Intern. J. Fuzzy Mathematical Archive
Vol. 5, No. 1, 2014, 1-9
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 5 August 2014
www.researchmathsci.org

International Journal of Fuzzy Mathematical Archive

k-Pseudo-Similar Interval–Valued Fuzzy Matrices

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Received 16 July 2014; accepted 26 July 2014

Abstract. In this paper, we have introduced the concept of k-pseudo-similar interval-valued fuzzy matrices (IVFM) as a generalization of k-pseudo-similar fuzzy matrices and as a special case for k=1, it reduces to pseudo-similar interval – valued fuzzy matrices (IVFM).

Keywords: pseudo-similar IVFM, k-pseudo-similar IVFM.

AMS mathematics Subject Classification (2010): 15A57, 15A09

1. Introduction

Throughout, we deal with IVFM, that is, matrices whose entries are intervals and all the intervals are subintervals of the interval [0, 1]. The concept of IVFM a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [7], by extending the max-min operations on fuzzy algebra F=[0, 1], for elements a,b∈F, a+b=max{a,b} and a·b=min{a,b}. In [3], Meenakshi and Kaliraja have represented an IVFM as an interval matrix of its lower and upper limit fuzzy matrices. $A\in F_{mn}$ is regular if there exists X such that $AXA = A$; X is called a generalized (g $^-$) inverse of A and is denoted as A^-. A{1} denotes the set of all g-inverses of a regular matrix A.

In [4], Meenakshi and Jenita have introduced the concept of k-regular fuzzy matrix as a generalization of regular fuzzy matrix developed in [1]. A matrix $A\in F_n$, the set of all n×n fuzzy matrices is said to be right (left) k-regular if there exists X (Y)$\in F_n$, such that

$$A^k XA = A^k \; (AYA^k = A^k)$$

X (Y) is called a right (left) k-g-inverse of A, where k is a positive integer. Recently, Meenakshi and Poongodi have extended the concept of k-regularity of fuzzy matrices to IVFM and determined the structure of k-regular IVFM in [5].

In section 2, some basic definitions and results required are given.

In section 3, we have introduced the concept of k-pseudo-similar interval-valued fuzzy matrices (IVFM) as a generalization of k-pseudo-similar fuzzy matrices [6] and as a special case for k=1, it reduces to pseudo-similar interval – valued fuzzy matrices (IVFM).

2. Preliminaries

Definition 2.1. [2] A∈ F_m and B∈ F_n are said to be pseudo-similar and denoted as $A \cong B$ if there exist X∈ F_{mn} and Y∈ F_{nm} such that $A = XBY$, $B = YAX$ and $XYX = X$.

Theorem 2.1. [2] Let A∈ F_m and B∈ F_n such that $A \cong B$. Then A is a regular matrix ⇔ B is a regular matrix.

Theorem 2.2. [2] Let A∈ F_m and B∈ F_n. Then the following are equivalent.

(i) $A \cong B$

(ii) $A^T \cong B^T$

(iii) $A^k \cong B^k$, for any integer k≥1.

(iv) $PAP^T \cong QBQ^T$, for some permutation matrices P∈ F_m and Q∈ F_n .

Lemma 2.1. [2] Let A∈ F_m and B∈ F_n . Then the following are equivalent:

(i) $A \cong B$

(ii) There exist X∈ F_{mn} , Y∈ F_{nm} such that $A = XBY$, $B = YAX$ and XY∈ F_m is idempotent.

(iii) There exist X∈ F_{mn} , Y∈ F_{nm} such that $A = XBY$, $B = YAX$ and YX∈ F_n is idempotent.

Definition 2.2. [3] Let A∈ $(IVFM)_{mn}$. If A is regular, then there exists a matrix X∈ $(IVFM)_{nm}$, such that $AXA = A$, for all X∈ A{1}.

Definition 2.3. [3] For a pair of fuzzy matrices E=(e_{ij}) and F=(f_{ij}) in F_{mn} such that $E \le F$, let us define the interval matrix denoted as [E, F], whose ij^th entry is the interval with lower limit e_{ij} and upper limit f_{ij}, that is ([e_{ij}, f_{ij}]). In particular for E=F, IVFM [E, E] reduces to E ∈ F_{mn}.

