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Controllability Result for Nonlocal Nonlinear Impulsive Fuzzy Stochastic Differential Systems

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Abstract. This paper is concerned with sufficient conditions for the controllability for nonlocal nonlinear impulsivefuzzy stochastic systems in Banach space by using the concept of Mild solution. And the conditions are drive by using Sadovskii fixed point theorem, Hausdorff measure of noncompactness and operator semigroup.

Keywords: Controllability, nonlinear impulsive fuzzy stochastic differential systems, nonlocal condition, mild solution, fixed point theorem

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1. Introduction

The problem of controllability is to show the existence ofa control function, which steers the solution of the system from its initial state to afinal state, where the initial and final states may vary over the entire space. A largeclass of scientific and engineering problems is modeled by partial differential equations, integral equations or coupled ordinary and partial differential, integrodifferential equations. So it becomes importantto study the controllability results of such systems using available techniques. Several authorshave studied the problem of controllability of semilinear and nonlinear systems represented by differential and integrodifferential equations in finite or infinite dimensional Banach spaces [7, 9]. A standard approach is to transform the controllabilityproblem into a fixed point problem for an appropriate operator in a functionalspace.In this paper we studied the controllability of impulsive nonlinear nonlocal fuzzy stochastic differential equation described by

$$dx(t) = Ax(t) + Bu(t)dt + f\big(t, x(t)\big)dt + g\big(t, x(t)\big)dw(t), t \in J := [0, a], t \neq t_i \quad (1)$$

$$\Delta x(t_i) = x(t_i^+) - x(t_i^-) = I_i(x(t_i)). \quad x \in X,\ i = 1,2,...,m \quad (2)$$

$$x(0) + \mu(t_1, t_2, ...t_p, x) = x_0. \quad (3)$$

Here, the state variable $x(\cdot)$ takes values in real separable Hilbert space H with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$ and the control function $u(\cdot)$ takes values in $L^2(J,U)$, a Banach space of admissible control functions for a separable Hilbert space U. Also,

$A(t,x)$ is the infinitesimal generator of a C_0 –semigroup in H and B is a bounded linear operator from U into H. Further, f, g and h are continuous and compact functions and $f: J \to X$ and $g:\Omega \to X$ are measurable mapping in H-norm. And here, the function $\mu: PC(J,X) \to X$ is continuous and the impulsive function $I_i : X \to X$ is compact. Furthermore, the fixed time t_i satisfies $0 = t_0 < t_1 < t_2 < ... < t_m < a$, $x(t_i^+)$ and $x(t_i^-)$ denoted the right and left limits of $x(t)$ *at* $t = t_i$, and $\Delta x(t_i) = x(t_i^+) - x(t_i^-)$ represents the jump in the state x at time t_i, where I_i determines the size of the jump. In [6] we studied the impulsive differential equations is much richer then the corresponding theory of differential equations without impulse effects. And, one of the important applications of introducing impulsive equations in stochastic setting is that it is possible to control the stochastic dynamical systems which are not exactly controllable. The nonlocal condition which is a generalization of the classical initial condition was motivated by physical problems. The basic work on nonlocal conditions is due to in [2,10,11]. And, also fixed point theorem is one of the most useful tool to obtain the controllability of various system of equations like integrodifferential, stochastic differential equation and etc., this is presented in [8].

In this paper motivated by [1,2,3,4,5], we establish the sufficient conditions for the controllability of impulsive nonlinear nonlocal fuzzy stochastic differential equations in general Banach space by using the measure of noncompactness, Sadovskii fixed-point theorem.

2. Preliminaries

Here first recall the basic notations on the fuzzy number space (E^1, D), mild solution of stochastic process, fuzzy process, controllable and so on.

