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Expected Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits

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Abstract. In this paper, the problem of time to recruitment is studied using a univariate max policy of recruitment for a single grade manpower system in which attrition takes places due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-exit times form an ordinary renewal process according as inter-policy decision times form an ordinary renewal process or a sequence of exchangeable and constantly correlated exponential random variables or a geometric process or an order statistics. The analytical results in closed form are derived by assuming specific distribution for loss of manpower and the breakdown threshold.

Keywords: Single grade manpower system, decision and exit epochs, geometric process, order statistics, correlated and exchangeable random variables, univariate max policy of recruitment and variance of time to recruitment

AMS Mathematics Subject Classification (2010): 60K05, 60K10, 62N05

1. Introduction

Attrition is a common phenomenon in many marketing organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the breakdown threshold. If the total (maximum) loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. Many researchers have studied several problems in manpower planning using different methods.In [2] and [3] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovianand renewal theoretic approach. In the study of the problem of time to recruitment several researchers have used univariate Max policy of recruitment which states that recruitment is done whenever the maximum loss of manpower exceeds a control limit, known as breakdown threshold. In [16,17] and [18] the authorshave studied the problem of time to recruitment for a single grade manpower system and obtained the variance of the time to recruitment by Laplace transform technique,using a univariate Max policy of recruitment when the loss of manpower and inter-decision times are independent exponential random variablesaccording as the breakdown threshold is a positive constant or an exponential random variable or a continuous random variable with SCBZ property. In [11] the authors have studied this work using a geometric breakdown threshold. In [12,13,14,15] the authors have extended the above cited work when the inter-decision times are exchangeable and constantly correlated exponential random variables.Recently, this research work is studied in [1] using a different probabilistic analysis which requires only the first two moments of the inter-exit times, unlike the conventional Laplace transform technique for which the knowledge of distribution of inter-exit times is a pre-requisite. In [9] the author has studied the problem of time to recruitment using optional and mandatory thresholds, thereby providing alertness in the context of threshold crossing. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. This aspect is taken into account for the first time in [4] and subsequently [5,6,7,8] the variance of the time to recruitment is obtained by using univariate Cum policy when the loss of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables according as the mandatory breakdown threshold is an exponential random variable or extended exponential random variable with shape parameter 2 or a continuous random variable with SCBZ property. The present paper extends the research work in [4,5,6,7,8] when the inter-decision times form (i) an ordinary renewal process (ii) a sequence of exchangeable and constantly correlated exponential random variables (iii) a geometric process and (iv) an order statistics using univariate Max policy of recruitment. Relevant conclusions based upon the analytical results are presented.

2. Model description and analysis

Consider an organization with single grade, taking decisions at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let X_i be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the i^{th}exit point and $\overline{X}_{i+1}$ be the maximum loss of manpower in the first (i+1) exit points. It is assumed that X_i's are independent and identically exponential distributed random variables with probability distribution function M(.) and density function m(.) and mean $\frac{1}{\alpha}(\alpha>0)$. Let A_i be the time between $(i-1)^{th}$ and i^{th} policy decisions, forming a sequence of independent and identically distributed random variables with probability distribution function F(.) , density function f(.). Let B_i be the time between $(i-1)^{th}$ and i^{th} exit times, forming a sequence of independent and identically distributed random variables with probability distribution function G(.) and density function g(.). Let D_{i+1} be the waiting time upto (i+1) exits. Let Y be the independent exponential threshold level for the depletion of manpower in the organization with probability distribution function H(.) and density function h(.). Let q be the probability that every policy decision has exit of personnel. As q=0 corresponds to the case where exits are impossible, it is assumed that q

$\neq 0$. Let χ(I) be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T). The univariate MAX policy of recruitment employed in this paper is stated as follows: Recruitment is done whenever the maximum loss of man hours in the organization exceeds the threshold for the loss of man hours in this organization.

