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An Algorithmic Approach on Finding Edge Dominating Set of Fuzzy Graph

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Abstract. In this paper strong adjacency matrix of a fuzzy graph is discussed. Some properties of the strong adjacency matrix are also given. An algorithm is discussed to find the edge dominating sets of a fuzzy graph. And another algorithm is also discussed to find a maximal edge dominating set of a fuzzy graph. Suitable examples are also given.

Keywords: Strong adjacency matrix, edge dominating set, maximal edge dominating set

AMS Mathematics Subject Classification (2010): 03E72, 05C40, 05C72

1. Introduction

The study of dominating sets in graphs was started by Ore and Berge [2,10] and the domination number was introduced by Cockayne and Hedetniemi [3]. The concept of fuzzy relation was introduced by Zadeh [13] in his classical paper in 1965. Rosenfeld [11] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Somasundram and Somasundram [12] discussed domination in fuzzy graphs using effective edges. NagoorGani and Chandrasekaran [4] discussed domination in fuzzy graph using strong arcs. Domination, independent domination and irredundance in fuzzy graphs using strong arcs was discussed by Nagoor Gani and Vadivel [7,8]. Nagoor Gani and Prasanna Devi [5,6] discussed edge domination and independence in fuzzy graphs and also discussed about accurate edge dominating set and maximal edge dominating set in fuzzy graphs. Here we formulate algorithms to find the edge dominating and maximal edge dominating sets.

2. Preliminaries

A fuzzy graph $G = <\sigma, \mu>$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all $x,y \in V$, we have $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$. A fuzzy graph $H = <\tau, \rho>$ is called a fuzzy subgraph of G if $\tau(v_i) \leq \sigma(v_i)$ for all $v_i \in V$ and $\rho(v_i, v_j) \leq \mu(v_i, v_j)$ for all $v_i, v_j \in V$. An edge in G is called an isolated edge if it is not adjacent to any edge in G. A fuzzy graph $G = <\sigma, \mu>$ is a complete fuzzy graph if $\mu(v_i, v_j) = \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \sigma^*$. An arc (x,y) in a fuzzy graph $G = <\sigma, \mu>$ is said to be strong if

$\mu^{\infty}(x,y) = \mu(x,y)$. The strong neighborhood of an edge e_i in a fuzzy graph G is $N_s(e_i) = \{e_j \in E(G) / e_j$ is a strong arc in G and adjacent to $e_i\}$. Let e_i and e_j be two edges of a fuzzy graph G. We say that e_i dominates e_j if e_i is a strong arc in G and adjacent to e_j. A subset D of E(G) is said to be an edge dominating set of G if for every $e_j \in E(G)–D$ there exist $e_i \in D$ such that e_i dominates e_j.The smallest number of edges in any edge dominating set of G is called its edge domination number and it is denoted by $\gamma'(G)$. An edge dominating set D of a fuzzy graph G is a maximal edge dominating set if $S - D$ is not an edge dominating set of G, where S is the set of all strong arcs in G. The maximal edge domination number, $\gamma'_m(G)$ of G is the minimum cardinality of a maximal edge dominating set of G. A square matrix is said to be a symmetric matrix if its transpose is equal to itself, that is, matrix A is symmetric if $A = A^T$.

3. Algorithms

Here we introduce strong adjacency matrix of a fuzzy graph. And we formulate two algorithms to find the edge dominating sets and maximal edge dominating sets of a fuzzy graph respectively.

3.1. Strong adjacency matrix

In this section we introduce strong adjacency matrix of a fuzzy graph. And some properties of the strong adjacency matrix are also discussed.

Definition 3.1.1. Let G be a fuzzy graph with nnodes and m arcs. Then the **strong adjacency matrix**$A(G) = A = [a_{ij}]_{m\times m}$is defined as

$$a_{ij} = \begin{cases} 1, & \text{if } e_i \in N_S(e_i) \\ 0, & \text{otherwise} \end{cases}, \quad i \neq j \text{ and } a_{ii} = \begin{cases} 1, & \text{if } e_i \in S \\ 0, & \text{otherwise} \end{cases}$$

where S is the set of all strong arcs in G.