Let $(X, \|\cdot\|)$ be a real Banach space. Denote by C([0, T],X) the space of X-valued continuous functions on [0, T] with the norm $\|x\|_\infty = \sup\{\|x(t)\| : t \in [0, T]\}$ and by L^1([0, T],X), the space of X-valued Bochner integrable functions on [0, T] with the norm $\|x\|_1 = \int_0^T \|x(t)\| dt.$ Throughout this paper, $(X, \|\cdot\|)$ is a Banach space, $\{A(t): t \in \mathrm{R}\}$ is a family of closed linear operators defined on a common domain D which is dense in X and we assume that the linear nonautonomous system

$$\begin{cases} u'(x) = A(x)u(x), & t \geq s \\ u(s) = u \in X, \end{cases} \tag{4}$$

has associated evolution family of operators $\{U(t,s): 0 \leq s \leq t \leq b\}$. In the next definition, L(X) is a space of bounded linear operators from X into X endowed with the uniform convergence topology.

Definition 2.1. A family of operators $\{U(t, s): 0 \leq s \leq t \leq b\} \subset L(X)$ is called an evolution family of operators for (4) if the following properties hold

i. $U(t, s)U(s, \tau) = U(t, \tau)$ and $U(t, t)x = x$ for every $s \leq \tau \leq t$ and all $x \in X$;
ii. For each $x \in X$ the functions for $(t, s) \to U(t, s)x$ is continuous and $U(t, s) \in L(X)$ for every $t \geq s$; and

iii. For $0 \le s \le t \le b$, the function $t \to U(t,s),(s,t] \in L(X)$, is differentiable with $\frac{\partial U(t,\ s)}{\partial t} = A(t)U(t,s).$

Definition 2.2. [10] A stochastic process x is said to be a mild solution of (1)-(3) if the following conditions are satisfied:

i. $x(t,\omega)$ is a measurable function from $J \times \Omega$ to H and $x(t)$ is F_t-adapted,

ii. $E\|x(t)\|^2 < \infty$ for each $t \in J$,

iii. $\Delta x(\tau_i) = x(\tau_i^+) - x(\tau_i^-) = I_i(x(\tau_i)). \quad x \in X, 1 \le i \le m.$

iv. For each $u \in L_2^F(J,U)$, the process x satisfies the following integral equation

$$x(t) = U(t,0)[x^1 - g(x)] + \int_0^t U(t,s)Bu(s)ds$$
$$+\int_0^t U(t,s)f(s,x(s))ds + \int_0^t U(t,s)g(s,x(s))dW(s)$$
$$+\sum_{0 \le t_i \le t} U(t,t_i)I_i(x(t_i^-)),\ t \in J.$$

Definition 2.3. Let $x, y \in C(I:E^n)$, here I be a real interval. A mapping $x: I \to E_N$ is called a fuzzy process. We denote, $[x(t)]^\alpha = [x_l^\alpha(t), x_r^\alpha(t)],\ t \in I,\ 0 < \alpha \le 1.$
The derivative $x'(t)$ of a fuzzy process x is defined by $[x'(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)],\ t \in I,\ 0 < \alpha \le 1$ provided that is equation defines a fuzzy $x'(t) \in E_N$.

Definition 2.4. The system (1)-(3) is said to be controllable on the interval J if, there exists u(t) such that the fuzzy solution x(t) of (1)-(3) satisfies $x(T) = x^1 - \mu(x)$ (i.e., $[x(T)]^\alpha = [x^1 - \mu(x)]^\alpha$) where x^1 is target set.Here, we assume the following hypotheses for study to controllability problem:H_1. A (t) generates a strongly continuous semigroup of a family of evolution operators U (t,s) and there exists a constant N_1>0 such that $\|[U(t,s)]^\alpha\| \le N_1^\alpha$ *for* $0 \le s \le t \le b.$

H_2. The linear operator $W: L^2(J,U) \to X$ defined by $Wu = \int_0^b U(b,s)Bu(s)ds$ has an inverse operator W^{-1}, which takes values in $L^2(J,U) \mid KerW$ and there exists apositive constant K_1 such that $\|BW^{-1}\| \le K_1$

H_3. i) For each t $\in$J, the function $f(t,\cdot,\cdot): X \times X \to X$ is continuous and, for each $(\phi, x) \in X \times X$, the function $f(\cdot, x, y): J \to X$ is strongly measurable.