In this section the variance of time to recruitment is obtained using a different probabilistic analysis instead of the conventional Laplace transform method. By the recruitment policy, recruitment is done whenever the maximum loss of manpower exceeds the threshold Y. When the first decision is taken, recruitment would not have been done for B_1 units of time. If the loss of manpower X_1(= $\overline{X}_1$) due to the first policy of decision is greater than Y, then the recruitment is done and in this case $T=B_1=D_1$. However, if $\overline{X}_1 \leq$Y, the non-recruitment period will continue till the next policy decision is taken. If the maximum $\overline{X}_2$ of the loss of manpower in the first two decisions exceeds Y, then recruitment is done and $T=B_1+B_2=D_2$.If $\overline{X}_2$ $\leq$Y, then the non-recruitment period will continue till the next policy decision is taken and depending on $\overline{X}_3$ >Y or $\overline{X}_3 \leq$ Y, recruitment is done or the non-recruitment period continues and so on. Hence

$$T = \sum_{i=0}^{\infty} D_{i+1}\, \chi\,(\overline{X}_i \leq Y < \overline{X}_{i+1}) \tag{1}$$

From (1) and from the definition of D_{i+1}, we get

$$E(T) = \sum_{i=0}^{\infty} (i+1)\, E(B)\, P\,(\overline{X}_i \leq Y < \overline{X}_{i+1}) \tag{2}$$

and

$$E(T^2) = \sum_{i=0}^{\infty} (i+1)\, [V(B)+ (i+1)E^2(B)]\, P\,(\overline{X}_i \leq Y < \overline{X}_{i+1}). \tag{3}$$

By using law of total probability

$$P\,(\overline{X}_i \leq Y < \overline{X}_{i+1}) = \int_0^{\infty} (M(y))^i h(y)dy . \int_0^{\infty} \overline{M}(y)h(y)dy \tag{4}$$

Using (4) in (2), (3) and on simplification we get

$$E(T) = E(B)\, J_1 J_2 \tag{5}$$

and

$$E(T^2) = V(B)\, J_1J_2 + (E(B))^2\, J_1\, [2\, J_3 - J_2] \tag{6}$$

where $J_1 = \int_0^{\infty} \overline{M}(y)\, h(y)dy$, $J_2 = \int_0^{\infty} (\overline{M}(y))^{-2} h(y)dy$ and $J_3 = \int_0^{\infty} (\overline{M}(y))^{-3}\, h(y)dy.$ (7)

We now obtain explicit analytical expressions for E(T) and V(T) by considering different cases for inter-decision times.

Case(i): F(x)=1-exp(-λx), $\lambda > 0, x > 0$

In this case the number of policy decisions announced is governed by an ordinary renewal process with independent and identically distributed exponential inter- decision times. For the present case $J_1 = \frac{\theta}{\alpha+\theta}$, $J_2 = \frac{\theta}{\theta-2\alpha}$ and $J_3 = \frac{\theta}{\theta-3\alpha}$ (8)

Therefore from (5) and (6) we get

$$E(T) = \frac{\theta^2}{\lambda q(\alpha+\theta)(\theta-2\alpha)} \tag{9}$$

and

$$V(T) = \frac{\theta^2[2(\alpha+\theta)(\theta-2\alpha)^2-\theta^2(\theta-3\alpha)]}{\lambda^2 q^2(\alpha+\theta)^2(\theta-2\alpha)^2(\theta-3\alpha)} \qquad (10)$$

(9) and (10) give the mean and variance of the time to recruitment for the present case.

Case (ii): Inter- policy decision times form a sequence of exchangeable and constantly correlated exponential random variableswith mean u (u>0). Let R be the correlation between A_i and A_j, i≠j and v=u(1-R).

In this case from (5), (6), (8) and [10] on simplification we get

$$E(T) = \frac{v\,\theta^2}{(1-R)q\,(\alpha+\theta)(\theta-2\alpha)} \qquad (11)$$

and $$V(T) = \frac{2(1-R)^2[1+u^2(R^2\overline{q}-1)]\theta^2(\alpha+\theta)(\theta-2\alpha)(\theta-3\alpha)+v^2[2\theta^2(\alpha+\theta)(\theta-2\alpha)^2-\theta^4(\theta-3\alpha)]}{(1-R)^2q^2(\alpha+\theta)^2(\theta-2\alpha)^2(\theta-3\alpha)} \qquad (12)$$

(11) and (12) give the mean and variance of the time to recruitment for thepresent case.