For A=(a_{ij})=[a_{ijL}, a_{ijU}]∈ $(IVFM)_{mn}$, let us define A_L=(a_{ijL}) and A_U=(a_{ijU}). Clearly A_L and A_U belongs to F_{mn} such that $A_L \le A_U$ and from Definition 2.3 A can be written as A=[A_L, A_U], where A_L and A_U are called lower and upper limits of A respectively.

The basic operations on IVFM are as follows [3]:

For A=$(a_{ij})_{m\times n}$ and B=$(b_{ij})_{m\times n}$, their sum A+B is defined by,

A+B= A=(a_{ij}+ b_{ij})=([(a_{ijL}+ b_{ijL}), (a_{ijU}+ b_{ijU})]) (2.1)

and their product is defined by,

$$AB=(c_{ij})=\sum_{k=1}^{n} a_{ik}b_{kj} = \left[\sum_{k=1}^{n}(a_{ikL}b_{kjL}), \quad \sum_{k=1}^{n}(a_{ikU}b_{kjU})\right] \text{ i=1,2,…..,m and j=1,2,….,p} \quad (2.2)$$

In particular if $a_{ijL} = a_{ijU}$ and $b_{ijL} = b_{ijU}$ then (2.2) reduces to the standard max-min composition of fuzzy matrices [1]. $A \le B \Leftrightarrow a_{ijL} \le b_{ijL}$ and $a_{ijU} \le b_{ijU}$. For A∈ $(IVFM)_{mn}$, A^T denote the transpose of A.

Lemma 2.2. [3] For A=[A_L, A_U]∈ $(IVFM)_{mn}$ and B=[B_L, B_U]∈ $(IVFM)_{np}$ the following hold:

(i) $A^T=[A_L^T, A_U^T]$
(ii) $AB=[A_L B_L, A_U B_U]$

Definition 2.4. [5] A matrix $A\in(IVFM)_n$, is said to be right k-regular if there exists a matrix $X\in(IVFM)_n$ such that $A^k XA = A^k$, for some positive integer k. X is called a right k-g-inverse of A.

Let $A_r\{1^k\} = \{X/ A^k XA = A^k\}$.

Definition 2.5. [5] A matrix $A\in(IVFM)_n$, is said to be left k-regular if there exists a matrix $Y\in(IVFM)_n$ such that $AYA^k = A^k$, for some positive integer k. Y is called a left k-g-inverse of A.

Let $A_\ell\{1^k\} = \{Y/ AYA^k = A^k\}$.

In general, right k-regular IVFM is different from left k-regular IVFM. Hence a right k-g-inverse need not be a left k-g-inverse [5].

Theorem 2.3. [5] Let $A=[A_L, A_U]\in(IVFM)_n$. Then A is right k-regular IVFM$\Leftrightarrow$ A_L and $A_U\in F_n$ are right k-regular.

Theorem 2.4. [5] Let $A=[A_L, A_U]\in(IVFM)_n$. Then A is left k-regular IVFM$\Leftrightarrow$ A_L and $A_U\in F_n$ are left k-regular.

3. *k*-pseudo-similar IVFM

Definition 3.1. $A=[A_L, A_U]\in(IVFM)_m$ and $B=[B_L, B_U]\in(IVFM)_n$ are said to be pseudo-similar IVFM, denoted by $A \overset{I}{\cong} B$ if there exist $X=[X_L, X_U]\in(IVFM)_{mn}$ and $Y=[Y_L, Y_U]\in(IVFM)_{nm}$ such that $A = XBY$, $B = YAX$ and $XYX = X$.

Remark 3.1. In particular, if $A_L=A_U$ and $B_L=B_U$ then Definition 3.1 reduces to pseudo-similar fuzzy matrices (Definition 2.1). Also we observe that $A \overset{I}{\cong} B \Leftrightarrow B \overset{I}{\cong} A$.

