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Solution of a Fuzzy Assignment Problem by Using a New Ranking Method

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Abstract: Assignment problem is a well-known topic and is used very often in solving problems of engineering and management science. In this problem c_{ij} denotes the cost for assigning the j^{th} job to the i^{th} person. This cost is usually deterministic in nature. In this paper c_{ij} has been considered to be triangular or trapezoidal fuzzy numbers denoted by $\tilde{c}_{ij}$which are more realistic and general in nature. A new ranking method is used for ranking the fuzzy numbers. The fuzzy assignment problem has been transformed into crisp assignment problem in the LPP form and solved by using LINGO 9.0. Numerical examples show that the fuzzy ranking method offers an effective way for handling the fuzzy assignment problem.

Keywords: Fuzzy Number, Assignment Problem, Fuzzy Ranking Method

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1. Introduction

Assignment Problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in industry and other applications. In an assignment problem, ***n*** jobs are to be performed by ***n*** persons depending on their efficiency to do the job. In this problem c_{ij} denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one person can be assigned exactly one job; also each person can do at most one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum.

Here we investigate a more realistic problem, namely the assignment problem with fuzzy costs or times $\tilde{c}_{ij}$. Since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy numbers to find the best alternative. On the basis of this idea a new ranking method is used to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve the assignment problem.

The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP form and to solve it by the simplex method. Other than the fuzzy assignment problem other applications of this method can be tried in project scheduling, maximal flow, transportation problem, etc.

1.1. Review of Literature

In recent years, fuzzy transportation and fuzzy assignment problems have received much attention.

Lin and Wen solved the assignment problem with fuzzy interval number costs by a labeling algorithm [2]. In the paper by Sakawa et al.[7], the authors dealt with actual problems on production and work force assignment in a housing material manufacturer and a subcontract firm and formulated two kinds of two-level programming problems. Applying the interactive fuzzy programming for two-level linear and linear fractional programming problems, they derived satisfactory solutions to the problems and thereafter compared the results. They examined actual planning of the production and the work force assignment of the two firms to be implemented. Chen[8] proved some theorems and proved a fuzzy assignment model that considers all individuals to have same skills. Wang[16] solved a similar model by graph theory. Dubois and Fortemps[3] surveys refinements of the ordering of solutions supplied by the max-min formulation. They have given a general algorithm which computes all maximal solutions in the sense of these relations. Different kinds of fuzzy transportation problems are solved in the papers [5,6,9,12,13,14].

Dominance of fuzzy numbers can be explained by many ranking methods [1,4,10,11,15,17]. Here we use a new ranking method.

2. Preliminaries

Zadeh [18] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

Definition 2.1. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range *i.e.* $\mu_{\tilde{A}}: X \to [0, 1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2. A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \to [0, 1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on [c, d].
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

Definition 2.3. A fuzzy number $\tilde{A}=(a,b,c,d)$ is a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \frac{x-a}{b-a}, \quad a \le x \le b$$
$$= 1, \qquad b \le x \le c$$
$$= \frac{x-d}{c-d}, \qquad c \le x \le d$$

Definition 2.4. A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \to [0,\omega]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a,b]$ and strictly decreasing on [c, d].
4. $\mu_{\tilde{A}}(x) = \omega$ for all $x \in [b,c]$, where $0 < \omega \le 1$.

Definition 2.5. A fuzzy number $\tilde{A}$=$(a,b,c,d;\omega)$ is a said to be a L-R type generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \omega L\left(\frac{b-x}{b-a}\right), \quad a \le x \le b$$
$$= \omega, \qquad b \le x \le c$$
$$= \omega R\left(\frac{x-c}{d-c}\right), \qquad c \le x \le d$$

where L and R are reference functions.

Definition 2.6. A L-R type generalized fuzzy number $\tilde{A}$=$(a,b,c,d;\omega)$ is a said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \omega\left(\frac{x-a}{b-a}\right), \quad a \le x \le b$$
$$= \omega, \qquad b \le x \le c$$
$$= \omega\left(\frac{x-d}{d-c}\right), \qquad c \le x \le d$$

where L and R are reference functions.

2.7. Arithmetic Operations

In this subsection, arithmetic operations between two L-R type generalized fuzzy numbers, defined on universal set of real numbers R, are reviewed.

Let $\tilde{A}$=$(a,b,c,d;\omega)$ and $\tilde{B}$=$(p,q,r,s;v)$ be any two L-R type generalized fuzzy numbers, then

(i) $\tilde{A}\oplus\tilde{B} = (a+p, b+q, c+r, d+s; \min(\omega, v))$

(ii) $\tilde{A} \ominus \tilde{B} = (a-s, b-r, c-q, d-p; \min(\omega, v))$

(iii) $\lambda\tilde{A} = (\lambda a, \lambda b, \lambda c, \lambda d; w)$ if $\lambda > 0$
$= (\lambda d, \lambda c, \lambda b, \lambda a; w)$ if $\lambda < 0$

(iv) $A\otimes\tilde{B} = (\min(ap, as, dp, ds), \min(bq, br, cq, cr),$
$max(bq, br, cq, cr), max(ap, as, dp, ds))$

3. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of an efficient ranking function $\mathfrak{R}: F(R) \to R$, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists *i.e.*,

(i) $A \succ \tilde{B}$ if and only if $\Re(\tilde{A}) > R(\tilde{B})$

(ii) $\tilde{A} \prec \tilde{B}$ if and only if $\Re(\tilde{A}) < R(\tilde{B})$

(iii) $\tilde{A} \sim \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

3.1. New Ranking Method

The centroid point of a trapezoid is considered to be the balancing point of the trapezoid. Divide the trapezoid into three plane figures. These three plane figures are a triangle APB, a rectangle BPQC and again a triangle CQD respectively. Let the centroids of the three plane figures be G_1, G_2 and G_3respectively.