Note: When q=1 our results agree with the results in [1] for the present case.

Case (iii): Inter- policy decision times form a geometric process of independent random variables with parameter c(c>0) and $P(A_1 \le x) = 1-\exp(-\lambda x)$, $\lambda > 0, x > 0$.

In this case from (5), (6),(8) and on simplification we get

$$E(T) = \frac{c\,\theta^2}{\lambda[c-1+q](\alpha+\theta)(\theta-2\alpha)} \qquad (13)$$

and

$$V(T) = \frac{[c^4+c^2(2c-1)(q-1)]\theta^2(\theta-3\alpha)(\alpha+\theta)(\theta-2\alpha)+c^2[c^2-1+q]\theta^4[(\theta^2-\alpha^2)(\theta-2\alpha)-(\theta-3\alpha)]}{\lambda^2[c-1+q]^2[c^2-1+q](\theta-3\alpha)(\alpha+\theta)^2(\theta-2\alpha)^2} \qquad (14)$$

(13) and (14) give the mean and variance of the time to recruitment for the present case.

Case (iv): Inter- policy decision times form an order statistics. Let $F_{a(j)}(.)$ and $f_{a(j)}(.)$ be the distribution and the probability density function of the j^{th} order statistics(j=1,2,…,r) selected from the sample of size r from the population $\{A_i\}_{i=1}^{\infty}$.

Suppose $f(t) = f_{a(1)}(t) = r[1-F(t)]^{r-1} f(t)$. (15)

Then from (5), (6),(8) and on simplification it is found that

$$E(T) = \left(\frac{\theta^2}{\lambda r q(\alpha+\theta)(\theta-2\alpha)}\right) \qquad (16)$$

and

$$V(T) = \frac{\theta^2[2(\alpha+\theta)(\theta-2\alpha)^2-(\theta-3\alpha)]}{\lambda^2 r^2 q^2(\alpha+\theta)^2(\theta-2\alpha)^2(\theta-3\alpha)}. \qquad (17)$$

(16) and (17) give the mean and variance of the time to recruitment when $f(t) = f_{a(1)}(t)$.

Suppose $f(t) = f_{a(r)}(t) = r[F(t)]^{r-1} f(t)$. (18)

Then from (5), (6),(8) andon simplification it is found that

$$E(T) = \left(\frac{\theta^2}{\lambda q(\alpha+\theta)(\theta-2\alpha)}\right)\left(\sum_{j=1}^{r}\frac{1}{j}\right) \qquad (19)$$

and

$$V(T)=\left(\frac{\theta^2}{\lambda^2 q(\alpha+\theta)(\theta-2\alpha)}\right)\left(\sum_{j=1}^{r}\frac{1}{j^2}\right)+\left(\frac{\theta^2[(\alpha+\theta)(\theta-2\alpha)(23\alpha-7\theta)-(\theta^3-3\alpha\theta^2)}{(\alpha+\theta)^2(\theta-2\alpha)^2(\theta-3\alpha)}\right)\left(\sum_{j=1}^{r}\frac{1}{j}\right)^2 \quad (20)$$

(19) and (20) give the mean and variance of the time to recruitment when $f(t)=f_{a(r)}(t)$.

3. Conclusion

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. In the context of attrition, the model developed in this paper can be utilized to plan for adequate provision of manpower in the organization. The goodness of fit for the distribution assumed in this paper can be tested by collecting relevant data. Further, the observations on the performance measures given in this paper will be useful to enhance the facilitation of the assessment of the manpower profile in future manpower development prediction, not only in industry but also in a broader domain.