Definition 3.2. $A=[A_L, A_U]\in(IVFM)_n$ is said to be right k-pseudo-similar to $B=[B_L, B_U]\in(IVFM)_n$ and it is denoted by $A \underset{r}{\overset{I(k)}{\cong}} B$ if there exist $X=[X_L, X_U]$, $Y=[Y_L, Y_U]\in(IVFM)_n$ such that $A = XBY$, $B = YAX^k$, $X^k YX = X^k$ and $YXY = Y$.

Definition 3.3. $A=[A_L, A_U]\in(IVFM)_n$ is said to be left k-pseudo-similar to $B=[B_L, B_U]\in(IVFM)_n$ and it is denoted by $A \underset{\ell}{\overset{I(k)}{\cong}} B$ if there exist $X=[X_L, X_U]$, $Y=[Y_L, Y_U]\in(IVFM)_n$ such that $A = X^k BY$, $B = YAX$, $XYX^k = X^k$ and $YXY = Y$.

Remark 3.2. In particular, if $A_L=A_U$ and $B_L=B_U$ then Definition 3.2 and Definition 3.3 reduce to right (left) k- pseudo-similar fuzzy matrices studied in [6]. Further, for k=1, Definitions 3.2 and 3.3 are identical and reduced to pseudo-similar IVFM [Definition

3.1]. We observe that the right (left) k-pseudo similarity relation on IVFM is not transitive as in the case of pseudo similar fuzzy matrices [2].

Lemma 3.1. Let A=[A_L, A_U], B=[B_L, B_U]∈(IVFM)$_n$. If $A \overset{I(k)}{\underset{r}{\cong}} B$ then we have the following:

(i) $A_L^k = X_L B_L^k Y_L$ and $A_U^k = X_U B_U^k Y_U$

(ii) $B_L Y_L X_L = Y_L X_L B_L = B_L$ and $B_U Y_U X_U = Y_U X_U B_U = B_U$

(iii) $A_L X_L Y_L = X_L Y_L A_L = A_L$ and $A_U X_U Y_U = X_U Y_U A_U = A_U$

(iv) $B_L^k = Y_L A_L^k X_L$ and $B_U^k = Y_U A_U^k X_U$

Proof: Since $A \overset{I(k)}{\underset{r}{\cong}} B$, $A = XBY$, $B = YAX^k$, $X^k YX = X^k$ and $YXY = Y$.

(i) $A = XBY \Rightarrow A^2 = (XBY)(XBY) = X(BYX)BY$

$BYX = (YAX^k)YX = YA(X^k YX) = YAX^k = B$

Therefore, $A^2 = (XBY)(XBY) = X(BYX)BY = XBBY = XB^2Y$.

Thus proceeding we get $A^k = XB^kY$.

$A^k = XB^kY \Rightarrow [A_L, A_U]^k = [X_L, X_U][B_L, B_U]^k[Y_L, Y_U]$

$\Rightarrow [A_L^k, A_U^k] = [X_L, X_U][B_L^k, B_U^k][Y_L, Y_U] \Rightarrow A_L^k = X_L B_L^k Y_L$ and $A_U^k = X_U B_U^k Y_U$.

Thus (i) holds.

(ii) $YXB = YX(YAX^k) = (YXY)AX^k = YAX^k = B$.

$YXB = B \Rightarrow [Y_L, Y_U][X_L, X_U][B_L, B_U] = [B_L, B_U] \Rightarrow Y_L X_L B_L = B_L$ and $Y_U X_U B_U = B_U$.

$BYX = B \Rightarrow [B_L, B_U][Y_L, Y_U][X_L, X_U] = [B_L, B_U] \Rightarrow B_L Y_L X_L = B_L$ and $B_U Y_U X_U = B_U$

(iii) $AXY = (XBY)XY = XB(YXY) = XBY = A$.

$XYA = XY(XBY) = X(YXB)Y = XBY = A$.

