set Ӿ is $\ddot{A} = \langle A', A'' \rangle$ where $A', A'' \subseteq$ Ӿ and $A' \cap A'' = \emptyset$.

Definition 2.2. [2] If $\ddot{A} = \langle A', A'' \rangle$, $\ddot{B} = \langle B', B'' \rangle$ and $\{\ddot{A}_i : i \in I\}$ are double sets in Ӿ, where $\ddot{A}_i = \langle A'_i, A''_i \rangle$. Then

1. $\ddot{A} \subseteq_Đ \ddot{B} \Leftrightarrow A' \subseteq B' \& B'' \subseteq A''$.

2. $\ddot{A} = \ddot{B} \Leftrightarrow \ddot{A} \subseteq_{Đ} \ddot{B} \& \ddot{B} \subseteq_{Đ} \ddot{A}$.
3. $\cup_{Đ} \ddot{A}_i = \langle \cup A'_i, \cap A''_i \rangle$.
4. $\cap_{Đ} \ddot{A}_i = \langle \cap A'_i, \cup A''_i \rangle$.
5. $\ddot{Ẋ}_{Đ} = \langle Ẋ, \emptyset \rangle$.
6. $\ddot{\emptyset}_{Đ} = \langle \emptyset, Ẋ \rangle$.
7. $cop.\ddot{A} = \langle A'', A' \rangle$.
8. $\ddot{A} \setminus \ddot{B} = \ddot{A} \cap_{Đ} \ddot{B}$.

Definition 2.3. [4] Double point of a non-void set Ẋ is $\ddot{p}_{Đ} = \langle \{p\}, \{p\}^c \rangle$, where $p \in Ẋ$.

Definition 2.4. [4] A collection $\mathfrak{I}_{Đ}$ of double sets of Ẋ called double topology (ĐT) if is satisfying the following axioms:

1. $\ddot{Ẋ}_{Đ}, \ddot{\emptyset}_{Đ} \in \mathfrak{I}_{Đ}$,
2. $\ddot{A} \cap_{Đ} \ddot{B} \in \mathfrak{I}_{Đ}$, for each $\ddot{A}, \ddot{B} \in \mathfrak{I}_{Đ}$,
3. $\cup_{Đ} \ddot{A}_i \in \mathfrak{I}_{Đ}$ for each $\ddot{A}_i \in \mathfrak{I}_{Đ}$.

the pair $(Ẋ, \mathfrak{I}_{Đ})$ is called a double topological space (ĐTS) and any double set belong to ĐT called Đ-open set. $cop.(\ddot{A})$ is Đ-closed if and only if Ä is Đ-open.

Definition 2.5. [6] Let $(Ẋ, \mathfrak{I}_{Đ})$ be an ĐTS and Ä is double set. Then
$int_{Đ}(\ddot{A}) = \cup_{Đ} \{\ddot{G} : \ddot{G} \in \mathfrak{I}_{Đ} \text{ and } \ddot{G} \subseteq_{Đ} \ddot{A}\}$.
$cl_{Đ}(\ddot{A}) = \cap_{Đ} \{\ddot{F} : \ddot{F} \text{ } is \text{ } Đ - closed \text{ and } \ddot{A} \subseteq_{Đ} \ddot{F}\}$.
Obvious

1. $int_{Đ}(\ddot{A})$ is Đ-open and $cl_{Đ}(\ddot{A})$ is Đ- closed.
2. Ä is Đ-open if and only if $int_{Đ}(\ddot{A}) = \ddot{A}$.
3. Ä is Đ-closed if and only if $cl_{Đ}(\ddot{A}) = \ddot{A}$.

3. Đɳɱα −open set in double bi-topological space

In this section we will define a new open set called Đɳɱα −Open Set and explain the most important characteristics of this set.