ii) There exists a function $K_f(\cdot) \in L^1(J, R^+)$ such that

$$\left\| f^\alpha(t,x_1,y_1) - f^\alpha(t,x_2,y_2) \right\|$$
$$\le K_f^\alpha(t,s)\left(\|x_1 - x_2\| + \|y_1 - y_2\|\right),\ for\ any\ t \in J,\ x_i, y_i \in X,\ for i = 1,2,..$$

iii) The function $f: J \times X \times X \to X$ is compact

H_4.i) For each t,s$\in$J, the function $g(t,s,\cdot): X \to X$ is continuous and, for each x$\in$X, the function $g(\cdot,\cdot,x): \Omega \to X$ is strongly measurable.

ii) There exists a function $Kg : J \times J \to [0,\infty)$ such that

$$\left\| g^{\alpha}(t,s,x) - g^{\alpha}(t,s,y) \right\| \le K_g^{\alpha}(t,s) \left\| x - y \right\|,$$

for any $t,s \in J,\ x,y \in X$.

H_5. The function $\mu : PC(J,X) \to X$ is continuous there exists aconstant $K_{\mu} > 0$ such that

$$\left\| \mu^{\alpha}(t_1,t_2,...,t_p,x(\cdot)) - \mu^{\alpha}(t_1,t_2,...,t_p,y(\cdot)) \right\| \le K_{\mu}^{\alpha} \left\| x - y \right\|,$$

for $x,y \in X$.

H_6. i) $I_i : X \to X$ and there exists a positive constant d_i such that

$$\left\| I_i^{\alpha}(u) - I_i^{\alpha}(v) \right\| \le d_i^{\alpha} \left\| u - v \right\|,$$

for $u,v \in X$.

ii) The function $I_i : X \to X$ is compact.

Lemma 2.1. (Sadovskii fixed-point theorem)
Let N be the condensing operator on a Banach space X, that is N is continuous and takes bounded sets into bounded sets and α (N(D)) < α(D), for every bounded set D of X with α(D) >0. If N(S)⊂S for a convex, closed and bounded set S of X, then N has a fixed point in S.

3. Controllability result

Theorem 3.1. If the hypotheses (H_1)-(H_6) are satisfied, then the system (1)-(3) is controllable on J. $(1 + bN_1^{\alpha}K_1^{\alpha})N_1^{\alpha}[K_{\mu}^{\alpha} + K_f^{\alpha} + K_g^{\alpha} + \sum_{0 \le t_i < t}^{m} d_i^{\alpha}] \le 1$ (5)

Proof: Using (H_2) for an arbitrary function $x(\cdot) \in PC(J,X)$, we define the control

$$\begin{aligned}
[u(t)]^{\alpha} &= [u_l^{\alpha}(t), u_r^{\alpha}(t)] \\
&= [W^{-1}\{[x^1 - \mu(t_1,t_2,...t_p,x)]_l^{\alpha}) - U_l^{\alpha}(b,0)\left([x_0 - \mu(t_1,t_2,...t_p,x)]_l^{\alpha}\right) \\
&\quad - \int_0^b U_l^{\alpha}(b,s) f_l^{\alpha}(s,x(s))ds - \int_0^b U_l^{\alpha}(b,s) g_l^{\alpha}(s,x(s))dW(s) - \sum_{0 \le t_i \le t} U_l^{\alpha}(b,t_i)(I_i)_l^{\alpha}(x(t_i^-))\}(t), \\
&\quad W^{-1}\{[x^1 - \mu(t_1,t_2,...t_p,x)]_r^{\alpha} - U_r^{\alpha}(b,0)\left([x_0 - \mu(t_1,t_2,...t_p,x)]_r^{\alpha}\right) \\
&\quad - \int_0^b U_r^{\alpha}(b,s) f_r^{\alpha}(s,x(s))ds - \int_0^b U_r^{\alpha}(b,s) g_r^{\alpha}(s,x(s))dW(s) - \sum_{0 \le t_i \le t} U_r^{\alpha}(b,t_i)(I_i)_r^{\alpha}(x(t_i^-))\}(t)],
\end{aligned} \tag{6}$$