$AXY = A \Rightarrow [A_L, A_U][X_L, X_U][Y_L, Y_U] = [A_L, A_U]$

$\Rightarrow [A_L X_L Y_L, A_U X_U Y_U] = [A_L, A_U] \Rightarrow A_L X_L Y_L = A_L$ and $A_U X_U Y_U = A_U$

$XYA = A \Rightarrow [X_L, X_U][Y_L, Y_U][A_L, A_U] = [A_L, A_U] \Rightarrow [X_L Y_L A_L, X_U Y_U A_U] = [A_L, A_U]$

$\Rightarrow X_L Y_L A_L = A_L$ and $X_U Y_U A_U = A_U$.

(iv)

$B = YXB \Rightarrow B^k = YXB^k \Rightarrow B^k = YX(B^k YX) = Y(XB^kY)X = YA^kX$.

$B^k = YA^kX \Rightarrow [B_L, B_U]^k = [Y_L, Y_U][A_L, A_U]^k[X_L, X_U]$ $B_L^k = Y_L A_L^k X_L$ and

$\Rightarrow [B_L^k, B_U^k] = [Y_L, Y_U][A_L^k, A_U^k][X_L, X_U] \Rightarrow B_U^k = Y_U A_U^k X_U$.

Lemma 3.2. Let A=[A_L, A_U], B=[B_L, B_U]$\in$(IVFM)$_n$. If $A \overset{I(k)}{\underset{\ell}{\cong}} B$ then we have the following:

(i) $B_L^k = Y_L A_L^k X_L$ and $B_U^k = Y_U A_U^k X_U$

(ii) $A_L X_L Y_L = X_L Y_L A_L = A_L$ and $A_U X_U Y_U = X_U Y_U A_U = A_U$

(iii) $B_L Y_L X_L = Y_L X_L B_L = B_L$ and $B_U Y_U X_U = Y_U X_U B_U = B_U$

(iv) $A_L^k = X_L B_L^k Y_L$ and $A_U^k = X_U B_U^k Y_U$

Proof: This can be proved along the same lines as that of Lemma 3.1 and hence omitted.

Theorem 3.1. Let A, B$\in$(IVFM)$_n$ such that $A \overset{I(k)}{\underset{r}{\cong}} B$. A is right (left) k-regular $\Leftrightarrow$ B is right (left) k-regular.

Proof:

Since $A \overset{I(k)}{\underset{r}{\cong}} B$, $A = XBY$, $B = YAX^k$, $X^k YX = X^k$ and $YXY = Y \Rightarrow$

$A_L = X_L B_L Y_L$, $B_L = Y_L A_L X_L^k$, $A_U = X_U B_U Y_U$ and $B_U = Y_U A_U X_U^k$.

By Lemma 3.1,

$A_L^k = X_L B_L^k Y_L$, $A_U^k = X_U B_U^k Y_U$, $B_L Y_L X_L = Y_L X_L B_L = B_L$,

$B_U Y_U X_U = Y_U X_U B_U = B_U$, $A_L X_L Y_L = X_L Y_L A_L = A_L$, $A_U X_U Y_U = X_U Y_U A_U = A_U$

, $B_L^k = Y_L A_L^k X_L$ and $B_U^k = Y_U A_U^k X_U$.

Let A be right k-regular. Then by Theorem 2.3, A_L and A_U are right k-regular.

Since, A_L and A_U are right k-regular there exists G=[G_L, G_U] $\in$(IVFM)$_n$ such that $A_L^k G_L A_L = A_L^k$ and $A_U^k G_U A_U = A_U^k$.Choose U=YGX, U=[U_L, U_U]$\in$(IVFM)$_n$.

$U = YGX \Rightarrow U_L = Y_L G_L X_L$ and $U_U = Y_U G_U X_U$.

To prove that, B is right k-regular, let us prove that B_L and B_U are right k-regular.

$B_L^k U_L B_L = (Y_L A_L^k X_L)(Y_L G_L X_L) B_L = Y_L (A_L^k X_L Y_L) G_L (X_L B_L)$

$= Y_L A_L^k G_L (X_L B_L Y_L X_L) = Y_L A_L^k G_L (A_L X_L) = Y_L (A_L^k G_L A_L) X_L = Y_L A_L^k X_L = B_L^k$.