Definition 3.1. Let $(Ẋ, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space. Then a double subset $\ddot{A}$ of Ẋ is called Đɳɱ$\alpha - open$ set if $\ddot{A} \subseteq_{Đ} ɳ - int_{Đ}\left(ɱ - cl_{Đ}\left(ɳ - int_{Đ}(\ddot{A})\right)\right)$, where

$$ɳ \neq ɱ, \qquad ɳ, ɱ = 1,2.$$

$Đɳɱ\alpha.O(Ẋ) = \{\ddot{A} : \ddot{A} \text{ } is \text{ } Đɳɱ\alpha - open \text{ } set\}$.

Example 3.2. Let $Ẋ = \{ẋ_1, ẋ_2, ẋ_3\}, \mathfrak{I}_{Đ1} = \{\ddot{Ẋ}_{Đ}, \ddot{\emptyset}_{Đ}, \langle \{ẋ_1\}, \{ẋ_2, ẋ_3\} \rangle\}$ and
$\mathfrak{I}_{Đ2} = \{\ddot{Ẋ}_{Đ}, \ddot{\emptyset}_{Đ}, \langle \{ẋ_1\}, \{ẋ_2, ẋ_3\} \rangle, \langle \{ẋ_1, ẋ_2\}, \{ẋ_3\} \rangle\}$. Then
$Đ12\alpha - open = Đ21\alpha.O(Ẋ) = \{\ddot{Ẋ}_{Đ}, \ddot{\emptyset}_{Đ}, \langle \{ẋ_1\}, \{ẋ_2, ẋ_3\} \rangle, \langle \{ẋ_1, ẋ_2\}, \{ẋ_3\} \rangle, \langle \{ẋ_1, ẋ_3\}, \{ẋ_2\} \rangle\}$.

Remark 3.3. Not necessary $Đ12\alpha.O(Ẋ) \neq Đ21\alpha - open$ set in general, for example.

Example 3.4. Let $(\mathrm{X}, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space where $\mathrm{X} = \{x_1, x_2, x_3\}$, $\mathfrak{I}_{Đ1} = \{\ddot{\mathrm{X}}_{Đ}, \ddot{\emptyset}_{Đ}, \langle\{x_1\}, \{x_2, x_3\}\rangle\}$ and
$\mathfrak{I}_{Đ2} = \{\ddot{\mathrm{X}}_{Đ}, \ddot{\emptyset}_{Đ}, \langle\{x_2\}, \{x_1, x_3\}\rangle, \langle\{x_3\}, \{x_1, x_2\}\rangle, \langle\{x_2, x_3\}, \{x_1\}\rangle\}$.
Then $\ddot{A} = \langle\{x_1\}, \{x_2, x_3\}\rangle$ is a $Đ12\alpha - open$ but not $Đ21\alpha - open$.

Lemma 3.5. Let $(\mathrm{X}, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space. Then a double subset $\ddot{A}$ of X is $Đ\eta\mathrm{m}\alpha - open$ if and only if there exists $\ddot{B} \in \mathfrak{I}_{Đ\eta}$ such that $\ddot{B} \subseteq_{Đ} \ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{B})\right)$.

Proof: Let $\ddot{A}$ be $Đ\eta\mathrm{m}\alpha - open$ set, then $\ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}(\ddot{A})\right)\right)$, since $\eta - int_{Đ}(\ddot{A}) \subseteq_{Đ} \ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}(\ddot{A})\right)\right)$ and since $\eta - int_{Đ}(\ddot{A})$ is $\mathfrak{I}_{Đi}$-open set, put $\ddot{B} = \eta - int_{Đ}(\ddot{A})$. Hence $\ddot{B} \subseteq_{Đ} \ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{B})\right)$.

Conversely, let $\ddot{B} \in \mathfrak{I}_{Đi}$ such that $\ddot{B} \subseteq_{Đ} \ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{B})\right)$, so $\ddot{B} = \eta - int_{Đ}(\ddot{B}) \subseteq_{Đ} \eta - int_{Đ}(\ddot{A})$ and we have $\ddot{B} \subseteq_{Đ} \eta - int_{Đ}(\ddot{A}) \subseteq_{Đ} \ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{B})\right)$ and $\mathrm{m} - cl_{Đ}(\ddot{B}) \subseteq_{Đ} \mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}(\ddot{A})\right)$, $\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{B})\right) \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}(\ddot{A})\right)\right)$. So $\ddot{B} \subseteq_{Đ} \ddot{A} \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{B})\right) \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}(\ddot{A})\right)\right)$. Hence $\ddot{A}$ is $Đ\eta\mathrm{m}\alpha - open$ set.

