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BD–Domination in Graphs

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Abstract. A vertex v is a boundary vertex of u if $d(u, w) \leq d(u, v)$ for all $w \in N(v)$. A vertex u have more than one boundary vertex at different distance levels. A vertex v is called a boundary neighbour of u if v is a nearest boundary of u. A set $S \subseteq V(G)$ is a bd – dominating set such that every vertex in V-S has at least one neighbour and at least one boundary neighbour in S. The cardinality of the minimum bd - dominating set is called the bd - domination number and is denoted by $\gamma_{bd}(G)$.In this paper we present several bounds on the bd - domination number and exact values of particular graphs.

Keywords: Boundary vertex, boundary neighbour, bd – dominating set

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1. Introduction and preliminaries

Let G be a finite, simple, undirected graph on n vertices with vertex set V(G) and edge set E(G). For graph theoretic terminology refer to Harary [5], Buckley and Harary [3].

Definition 1.1. The **open neighborhood**N(u) of a vertex v is the set of all vertices adjacent to v in G. $N[v] = N(v) \cup \{v\}$ is called the **closed neighborhood** of v.

Definition 1.2. A **bigraph or bipartite graph** G is a graph whose point set V can be partitioned into two subsets V_1 and V_2 such that every line of G joins V_1 with V_2. If further G contains every line joining the points of V_1 to the points of V_2 then G is called a **complete bigraph**. If V_1 contains m points and V_2 contains n points then the complete bigraph G is denoted by $K_{m,n}$.

Definition 1.3. A **star** is a complete bi graph $K_{1,n.}$

Definition 1.4. [8] A set $D \subseteq V$ is said to be a **dominating set** in G, if every vertex in V−D is adjacent to some vertex in D. The cardinality of minimum dominating set is called the **domination number** and is denoted by **$\gamma(G)$**.

Definition 1.5. [6] A set D $\subseteq$ V(G) is an **eccentric dominating set** if D is a dominating set of G and for every v $\in$ V−D, there exists at least one eccentric point of v in D. The cardinality of minimum eccentric dominating set is called the **eccentric domination number** and is denoted by $\gamma_{ed}(G)$.

If D is an eccentric dominating set, then every superset D′⊇D is also an eccentric dominating set. But D″⊆D is not necessarily an eccentric dominating set.
An eccentric dominating set D is a **minimal eccentric dominating set** if no proper subset D″⊆D is an eccentric dominating set.

Definition 1.6. [4] A vertex v is a **boundary vertex** of u if d(u, w) ≤ d(u, v) for all w $\in$ N(v). A vertex u have more than one boundary vertex at different distance levels.
A vertex v is called a **boundary neighbour** of u if v is a nearest boundary of u. The number of boundary neighbour of u is called the **boundary degree** of u.
In 2010, Janakiraman el al. have defined Eccentric domination in graphs. Motivated by this, here we have defined bd – domination number of a given graph and study that parameter.

Theorem 1.1. [6] $\gamma_{ed}(K_n) = 1$.
Theorem 1.2. [6] $\gamma_{ed}(K_{m,n}) = 2$.
Theorem 1.3. [6] $\gamma_{ed}(K_{1,n}) = 2$, $n \geq 2$.
Theorem 1.4. [6] $\gamma_{ed}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil, & if\ n = 3k+1 \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ n = 3k\ or\ 3k+2. \end{cases}$
Theorem 1.5. [6] $\gamma_{ed}(W_3) = 1$, $\gamma_{ed}(W_4) = 2$, $\gamma_{ed}(W_n) = 3$ for $n \geq 7$.
Theorem 1.6. [6] (i) $\gamma_{ed}(C_n) = n/2$ if n is even.

$$\text{(ii) } \gamma_{ed}(C_n) = \begin{cases} \frac{n}{3}\ if\ n=3m\ and\ is\ odd \\ \left\lceil \frac{n}{3} \right\rceil if\ n=3m+1 and\ is\ odd \\ \left\lceil \frac{n}{3} \right\rceil +1 if\ n=3m+2\ and\ is\ odd \end{cases}$$

2. BD – Domination

Definition 2.1. A set S ⊆V(G) is a **bd – dominating set** such that every vertex in V-S has at least one neighbour and at least one boundary neighbour in S. The cardinality of the minimum bd - dominating set is called the **bd - domination number** and is denoted by $\gamma_{bd}(G)$.