REFERENCES

1. A.Amala Nancy, A.Devi and A.Srinivasan, A stochastic model on the time to recruitment for a single grade manpower system having correlated inter-decision times, *International Journal of Physics and Mathematical Sciences,* 4(2) (2014)103-108.
2. D.J.Bartholomew, Stochastic Model for Social Processes, Second Edition, New York, John Wiley and Sons, 1973.
3. D.J.Bartholomew and F.A.Forbes, Statistical Techniques for Manpower Planning, New York, John Wiley and Sons, 1979.
4. A.Devi and A.Srinivasan, Variance of time to recruitment for a single grade manpower system with different epochs for decisions and exits, *International Journal of Research in Mathematics & Computation,* 2(1) (2014) 23-27.
5. A.Devi and A.Srinivasan, Probabilistic analysis on time to recruitment for a single grade manpower system with different epochs for decisions and exits, *International Journal of Revolution in Science and Humanity,* 4(2) (2014) 59-64.
6. A.Devi and A.Srinivasan, Expected time to recruitment in a single grade manpower system with different epochs for decisions and exits having correlated inter-decision times using univariate policy of recruitment, *Asian Academic Research Journal of Multidisciplinary,*1(23) (2014) 571-580.
7. A.Devi and A.Srinivasan, Variance of time to recruitment for a single grade manpower system with different epochs for decisions and exits having correlated inter-decision times, *Annals of Pure and Applied Mathematics,* 6(2) (2014)185-190.
8. A.Devi and A.Srinivasan, Expected time to recruitment for a single grade manpower system with different epochs for decisions and exits having inter-decision times as an order statistics, *Mathematical Science International Research Journal,* 3(2) (2014) 887-890.
9. J.B.Esther Clara, Contributions to the study on some stochastic models in manpower planning, Ph.D Thesis, Bharathidasan University, 2012.

10. J.Gurland, Distribution of the maximum of the arithmetic mean of correlated random variables, *Ann. Mathematical Statistics*, 26 (1955) 294-300.
11. N.Muthulakshmi, K.Kasturi and A.Srinivasan, Mean and variance of the time for recruitmentin a single graded manpower system with anunivariate policy of recruitment involving geometricthreshold, *Acta Ciencia Indica,*34 M (4) (2088) 1637-1641.
12. N.Poorni Perianayaki and A.Srinivasan, Mean and variance of the time to recruitment in a single graded manpower having correlated inter-decision times associated with an univariate policy of recruitment when the threshold distribution is continuous, *Acta Ciencia Indica*, 34 M (1) (2008) 65-68.
13. N.Poorni Perianayaki and A.Srinivasan, Mean and variance of the time to recruitment in a single graded manpower having correlated inter-decision times associated with an univariate policy of recruitment when the threshold distribution is discrete, *Acta Ciencia Indica*, 34 M (1) (2008) 333-338.
14. N.Poorni Perianayaki and A.Srinivasan, Mean and variance of the time to recruitment in a single graded manpower having correlated inter-decision times associated with an univariate policy of recruitment involving a constant threshold, *Acta Ciencia Indica*, 34 M (1) (2008) 441-445.
15. N.Poorni Perianayaki and A.Srinivasan, Mean and variance of the time to recruitment in a single graded manpower having correlated inter-decision times associated with an univariate policy of recruitment when the threshold distribution has SCBZ property, *Acta Ciencia Indica*, 34 M (3) (2008) 983-986.
16. W.Ritha and W.Lilly Merline, An analysis on risk factors of coronary heart disease using simple and weighted fuzzy cognitive map, *Intern. J. Fuzzy Mathematical Archive*, 3 (2013) 76-85.
17. B.Venkatesh and A.Srinivasan, Mean and variance of time to recruitment in a single graded manpower system using an univariate policy of recruitment involving a constant threshold, *Acta Ciencia Indica*, 33 M (4) (2007) 1671-1674.
18. B.Venkatesh and A.Srinivasan, Mean and variance of time to recruitment in a single graded manpower system using an univariate policy of recruitment when the threshold distribution has SCBZ property, *Acta Ciencia Indica*, 33 M(4) (2008) 1679-1682.
19. B.Venkatesh and A.Srinivasan, Mean and variance of time to recruitment in a single graded manpower system using an univariate policy of recruitment when the threshold distribution is continuous, *ActaCienciaIndica*, 33 M (4) (2007) 1675-1678.