Lemma 3.6. Let $(\mathrm{X}, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space. Then

$$\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right)\right) = \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right).$$

Proof: Since $\eta - int_{Đ}\left(cl_{Đ}(\ddot{A})\right) \subseteq_{Đ} \mathrm{m} - cl_{Đ}(\ddot{A})$,
then $\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right) \subseteq_{Đ} \mathrm{m} - cl_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right) = \mathrm{m} - cl_{Đ}(\ddot{A})$ and we have

$$\eta - \eta nt_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right)\right) \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right) \quad (1)$$

Also $\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right) \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)$, then $\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right) \subseteq_{Đ} \mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right)$, so $\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right) = \eta - int_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right) \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right)\right)$ and we have

$$\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right) \subseteq_{Đ} \eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right)\right) \quad (2)$$

From (1) and (2) we get $\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}\left(\eta - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)\right)\right) = i - int_{Đ}\left(\mathrm{m} - cl_{Đ}(\ddot{A})\right)$.

Lemma 3.7. If $\ddot{B}$ is $Ð\eta m\alpha - open$ set and $\ddot{B} \subseteq_{Ð} Ä \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{B})\right)$, then $Ä$ is also $Ð\eta m\alpha - open$ set.

Proof: Let $\ddot{B}$ be $Ð\eta m\alpha - open$ set, then there exist $\ddot{C} \in \Im_{Ð\eta}$ such that $\ddot{C} \subseteq_{Ð} \ddot{B} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)$ and $\ddot{B} \subseteq_{Ð} \ddot{A} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{B})\right)$, there fore $\ddot{C} \subseteq_{Ð} \ddot{B} \subseteq_{Ð} \ddot{A} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)$.

But $\eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right) \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{B})\right)$, and $\ddot{B} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)$,

then $\eta - int_{Ð}\left(m - cl_{Ð}(\ddot{B})\right) \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)\right)\right)$, so

$\ddot{C} \subseteq_{Ð} \ddot{B} \subseteq_{Ð} \ddot{A} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{B})\right) \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)\right)\right) = \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)$, there fore we have $\ddot{C} \subseteq_{Ð} \ddot{A} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}(\ddot{C})\right)$. Hence $\ddot{A}$ is $Ð\eta m\alpha - open$ set.

Lemma 3.8. Every $\Im_{Ð\eta} - open$ set is $Ð\eta m\alpha - open$ set.

Proof: Let $\ddot{A}$ be $\Im_{Ð\eta} - open$ set, then $\eta - int_{Ð}(\ddot{A}) = \ddot{A}$, but $\eta - int_{Ð}(\ddot{A}) \subseteq_{Ð} m - cl_{Ð}\left(\eta - int_{Ð}(\ddot{A})\right)$, so $\eta - int_{Ð}(\ddot{A}) = \eta - int_{Ð}\left(\eta - int_{Ð}(\ddot{A})\right) \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}(\ddot{A})\right)\right)$, so $\ddot{A} \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}(\ddot{A})\right)\right)$. Hence $\ddot{A}$ is $Ð\eta m\alpha - open$ set.

Remark 3.9. The converse of (Lemma 3.8) is not true in general for (Example 3.2), $\langle\{x_1, x_3\}, \{x_2\}\rangle$ is a $Ð12\alpha - open$ set but not $\Im_{Ð1} - open$ set.

Proposition 3.10. The double union of $Ð\eta m\alpha - open$ set is $Ð\eta m\alpha - open$ set.