Let S ⊆V(G). Then S is known as a **boundary neighbour set** of G if for every vertex v $\in$ V – S, S has at least one vertex u such that u $\in$ E(v).
A boundary neighbour set S of G is a **minimal boundary neighbour set** if no proper subset S′ of S is a boundary neighbour set of G.

We define S is a **minimum boundary neighbour set** if S is a boundary neighbour set with minimum cardinality and **b(G)** be the cardinality of a minimum boundary neighbour set of G and b(G) can be called as boundary number of G.
Let D be a minimum dominating set of a graph G and S be a minimum boundary neighbour set of G.

Clearly, D∪S is a bd-dominating set of a graph G. Hence, $\gamma_{bd}(G) \leq \gamma(G) + b(G)$.

Example 2.1.

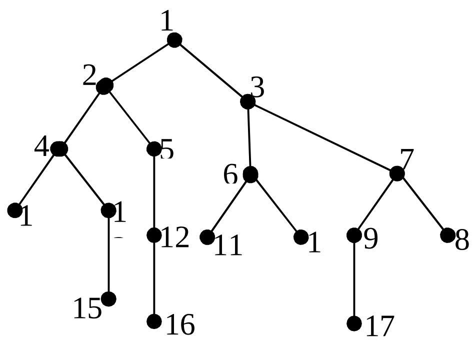

Figure 1: G

D_1= {1, 4, 5, 6, 7, 15, 16, 17} is a minimum dominating set. $\gamma(G) = 8$.
D_2 = {1, 4, 5, 6, 7, 8, 10, 15, 16, 17} is a minimum bd – dominating set. $\gamma_{bd}(G) = 10$.
D_3 = {1, 4, 5, 6, 7, 15, 16, 17} is a minimum eccentric dominating set. $\gamma_{ed}(G) = 8$.

Theorem 2.1.

(i) $\gamma_{bd}(K_n) = \gamma_{ed}(K_n)$
(ii) $\gamma_{bd}(K_{1, n}) = \gamma_{ed}(K_{1, n})$, $n \geq 2$
(iii) $\gamma_{bd}(K_{m, n}) = \gamma_{ed}(K_{m, n})$
(iv) $\gamma_{bd}(C_n) = \gamma_{ed}(C_n)$
(v) $\gamma_{bd}(W_n) = \gamma_{ed}(W_n)$

Proof: In these particular graphs, the boundary neighbours are the eccentric vertices. Therefore, eccentric dominating set is equal to the boundary dominating set. Hence, we get the above results.

Theorem 2.2. $\gamma_{bd}(P_n) = \gamma_{ed}(P_n) = \gamma(P_n)$ or $\gamma(P_n) + 1$.
Proof: The bd – dominating set of P_n must contain two end vertices. Therefore, bd – dominating set is also the eccentric dominating set. Hence, $\gamma_{bd}(P_n) = \gamma_{ed}(P_n) = \gamma(P_n)$ or $\gamma(P_n) + 1$.

Note. For a graph $G = K_n + K_1 + K_1 + K_m$, $n, m \geq 2$, $\gamma_{bd}(G) = 2$.

Theorem 2.3. A bd – dominating set D is a minimal bd – dominating set if and only if for each vertex $u \in D$, one of the following is true.
(i) u is an isolated vertex of D or u has no boundary vertex in D.
(ii) There exists some $v \in V-D$ such that $N(v) \cap D = \{u\}$ or $b(v) \cap D = \{u\}$. Where b(v) is the boundary neighbour set of v.

Proof: Assume that D is a minimal dominating set of. Then for every vertex $u \in D$,D-{u} is not a bd – dominating set. That is there exists some vertex v in (V-D) $\cup$ {u} which is not dominated by any vertex in D – {u} or there exists v in (V-D) $\cup$ {u} such that v has no boundary neighbour in D-{u}.

Case (i) Suppose u = v, then u is an isolate of D or u has no boundary neighbour in D.

Case (ii) Suppose $v \in V- D$

(a) If v is not dominated by D-{u}, but is dominated by D, then v is adjacent to only u in D, that is $N(v) \cap D = \{u\}$.

(b) Suppose v has no boundary neighbour in D – {u} but v has a boundary neighbour in D. Then u is the only boundary neighbour of v in D. That is $b(v) \cap D = \{u\}$.