Consider the Banach space Ƶ=PC(J,X) with the norm $\|x\| = \sup\{|x(t)| : t \in J\}$.We prove that when using the control u(t) , the operator Ψ: Ƶ → Ƶ defined by,

$$\begin{aligned}
[\Psi x(t)]^{\alpha} &= [U(t,0)[x^1 - \mu(t_1,t_2,...t_p,x)] + \int_0^t U(t,s)BW^{-1}\{[x^1 - \mu(t_1,t_2,...t_p,x)] \\
&\quad - U(b,0)\left([x_0 - \mu(t_1,t_2,...t_p,x)]\right) - \int_0^b U(b,s) f(s,x(s))ds \\
&\quad - \int_0^b U(b,s) g(s,x(s))dW(s) - \sum_{0 \le t_i \le t} U(b,t_i) I_i(x(t_i^-))\}(s)ds + \int_0^t U(t,s) f(s,x(s))ds \\
&\quad + \int_0^t U(t,s) g(s,x(s))dW(s) + \sum_{0 \le t_i \le t} U(t,t_i) I_i(x(t_i^-))]^{\alpha}
\end{aligned}$$

has a fixed point x(.). This fixed point is the mild solution to system (1)-(3), which implies that the system is controllable on J.

Now we prove that operator Ψ is a completely continuous operator. Set

$$B_m = \sup\{x \in \mathbb{Z} : \|x\| \le m\} \quad \text{for some} \quad m \ge 1$$

Then, for each *m*, B_m is a obviously a bounded closed convex set in Ƶ.

Step 1: Here we show that there exists a positive constant *m* such that, $\Psi(B_m) \subseteq B_m$. If this is not true, then for each positive number *m*, there exists a function $x^m \in B_m$, but Ψ B_m does not belongs to B_m, that is $\|[\Psi x^m(t)]^\alpha\| > m$ for some $t \ge J$ .However, we have,

$$\begin{aligned}
m &< \|[\Psi x^m(t)]^\alpha\| \\
&\le \|[U(t,0)[x^1 - \mu(t_1,t_2,\dots t_p,x^m)] + \int_0^t U(t,s)BW^{-1}\{[x^1 - \mu(t_1,t_2,\dots t_p,x^m)] \\
&\quad - U(b,0)\big([x_0 - \mu(t_1,t_2,\dots t_p,x^m)]\big) - \int_0^b U(b,s)f(s,x^m(s))ds \\
&\quad - \int_0^b U(b,s)g(s,x^m(s))dW(s) - \sum_{0\le t_i\le t} U(b,t_i)I_i(x^m(t_i^-))\}(s)\,ds + \int_0^t U(t,s)f(s,x^m(s))ds \\
&\quad + \int_0^t U(t,s)g(s,x^m(s))dW(s) + \sum_{0\le t_i\le t} U(t,t_i)I_i(x^m(t_i^-))]^\alpha \| \\
&\le N_1^\alpha[\|x_0\| + \|\mu^\alpha(t_1,t_2,\dots t_p,x^m)\|] + bN_1^\alpha K_1^\alpha\{\|x^1\| + \|\mu^\alpha(t_1,t_2,\dots t_p,x^m)\|\; N_1^\alpha[\|x_0\| + \|\mu^\alpha(t_1,t_2,\dots t_p,x^m)\|] \\
&\quad + N_1^\alpha\int_0^b \|f^\alpha(s,x^m(s))\|ds + N_1^\alpha\int_0^b \|g^\alpha(s,x^m(s))\|dW(s) + N_1^\alpha \sum_{0\le t_i\le t}\|I_i^\alpha(x^m(t_i^-))\|\} \\
&\quad + N_1^\alpha\int_0^t \|f^\alpha(s,x^m(s))\|ds + N_1^\alpha\int_0^t \|g^\alpha(s,x^m(s))\|dW(s) + N_1^\alpha \sum_{0\le t_i\le t}\|I_i^\alpha(x^m(t_i^-))\|
\end{aligned}$$