Proof: Let $\ddot{A}_\lambda$ be $Ð\eta m\alpha - open$ set foe every $\lambda \in \Lambda$, then $\ddot{A}_\lambda \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}(\ddot{A}_\lambda)\right)\right)$, for every $\lambda \in \Lambda$. Since $\cup_{Ð}\, int_{Ð}(\ddot{A}_\lambda) \subseteq_{Ð} int_{Ð}(\cup_{Ð}\, \ddot{A}_\lambda)$ and $\cup_{Ð}\, cl_{Ð}(\ddot{A}_\lambda) \subseteq_{Ð} cl_{Ð}(\cup_{Ð}\, \ddot{A}_\lambda)$, then we have

$\bigcup_{\lambda\in\Lambda} \ddot{A}_\lambda \subseteq_{Ð} \bigcup_{\lambda\in\Lambda} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}(\ddot{A}_\lambda)\right)\right) \subseteq_{Ð} \eta - int_{Ð}\left(\bigcup_{\lambda\in\Lambda} m - cl_{Ð}\left(\eta - int_{Ð}(\ddot{A}_\lambda)\right)\right) \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\bigcup_{\lambda\in\Lambda} \eta - int_{Ð}(\ddot{A}_\lambda)\right)\right) \subseteq_{Ð} \eta - int_{Ð}\left(m - cl_{Ð}\left(\eta - int_{Ð}\left(\bigcup_{\lambda\in\Lambda} \ddot{A}_\lambda\right)\right)\right)$.

Hence $\bigcup_{\lambda\in\Lambda} \ddot{A}_\lambda$ is $Ð\eta m\alpha - open$ set.

Remark 3.11. The double intersection of two $Đɳɱ\alpha - open$ sets not necessary $Đɳɱ\alpha - open$ set for example

Example 3.12. Let $(Ẋ, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space where $Ẋ = \{ẋ_1, ẋ_2, ẋ_3, ẋ_4\}$,
$\mathfrak{I}_{Đ1} = \{\ddot{Ẋ}_Đ, \ddot{\emptyset}_Đ, \langle\{ẋ_1\}, \{ẋ_2, ẋ_3\}\rangle, \langle\{ẋ_4\}, \{ẋ_1, ẋ_2, ẋ_3\}\rangle, \langle\{ẋ_1, ẋ_4\}, \{ẋ_2, ẋ_3\}\rangle, \langle\{ẋ_2, ẋ_3\}, \{ẋ_1, ẋ_4\}\rangle, \langle\{ẋ_1, ẋ_2, ẋ_3\}, \{ẋ_4\}\rangle, \langle\{ẋ_2, ẋ_3, ẋ_4\}, \{ẋ_1\}\rangle\}$
and $\mathfrak{I}_{Đ2} = \{\ddot{Ẋ}_Đ, \ddot{\emptyset}_Đ, \langle\{ẋ_1\}, \{ẋ_2, ẋ_3\}\rangle, \langle\{ẋ_4\}, \{ẋ_1, ẋ_2, ẋ_3\}\rangle, \langle\{ẋ_1, ẋ_4\}, \{ẋ_2, ẋ_3\}\rangle\}$.
Hence $\langle\{ẋ_1, ẋ_3\}, \{ẋ_2, ẋ_4\}\rangle$ and $\langle\{ẋ_2, ẋ_3\}, \{ẋ_1, ẋ_4\}\rangle$ are $Đ12\alpha - open$ sets but $\langle\{ẋ_1, ẋ_3\}, \{ẋ_2, ẋ_4\}\rangle \cap_Đ \langle\{ẋ_2, ẋ_3\}, \{ẋ_1, ẋ_4\}\rangle = \langle\{ẋ_3\}, \{ẋ_1, ẋ_2, ẋ_4\}\rangle$ is not $Đ12\alpha - open$ set.

4. $Đɳɱ\alpha$-neighborhood of double point $\ddot{p}_Đ$

In this section, we will know a new neighborhood around a point called ($Đɳɱ\alpha - \text{nhd}$) and explain the most important characteristics of this set

Definition 4.1. Let $(Ẋ, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space, and $\ddot{p}_Đ$ double point of $Ẋ$. $\ddot{N}$ is $Đɳɱ\alpha - nhd$ of $\ddot{p}_Đ$ if and only if there is $Đɳɱ\alpha - open$ set $\ddot{U}$ such that $\ddot{p}_Đ \in \ddot{U} \subseteq_Đ \ddot{N}$.
$Đɳɱ\alpha.N(\ddot{p}_Đ) = \{\ddot{N} : \ddot{N}\ is\ Đɳɱ\alpha - nhd\ of\ \ddot{p}_Đ\}$.