Conversely, suppose that D is bd – dominating set and for each $u \in D$ one of the conditions holds, we show that D is a minimal bd – dominating set.

Suppose that D is not a minimal bd – dominating set, (ie) there exists a vertex $u \in D$ such that D – {u} is a bd – dominating set. Hence u is adjacent to at least one vertex v in D – {u} and u has a boundary neighbour in D – {u}.

Therefore, condition (i) does not hold.

Also, if D – {u} is a bd – dominating set, every element x in V – D is adjacent to at least one vertex in D – {u} and x has a boundary neighbour in D – {u}.

Hence, condition (ii) does not hold. This is a contradiction to our assumption that for each $u \in D$, one of the conditions holds.

This proves the theorem.

Observation: 2.1. (i) The pendent vertices are the boundary neighbours of their support vertices.

(ii) Eccentric vertices are also boundary vertices.

Theorem 2.4. Let T be a tree of order n with n_1 pendent vertices. Then $\gamma_{bd}(T) \leq \gamma(T) + n_1$.

Proof: Let T be a tree of order n.

Case (i) Assume D be a dominating set of T. If D contains all the pendent vertices of T, then D becomes a bd – dominating set. Hence $\gamma_{bd}(T) = \gamma(T)$.

Case (ii) a

If D does not contain the pendent vertices, then we add the boundary neighbours with D. Then we have $\gamma_{bd}(T) < \gamma(T) + n_1$.

Case (ii) b

All the pendent vertices are the boundary neighbours of T then $\gamma_{bd}(T) < \gamma(T) + n_1$. Since the support vertices of the pendant vertices can be removed from D and the pendant vertices can be added. Hence, $\gamma_{bd}(T) < \gamma(T) + n_1$.

Theorem 2.5. If G is a connected graph with n vertices then $\gamma_{bd}(G) \leq \lfloor 2n/3 \rfloor$.

Proof: If D is a minimum bd – dominating set, then for $v \in V-D$ there exists $u \in D$ and $w \in D$ such that u is adjacent to v in G and w is boundary dominate a vertex v in G. Hence D contains at most 2n/3 vertices. Hence $\gamma_{bd}(G) \leq \lfloor 2n/3 \rfloor$.

Theorem 2.6. If G is a caterpillar such that each non pendent vertices is of degree three then $\gamma_{bd}(G) = (n/2) +1$.

Proof: Since degree of each non pendent vertex is three, then G is of the following form.

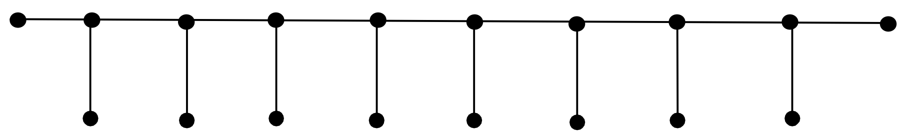

Figure 2:

Pendent vertex set form a bd – dominating set S and it is also a minimum set. Then every vertex v in V-S has a adjacent vertex in S and adjacent pendent vertex is a boundary neighbour of v. Hence $\gamma_{bd}(G) = (n/2) +1$.

Theorem 2.7. If G is a spider then $\gamma_{bd}(G) = \Delta(G) +1 = N(u) +1 = n -\Delta(G)$.

Proof: Let G be a spider, and u be a vertex of maximum degree $\Delta(G)$. N(u) vertices form a dominating set. Adding any one end vertex form a bd - dominating set. Hence $\gamma_{bd}(G) = |N(u)| +1$.That is, $\gamma_{bd}(G) = \Delta(G) +1 = n -\Delta(G)$.

Theorem 2.8. If G is a wounded spider then $\gamma_{bd}(G) \leq \Delta(G)$.

Proof: Let G be a wounded spider. Let u be the vertex of maximum degree $\Delta(G)$.

Case (i) If G has one or two wounded legs.

N(u) vertices form a bd – dominating set, since the end vertex of the wounded leg is a boundary neighbour of the other vertices. Hence $\gamma_{bd}(G) = \Delta(G)$.

Case (ii) If G has more than two wounded leg.

The end vertices of the non wounded legs and the central vertex u form a dominating set of G. Adding any one end vertex of the wounded leg form a bd - dominating set. Hence $\gamma_{bd}(G) \leq \Delta(G)$.

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