Since

$$\begin{aligned}
\int_0^t \|[f(s,x^m(s))]^\alpha\|ds &\le \int_0^b \Big[\|f^\alpha(s,x^m(s)) - f^\alpha(s,0)\| + \|f^\alpha(s,0)\|\Big]ds \\
&\le \int_0^b \{K_f^\alpha(s)\|x^m(s)\| + \|f^\alpha(s,0)\|\}ds
\end{aligned}$$

Then

$$\begin{aligned}
m &< bN_1^\alpha K_1^\alpha\|x^1\| + (1 + bN_1^\alpha K_1^\alpha + bN_1^\alpha K_1^\alpha)N_1^\alpha\|x_0\| \\
&\quad + (1 + bK_1^\alpha + bN_1^\alpha K_1^\alpha)N_1^\alpha\|\mu^\alpha(t_1,t_2,\dots t_p,0)\| \\
&\quad + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha K_\mu^\alpha\|x^m\| + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha\sum_{i=1}^m I_i^\alpha(0) \\
&\quad + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha K_f^\alpha\|x^m\| + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha\int_0^b \|f^\alpha(s,0)\|ds \\
&\quad + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha K_g^\alpha\|x^m\| + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha\int_0^b \|g^\alpha(s,0)\|dW(s) \\
&\quad + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha\sum_{0\le t_i<t}^m d_i^\alpha\|x^m(t_i)\|.
\end{aligned}$$

$$m < M + (1 + bN_1^\alpha K_1^\alpha)N_1^\alpha[K_\mu^\alpha + K_f^\alpha + K_g^\alpha + \sum_{0\le t_i<t}^m d_i^\alpha]m$$

Dividing both sides by *m* and taking the limit as $m\to\infty$. And M is independent of *m*,then we have $(1 + bN_1^\alpha K_1^\alpha)N_1^\alpha[K_\mu^\alpha + K_f^\alpha + K_g^\alpha + \sum_{0\le t_i<t}^m d_i^\alpha] \ge 1$

This contradicts (5). Hence for some positive number *m*, $\Psi(B_m) \subseteq B_m$

Now, we have to prove that Ψ is a condensing operator, and we introduce the decomposition $\Psi = \Psi_1 + \Psi_2$, where

$$\Psi_1 x(t) = U(t,0)[x_0 - \mu(x)] + \int_0^t U(t,s)Bu(s)ds$$

$$\Psi_2 x(t) = \int_0^t U(t,s) f(s,x(s))ds + \int_0^t U(t,s) g(s,x(s))dW(s) + \sum_{0\le t_i\le t} U(t,t_i) I_i(x(t_i^-))$$

For $t \in J$, respectively

Step 2: We want to show that Ψ_1 is contraction on Bm. Let x,y$\in$ B_m. Then, for each t$\in$J, from (6), we see that

$$\left\|[\Psi_1 x(t)]^\alpha - [\Psi_1 y(t)]^\alpha\right\| \le U(t,0)[\left\|\mu^\alpha(t_1,t_2,...t_p,x) - \mu^\alpha(t_1,t_2,...t_p,y)\right\| + \int_0^t \left\|U^\alpha(t,s)\right\| \left\|BW^{-1}\right\| \Big[\left\|\mu^\alpha(t_1,t_2,...t_p,x) - \mu^\alpha(t_1,t_2,...t_p,y)\right\|$$
$$+\left\|U^\alpha(b,0)\right\| \Big[\left\|\mu^\alpha(t_1,t_2,...t_p,x) - \mu^\alpha(t_1,t_2,...t_p,y)\right\|\Big] + \left\|U^\alpha(b,s)\right\| \left\|f^\alpha(s,x(s)) - f^\alpha(s,y(s))\right\| ds$$
$$+\left\|U^\alpha(b,s)\right\| \left\|g^\alpha(s,x(s)) - g^\alpha(s,y(s))\right\| dW(s) + \sum_{0\le t_i\le t} \left\|U^\alpha(b,t_i)\right\| \left\|I_i^\alpha[x(t_i^-) - y(t_i^-)\right\| \Big] (s)ds.$$

$$\le \left\{ K_\mu^\alpha + bN_1^\alpha K_1^\alpha [K_\mu^\alpha + K_f^\alpha + K_g^\alpha + \sum_{0\le t_i < t}^{m} d_i^\alpha] N_1^\alpha \right\} \|x - y\|$$

From (5), which implies that Ψ_1(•)is a contraction on B_m. Next, we prove that Ψ_2(•) is completely continuous from $B_m$into B_m.