Example 3.1.1.

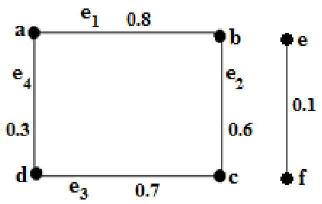

Figure 1: G

Here $S = \{e_1, e_2, e_3, e_5\}$. The strong adjacency matrix A(G) is given as

$$A(G) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Properties of strong adjacency matrix

1. The strong adjacency matrix A(G) need not to be a symmetric matrix.

2. If the fuzzy graph G has only strong arcs then $A(G)$ is a symmetric matrix.
3. If G is a complete fuzzy graph then $a_{ii} = 1, \ \forall i$ but the converse needs not to be true.

3.1.1. Remark

1. If an edge $e_i$is not a strong arc then the row corresponding to the edgee_i, has all zero entries and vice versa.
2. If an edge e is an isolated edge then there will be only one ‘1’ entry in the row corresponding to the edgee.

Example 3.1.2. In the above example, Fig. 1, e_4 is not a strong arc thus the row corresponding toe_4 has all entries as zero and e_5 is an isolated edge thus the row corresponding toe_5 have only one $'1'$ entry.

3.2. Algorithms

Here in this section we introduce two algorithms. One algorithm to find the edge dominating sets of a fuzzy graph and another to find the maximal edge dominating sets of the fuzzy graphs. And suitable examples are given.

Algorithm 3.2.1 Let G be a fuzzy graph and let S be the set of all strong arcs of G.
Step 1: Write the strong adjacency matrix $A = [a_{ij}]$ for the given fuzzy graph G.
Let D = { }
Step 2: Choose an edge $e_i \in S - D$ (arbitrarily) and pute_i in D i.e., D = D∪ {e_i}.
Step 3: Let the sub matrix Bof Abe formed by the rows corresponding to the edges inD.
Step 4: If the sub matrix Bhas atleast one ‘1’ entry in each column of Bthen D is an edge dominating set of G.
Otherwise goto Step 2 and repeat the process.

Note 3.2.1 Every superset of an edge dominating set of G with only strong arcs, is also an edge dominating set of G.

Example 3.2.1.

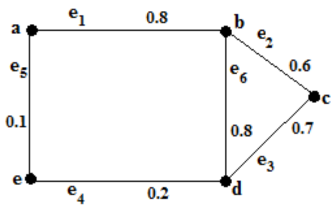

Figure 2: G

Here $S = \{e_1, e_3, e_4, e_6\}$
Step 1: The strong adjacency matrix $A(G)$ is given as

$$A(G) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) & N_S(e_6) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

And let $D = \{\ \}$

Step 2: $e_1 \in S - D$ and $D = \{e_1\}$

Step 3: The sub matrix B is

$$B = \begin{array}{c} \\ e_1 \end{array} \begin{array}{c} \begin{array}{cccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) & N_S(e_6) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Step 4: The sub matrix Bdoes not has atleast one $'1'$ entry in each column of B.

Now goto Step 2.

Step 2: $e_3 \in S - D$ and $D = \{e_1, e_3\}$

Step 3: $B = \begin{array}{c} \\ e_1 \\ e_3 \end{array} \begin{array}{c} \begin{array}{cccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) & N_S(e_6) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$

Step 4: The sub matrix Bhas atleast one $'1'$ entry in each of its column.

Thus $D = \{e_1, e_3\}$ is an edge dominating set of the given fuzzy graph G.

Algorithm 3.2.2 Let G be a fuzzy graph and let S be the set of all strong arcs of G.

Step 1: Write the strong adjacency matrix $A = [a_{ij}]$ for the given fuzzy graph G.