Lemma 4.2. Let $(Ẋ, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space, and $\ddot{p}_Đ$ double point of $Ẋ$. Then every $\mathfrak{I}_{Đɳ}$-nhd of $\ddot{p}_Đ$ is $Đɳɱ\alpha - nhd$ of $\ddot{p}_Đ$.
Proof: Assume $\ddot{N}$ is $\mathfrak{I}_{Đɳ}$-nhd of $\ddot{p}_Đ$, then there is $\mathfrak{I}_{Đɳ}$-open set $\ddot{U}$ such that $\ddot{p}_Đ \in \ddot{U} \subseteq_Đ \ddot{N}$, by (Lemma 3.8) we get $\ddot{U}$ is $Đɳɱ\alpha - open$ set. Hence $\ddot{N}$ is $Đɳɱ\alpha - nhd$ of $\ddot{p}_Đ$.

Proposition 4.3. If $\ddot{N} \in Đɳɱ\alpha.O(Ẋ)$, then $\ddot{N}$ is $Đɳɱ\alpha - nhd$ of its double points.
Proof: Let $\ddot{N}$ be $Đɳɱ\alpha - open$ set then by (Definition 4.1) $\ddot{N}$ is $Đɳɱ\alpha - nhd$ of its double points.

Theorem 4.4. Let $(Ẋ, \mathfrak{I}_{Đ1}, \mathfrak{I}_{Đ2})$ be a double bi-topological space. Then

1. $\forall\ \ddot{p}_Đ \in Ẋ, Đɳɱ\alpha.N(\ddot{p}_Đ) \neq \ddot{\emptyset}_Đ$.
2. $\forall\ \ddot{N} \in Đɳɱ\alpha.N(\ddot{p}_Đ)$, then $\ddot{p}_Đ \in \ddot{N}$.
3. If $\ddot{N} \in Đɳɱ\alpha.N(\ddot{p}_Đ), \ddot{N} \subseteq_Đ \ddot{M}$, then $\ddot{M} \in Đɳɱ\alpha.N(\ddot{p}_Đ)$.
4. If $\ddot{N} \in Đɳɱ\alpha.N(\ddot{p}_Đ)$, then there exists $\ddot{M} \in Đɳɱ\alpha.N(\ddot{p}_Đ)$ such that $\ddot{M} \subseteq_Đ \ddot{N}$ and $\ddot{M} \in Đɳɱ\alpha.N(\ddot{q}_Đ), \forall\ \ddot{q}_Đ \in \ddot{M}$.

Proof:

1. Since $Ẋ$ is $Đɳɱ\alpha - open$ set, it is an $Đɳɱ\alpha.nhd$ of every $\ddot{p}_Đ \in Ẋ$. Hence $Ẋ \in Đɳɱ\alpha.N(\ddot{p}_Đ)$. Hence $Đɳɱ\alpha.N(\ddot{p}_Đ) \neq \ddot{\emptyset}_Đ$, $\forall\ \ddot{p}_Đ \in Ẋ$.
2. Direct from definition
3. Obvious
4. If $\ddot{N} \in Đɳɱ\alpha.N(\ddot{p}_Đ)$, then there exists a $Đɳɱ\alpha - open$ set $\ddot{M}$ such that $\ddot{p}_Đ \in \ddot{M} \subseteq_Đ \ddot{N}$. Since $\ddot{M}$ is $Đɳɱ\alpha - open$ set, then is $Đɳɱ\alpha - nhd$ of each point. Therefore $\ddot{M} \in Đɳɱ\alpha.N(\ddot{q}_Đ), \forall\ \ddot{q}_Đ \in \ddot{M}$.

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