Step 3: Next to show that Ψ_2 is completely continuous. First, we show that $\Psi_2(\cdot)$is continuous on B_m. Let $\{x_n(t)\}_0^\infty \le B_m$ with $x_n \to x$ in B_m. Then, there exists a number $m>0$ such that $\|x_n(t)\| \le m$ for all n and t$\in$J, so $x_n \in B_m$ and x $\in B_m$.

From the assumptions (H_3) and (H_4), we get

I_i, i=1,2,3,...,m is continuous.

(ii) $f^\alpha(t,x_n(t)) \to f^\alpha(t,x(t))$ for each t$\in$J and since $\left\|f^\alpha(t,x_n(t)) \to f^\alpha(t,x(t))\right\| \le mK_f^\alpha(t)$.

From the Dominated Converges theorem,

$$\|\Psi_2 x_n - \Psi_2 x\| \le N_1^\alpha \int_0^b \left\|f^\alpha(s,x_n(s)) - f^\alpha(s,x(s))\right\| ds$$
$$+ N_1^\alpha \int_0^b \left\|g^\alpha(s,x_n(s)) - g^\alpha(s,x(s))\right\| dW(s)$$
$$+ N_1^\alpha \sum_{0\le t_i<t} \left\|I_i^\alpha(x_n(t_i^-)) - I_i^\alpha(x(t_i^-))\right\|$$
$$\to 0 \ as\ n \to \infty.$$

Thus Ψ_2 is continuous on B_m.

Second, we prove that Ψ_2 (•) is relatively compact and equicontinuous on X for every t$\in$[0,b]. We need to prove that $\Psi_2(B_m) \subseteq PC(J,X)$ is equicontinuous and $\Psi_2(B_m)(t)$ is precompact for any $m>0$,t$\in$J. For any x$\in B_m$ with (t+l)$\in$[0,b],we have

$$\left\|[\Psi_2 x(t+l)]^\alpha - [\Psi_2 x(t)]^\alpha\right\| \le N_1^\alpha l[K_f^\alpha(t) + K_g^\alpha(t) + \sum_{i=1}^{m} d_i^\alpha]m$$
$$+N_1^\alpha \int_0^{t+l} \left\| f^\alpha(s,0)\right\| ds + N_1^\alpha \int_0^{t+l} \left\| g^\alpha(s,0)\right\| dW(s)$$
$$+N_1^\alpha \sum_{i=1}^{m} \left\| I_i(0)\right\| + N_1^\alpha \sum_{i=1}^{m} \left\| [U^\alpha(t+l,t_i) - I] \times I_i^\alpha(x(t_i^-))\right\|$$
$$+N_1^\alpha \int_0^t \left\| [U^\alpha(t+l,t) - I] \times f^\alpha(s,x(s))\right\| ds$$
$$+N_1^\alpha \int_0^t \left\| [U^\alpha(t+l,t) - I] \times g^\alpha(s,x(s))\right\| dW(s)$$

Since f, g and I_i are compact

$$\left\| [U^\alpha(t+l,t) - I]\{ f^\alpha(s,x(s)) + g^\alpha(s,x(s)) + \sum_{0 \le t_i < t}^{m} I_i^\alpha(x(t_i^-))\}\right\| \to 0 \ as \ l \to 0.$$