Let D = {}

Step 2: Choose an edge$e_i \in S{-}D$ (arbitrarily) and pute_i in D i.e., D = D∪ {e_i}.

Step 3: Let the sub matrix Bof Abe formed by the rows corresponding to the edges inD.

Step 4: If the sub matrix Bhas atleast one $'1'$in each column then goto Step 5.

Otherwise goto Step 2.

Step 5: Define a new matrix $C = [c_{ij}]$ as follows

$$c_{ij} = \begin{cases} a_{ij}, & \text{if} e_i \notin D \\ 0, & \text{otherwise} \end{cases}$$

If at least one column of C has all entries as zero then D is a maximal edge dominating set of G. Otherwise goto Step 2 and repeat the process.

Example 3.2.2.

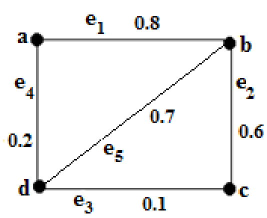

Figure 3: G

Here $S = \{e_1, e_2, e_5\}$

Step 1: The strong adjacency matrix A(G) is given as

$$A(G) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

And let D = { }

Step 2: $e_1 \in S - D$ and then $D = \{e_1\}$

Step 3: The sub matrix Bis

$$B = \begin{array}{c} \\ e_1 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ [\,1 \quad 1 \quad 0 \quad 1 \quad 1\,] \end{array}$$

Step 4: The sub matrix B does not has $'1'$ entry in 3rd column. Then goto Step 2.

Step 2: $e_2 \in S - D$ and then $D = \{e_1, e_2\}$

Step 3: Now $B = \begin{array}{c} \\ e_1 \\ e_2 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$

Step 4: Each column has at least $'1'$one entry in the sub matrix B. Goto Step 5.

Step 5: The new matrix Cis as follows

$$C = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

No column of Chas all entries as zero. Then goto Step 2.

Step 2: $e_5 \in S - D$ and then $D = \{e_1, e_2, e_5\}$

Step 3: Now $B = \begin{array}{c} \\ e_1 \\ e_2 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$

Step 4: Bhas atleast one $'1'$ entry in each column. Therefore goto Step 5.

Step 5: Now the matrix C is as follows

$$C = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Column of Chas all entries as zero.

Thus $D = \{e_1, e_2, e_5\}$ is a maximal edge dominating set of G.

Example 3.2.3. For the same fuzzy graph G in the above example (Fig. 3) let us find another maximal edge dominating set. Here $S = \{e_1, e_2, e_5\}$

Step 1: The strong adjacency matrix A(G) is given as

$$A(G) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

And let $D = \{\ \}$

Step 2: $e_1 \in S - D$ and then $D = \{e_1\}$

Step 3: The sub matrix Bis

$$B = \begin{array}{c} \\ e_1 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ [\,1 \quad 1 \quad 0 \quad 1 \quad 1\,] \end{array}$$

Step 4: The sub matrix B does not has $'1'$ entry in 3rd column. Then goto Step 2.

Step 2: $e_5 \in S - D$ and then $D = \{e_1, e_5\}$

Step 3: Now $B = \begin{array}{c} \\ e_1 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$

Step 4: Each column of B has at least $'1'$ one entry. Goto Step 5.

Step 5: The new matrix C is as follows

$$C = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \begin{array}{ccccc} N_S(e_1) & N_S(e_2) & N_S(e_3) & N_S(e_4) & N_S(e_5) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Here the 4th column has only zero entries.

Therefore $D = \{e_1, e_5\}$ is a maximal edge dominating set of G.

6. Conclusion

We defined strong adjacency matrix of a fuzzy graph. Some properties of strong adjacency matrix are also given. Using this strong adjacency matrix we formulated two algorithm one to find the edge dominating sets and another one to find the maximal edge dominating sets of a fuzzy graph. Some examples are also given. Further works are to formulate algorithm to find the accurate edge dominating set of a fuzzy graph.

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