$B_U^k U_U B_U = (Y_U A_U^k X_U)(Y_U G_U X_U) B_U = Y_U (A_U^k X_U Y_U) G_U (X_U B_U)$

$= Y_U A_U^k G_U (X_U B_U Y_U X_U) = Y_U A_U^k G_U (A_U X_U) = Y_U (A_U^k G_U A_U) X_U = Y_U A_U^k X_U = B_U^k$.

Therefore, B_L and B_U are right k-regular. Hence by Theorem 2.3, B is right k-regular.

Converse part follows by replacing A by B in the above proof.

A is left k-regular $\Leftrightarrow$ B is left k-regular can be proved in the same manner and hence omitted.

Hence the Theorem..

Theorem 3.2. Let A, B$\in$(IVFM)$_n$ such that $A \overset{I(k)}{\underset{\ell}{\cong}} B$. A is right (left) k-regular $\Leftrightarrow$ B is right (left) k-regular.

Proof: This can be proved as that of Theorem 3.1 and hence omitted.

Remark 3.3. For k=1, Theorems 3.1 and 3.2 reduces to the following theorem.

Theorem 3.3. Let A∈(IVFM)$_m$ and B∈(IVFM)$_n$ such that $A \overset{I}{\cong} B$. Then A is a regular matrix ⇔ B is a regular matrix.

Remark 3.4. In particular, for fuzzy matrices, $A_L=A_U$ and $B_L=B_U$, Theorem 3.3 reduces to Theorem 2.1.

Lemma 3.3. Let A, B∈(IVFM)$_n$. If $A \underset{r}{\overset{I(k)}{\cong}} B$ then there exist X, Y∈(IVFM)$_n$ such that $A = XBY$, $B = YAX^k$ and XY is k-potent.

Proof:

Since $A \underset{r}{\overset{I(k)}{\cong}} B$, $A = XBY$, $B = YAX^k$, $X^kYX = X^k$ and $YXY = Y$.

$$(XY)^k = (XY)^{k-1}XY = (XY)^{k-2}XYXY = (XY)^{k-2}X(YXY) = (XY)^{k-2}XY = \ldots\ldots\ldots\ldots = XY.$$

Hence the proof.

Lemma 3.4. Let A, B∈(IVFM)$_n$. If $A \underset{\ell}{\overset{I(k)}{\cong}} B$ then there exist X, Y∈(IVFM)$_n$ such that $A = X^kBY$, $B = YAX$ and YX is k-potent.

Proof:

Since $A \underset{\ell}{\overset{I(k)}{\cong}} B$, $A = X^kBY$, $B = YAX$, $XYX^k = X^k$ and $YXY = Y$.

$$(YX)^k = (YX)^{k-1}YX = (YX)^{k-2}YXYX = (YX)^{k-2}(YXY)X = (YX)^{k-2}YX = \ldots\ldots\ldots\ldots = YX.$$

Hence the proof.

Remark 3.5. For k=1, from Lemma 3.3 and Lemma 3.4 we get an equivalence condition for pseudo similar IVFM in the following:

Lemma 3.5. Let A∈(IVFM)$_m$ and B∈(IVFM)$_n$. Then the following are equivalent:

(i) $A \overset{I}{\cong} B$

(ii) There exist X∈(IVFM)$_{mn}$, Y∈(IVFM)$_{nm}$ such that $A = XBY$, $B = YAX$ and XY∈(IVFM)$_m$ is idempotent.

(iii) There exist X∈(IVFM)$_{mn}$, Y∈(IVFM)$_{nm}$ such that $A = XBY$, $B = YAX$ and YX∈(IVFM)$_n$ is idempotent.

Proof:

(i)⇒(ii) and (i)⇒(iii) are trivial, since $XYX = X$ ⇒ XY∈(IVFM)$_m$ and YX∈(IVFM)$_n$ are idempotent matrices.

(ii)⇒(i): $A = XBY = X(YAX)Y = (XY)A(XY) = (XY)XBY(XY) = (XYX)B(YXY)$.