Uniformly for $\delta \in J$ and $x \in B$. This implies that for any $\varepsilon>0$, there exists $\delta>0$ such that the above equation is $\le\varepsilon$, for some $0 \le l \le \delta$ *and* $x \in B_m$ .Therefore, we get

$$\left\|[\Psi_2 x(t+l)]^\alpha - [\Psi_2 x(t)]^\alpha\right\| \le N_1^\alpha l[K_f^\alpha(t) + K_g^\alpha(t) + \sum_{i=1}^{m} d_i^\alpha]m$$
$$+ N_1^\alpha \int_0^{t+l} \left\| f^\alpha(s,0)\right\| ds + N_1^\alpha \int_0^{t+l} \left\| g^\alpha(s,0)\right\| dW(s)$$
$$+ N_1^\alpha \sum_{i=1}^{m} \left\| I_i(0)\right\| + bN_1^\alpha \varepsilon$$

for some $0 \le l \le \delta$ *and* $x \in B_m$ .Thus, Ψ_2 maps B_m into an equicontinuous family of functions. The set, $\{U^\alpha(t,s) f^\alpha(s,x(s)) : t,s \in J, x \in B_m\}$ and $\{U^\alpha(t,s) g^\alpha(s,x(s)) : t,s \in J, x \in B_m\}$ is precompact as f, g are compact and $U(\cdot,\cdot)$ is a semigroup.

Finally, Ψ_2 maps B_m into precompact set in X as

$$\Psi_2(B_m)(t) \subset t\overline{conv}\{U^\alpha(t,s)\{ f^\alpha(s,x(s)) + g^\alpha(s,x(s)) : t,s \in J, x \in B_m\}.$$

Therefore, by the above steps1-3, we conclude that $\Psi = \Psi_1 + \Psi_2$is a condensing operation on B_m. By lemma there exists a fixed point $x(\cdot) \in B_m$ such that $[\Psi x(t)]^\alpha = [x(t)]^\alpha$ and this point $x(\cdot)$ is a mild solution to system (1)-(3). Clearly $\Psi x(b) = x(b) = x^1 - \mu(x), (ie.[x(b)]^\alpha = \left[x^1 - \mu(x)\right]^\alpha)$ this implies that the system (1)-(3) is controllable. Hence the proof.

4. Conclusion

This paper contains controllability results for nonlocal impulsive fuzzy stochastic differential equations in Banach space by using the Sadovskii fixed point theorem. It is proved that, the system is controllable on some hypotheses. Our result shows that Sadovskii fixed point theorem can strongly be used in control problems to obtain the sufficient conditions.

REFERENCES

1. K. Balachandran and S. Karthikeyan, Controllability of stochastic integrodifferential systems, *Int. J. Control*, 80 (2007) 486-491.

2. J.H.Park, J.S.Park and Y.C.Kwun, Controllability for the Semilinear Fuzzy Integrodifferential Equations with Nonlocal Conditions, Springer-Verlag Berlin Heidelberg, 2006, pp. 221–230.
3. J. Klamka, Stochastic controllability of linear systems with state delays, *Int. J. Appl. Math. Comput. Sci.*, 17 (2007) 5-13.
4. J.Klamka, Stochastic controllability of linear systems with delay in control, *Bull. Pol. Ac. Tech.*, 55 (2007) 23-29.
5. J.Klamka, Stochastic controllability of systems with variable delay in control, *Bull. Pol. Ac. Tech.*, 56 (2008) 279-284.
6. V.Lakshmikantham, D.D.Bainov and P.S.Simeonov, Theory of impulsive differential equations, World Scientific, Singapore, 1989.
7. N.I. Mahmudov, Controllability of semilinear stochastic systems in Hilbert spaces, *J. Math. Anal. Appl.*, 288 (2003) 197-211.
8. B.Sadaovskii. On a fixed point principle, *Functional Analysis and Applications*, (2) (1967) 151 – 153.
9. R.Sakthivel, N.I.Mahmudov and S.-G.Lee, Controllability of non-linear impulsive stochastic systems, *International Journal of Control*, 82(5) (2009) 801–807.
10. R.Sathya, controllability of nonlocal impulsive stochastic quasilinear integrodifferential systems, *Electronic Journal of Qualitative Theory of Differential Equations*, (50) (2011) 1-16.
11. R.Sathya, Controllability of stochastic impulsive integrodifferential systems with infinite delay, *Electronic Journal of Qualitative Theory of Differential Equations*, (50) (2011) 1-16.