Similarly, $B = YAX = (YXY)A(XYX)$. Put $XYX = X'$ and $YXY = Y'$. Then, $A = X'BY'$ and $B = Y'AX'$. Further using XY is idempotent, we get $X'Y' = (XYX)(YXY) = XY$ and $(X'Y')(X'Y') = (XY)(XY) = X'Y'$. Thus $X'Y'$ is idempotent. Set $X'Y'X' = X''$ and $Y'X'Y' = Y''$. Then, $A = X'BY' = X'Y'AX'Y' = (X'Y'X')B(Y'X'Y')$, therefore $A = X''BY''$. Similarly, $B = Y''AX''$. By using $X'Y'$ is idempotent, we have $X''Y''X'' = (X'Y'X')(Y'X'Y')(X'Y'X') = X'Y'X' = X''$. Therefore, $A \overset{I}{\cong} B$. Thus (i) holds.

(iii)⇒(i): Can be proved in the same manner and hence omitted.

Remark 3.6. In particular, if $A_L=A_U$ and $B_L=B_U$, Lemma 3.5 reduces to Lemma 2.1.

Lemma 3.6. Let A∈ $(IVFM)_m$ and B∈ $(IVFM)_n$. Then the following are equivalent:

(i) $A \overset{I}{\cong} B$

(ii) There exist X∈ $(IVFM)_{mn}$, Y∈ $(IVFM)_{nm}$ such that $A = XBY$, $B = YAX$, $XYX = X$ and $YXY = Y$.

(iii) There exist X∈ $(IVFM)_{mn}$, Y∈ $(IVFM)_{nm}$ such that $A = XBY$, $B = ZAX$, $XYX = X = XZX$.

Proof:

(i)⇒(iii): Since $A \overset{I}{\cong} B$, $A = XBY$, $B = YAX$ and $XYX = X$.

Let Y=Z, then $B = ZAX$ and $X = XZX$ as required. Thus (iii) holds.

(iii)⇒(ii): Suppose there exist X∈ $(IVFM)_{mn}$, Y, Z∈ $(IVFM)_{nm}$ such that $A = XBY$, $B = ZAX$, $XYX = X = XZX$, then

$A = XBY = X(ZAX)Y = XZ(XBY)(XY) = (XZX)B(YXY) = XB(YXY)$ and

$B = ZAX = Z(XBY)X = ZX(ZAX)YX = (ZXZ)A(XYX) = (ZXZ)AX$. Set $YXY = Y'$ and $Z' = ZXZ$. Then, $X = XYX = XY(XYX) = XY'X$ and $X = XZX = XZ(XZX) = XZ'X$. In addition, we have $A = XBY'$ and $B = Z'AX$. Set $Y'' = Z'XY'$.

Then $XY''X = XZ'(XY'X) = XZ'X = X$ and

$Y''XY'' = Z'(XY'X)Z'XY' = Z'XY' = Y''$. We directly check that $XBY'' = A$, $Y''AX = B$. Thus there exist X∈ $(IVFM)_{mn}$, Y’’∈ $(IVFM)_{nm}$ such that $A = XBY''$, $B = Y''AX$, $XY''X = X$ and $Y''XY'' = Y''$.

Thus (ii) holds. (ii)⇒(i): This is trivial.

Theorem 3.4. Let A, B∈ $(IVFM)_n$. Then the following are equivalent.

(i) $A \underset{r}{\overset{I(k)}{\cong}} B$

(ii) $B^T \underset{\ell}{\overset{I(k)}{\cong}} A^T$

(iii) $PAP^T \overset{I(k)}{\underset{r}{\cong}} PBP^T$ for some permutation matrix P=[P_L, P_U]∈(IVFM)$_n$ with P_L=P_U=P.

Proof:

(i)⇔(ii): This is direct by taking transpose on both sides and by using $(A^T)^T = A$ and $(AX)^T = X^T A^T$.

(ii)⇔(iii): Suppose $A \overset{I(k)}{\underset{r}{\cong}} B$ then $A = XBY$, $B = YAX^k$, $X^kYX = X^k$ and $YXY = Y$.

$$A = XBY \Rightarrow PAP^T = PXBYP^T = (PXP^T)(PBP^T)(PYP^T) \quad (3.1)$$

$$B = YAX^k \Rightarrow PBP^T = PYAX^kP^T = (PYP^T)(PAP^T)(PX^kP^T)^k$$
$$= (PYP^T)(PAP^T)(PXP^T) \quad (3.2)$$

$$X^kYX = X^k \Rightarrow PX^kP^T = PX^kYXP^T \Rightarrow PX^kP^T = (PX^kP^T)(PYP^T)(PXP^T)$$
$$\Rightarrow (PXP^T)^k = (PXP^T)^k(PYP^T)(PXP^T) \quad (3.3)$$

$$Y = YXY \Rightarrow PYP^T = PYXYP^T \Rightarrow PYP^T = (PYP^T)(PXP^T)(PYP^T) \quad (3.4)$$

Hence $PAP^T \overset{I(k)}{\underset{r}{\cong}} PBP^T$.

Conversely, suppose $PAP^T \overset{I(k)}{\underset{r}{\cong}} PBP^T$.

Pre multiply by P^T and post multiply by P in Equations (3.1) to (3.4), we get $A = XBY$, $B = YAX^k$, $X^kYX = X^k$ and $YXY = Y$. Hence $A \overset{I(k)}{\underset{r}{\cong}} B$.

Hence the proof.

Theorem 3.5. Let A, B∈(IVFM)$_n$. Then the following are equivalent.

(i) $A \overset{I(k)}{\underset{\ell}{\cong}} B$

(ii) $B^T \overset{I(k)}{\underset{r}{\cong}} A^T$

(iii) $PAP^T \overset{I(k)}{\underset{\ell}{\cong}} PBP^T$ for some permutation matrix P=[P_L, P_U]∈(IVFM)$_n$ with P_L=P_U=P.

Proof: Proof of the theorem is similar to Theorem 3.4 and hence omitted.

Theorem 3.6. Let A, B∈(IVFM)$_n$. If $A \overset{I(k)}{\underset{r}{\cong}} B$ then $A^k \overset{I(k)}{\underset{r}{\cong}} B^k$.

Proof:

Suppose $A \overset{I(k)}{\underset{r}{\cong}} B$ then $A = XBY$, $B = YAX^k$, $X^kYX = X^k$ and $YXY = Y$.

Prove that, $A^k \overset{I(k)}{\underset{r}{\cong}} B^k$.

By Lemma [3.1] (i), $A^k = XB^kY$.

Next,let us prove that, $B^k = YA^k X^k$.
By Lemma [3.1] (ii), $BYX = YXB = B \Rightarrow$

$$B^K = YXB^k = YXB^{k-1}B = YXB^{k-1}(YAX^k) = Y(XB^{k-1}Y)AX^k = Y(A^{k-1})AX^k = YA^k X^k$$

.

Hence $A^k \underset{r}{\overset{k}{\cong}} B^k$.

Theorem 3.7. Let A, B$\in$(IVFM)$_n$. If $A \underset{\ell}{\overset{I(k)}{\cong}} B$ then $A^k \underset{\ell}{\overset{I(k)}{\cong}} B^k$.

Proof:
This is similar to Theorem 3.6 and hence omitted.

Remark 3.7: As a special case of Theorem 3.5,Theorem 3.6 and Theorem 3.7 for k=1,we have the following:

Theorem 3.8. Let A$\in$(IVFM)$_m$ and B$\in$(IVFM)$_n$. Then the following are equivalent.

(i) $A \overset{I}{\cong} B$

(ii) $A^T \overset{I}{\cong} B^T$

(iii) $A^k \overset{I}{\cong} B^k$, for any integer k≥1.

(iv) $PAP^T \overset{I}{\cong} P\,BP^T$, for some permutation matrix P=[P$_L$, P$_U$]$\in$(IVFM)$_n$ with P$_L$=P$_U$=P.

(v)

Remark 3.8. In particular, if A$_L$=A$_U$ and B$_L$=B$_U$, Theorem 3.8 reduces to Theorem 2.2.

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