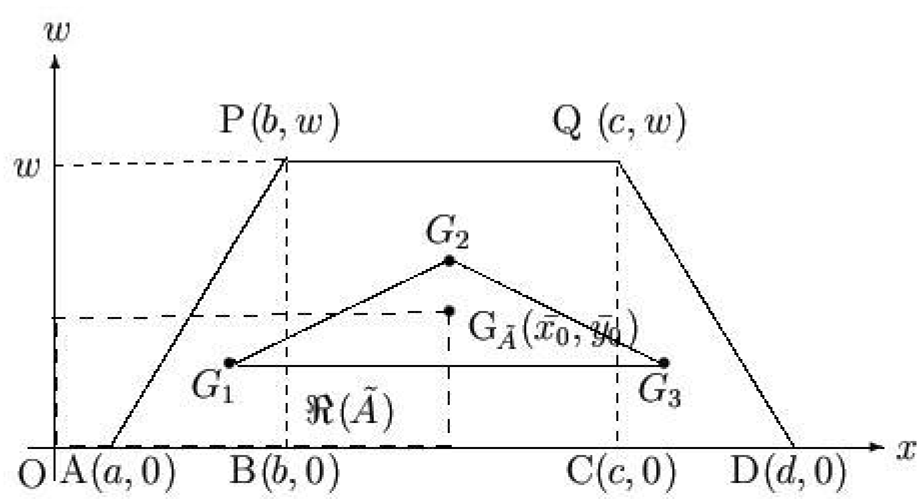

Figure 3.1. Trapezoidal fuzzy number

The centroid of these centroids G_1, G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point $G_1$of triangle APB, $G_2$of rectangle BPQC and $G_3$of triangle CQD are balancing points of each individual plane figure and the centroid of these centroid points is a much more balancing point for a general trapezoidal fuzzy number.

Consider a generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; \omega)$. The centroids of these plane figures are $G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right)$; $G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)$ and $G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right)$ respectively. Equation of the line G_1G_3 is $y = \frac{w}{3}$ and G_2does not lie on the line G_1G_3. Thus G_1, G_2 and G_3 are non collinear and they form a triangle.

We define the centroid $G_{\tilde{A}}(\overline{x_0}, \overline{y_0})$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; \omega)$ as $G_{\tilde{A}}(\overline{x_0}, \overline{y_0}) = \left(\frac{2a+7b+7c+2d}{18}, \frac{7w}{18}\right)$.

As a special case, for triangular fuzzy numbers $\tilde{A}$=$(a, b, d; \omega)$ $i.e., c = b$, the centroid of centroids is given by $G_{\tilde{A}}(\overline{x_0}, \overline{y_0}) = \left(\frac{2a+7b+d}{9}, \frac{7w}{18}\right)$.

The ranking function of the generalized trapezoidal fuzzy number $\tilde{A}$=$(a, b, c, d; \omega)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as

$$\Re(\tilde{A}) = \overline{x_0}.\overline{y_0} = \left(\frac{2a+7b+7c+2d}{18}\right)\left(\frac{7w}{18}\right) \tag{3.1}$$

This is the area between the centroid of the centroids $G_{\tilde{A}}(\overline{x_0}, \overline{y_0})$ and the original point.
The mode of the generalized trapezoidal fuzzy number $\tilde{A}$=$(a, b, d; \omega)$is defined as:

$$\textbf{\textit{mode}} = \frac{1}{2}\int_0^w (b + c)dx = \frac{w}{2}(b + c) \tag{3.2}$$

The average spread of the generalized trapezoidal fuzzy number $\tilde{A}$=$(a, b, d; \omega)$is defined as:

$$\textbf{\textit{AS}} = \frac{1}{2}\left[\int_0^w (b - a)dx + \int_0^w (d - c)dx\right] = \frac{1}{2}[w(b - a) + w(d - c)] \tag{3.3}$$

Using the above definitions we now define the ranking procedure of two generalized trapezoidal fuzzy numbers. Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B}$=$(a_2, b_2, c_2, d_2; w_2)$ be any two generalized fuzzy numbers. The working procedure to compare $\tilde{A}$ and $\tilde{B}$ is as follows:

Step 1: Find $\Re(\tilde{A})and\ \Re(\tilde{B})$.
Case (***i***) If $\Re(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (***ii***) If $\Re(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (***iii***) If $\Re(\tilde{A}) = R(\tilde{B})$ then comparison is not possible, then go to step 2.

Step 2: Find $mode(\tilde{A})and\ mode(\tilde{B})$.
Case (***i***) If $mode(\tilde{A}) > mode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (***ii***) If $mode(\tilde{A}) < mode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (***iii***) If $mode(\tilde{A}) = mode(\tilde{B})$ then comparison is not possible, then go to step 3.

Step 3: Find $AS(\tilde{A})and\ AS(\tilde{B})$.
Case (***i***) If $AS(\tilde{A}) > AS(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$
Case (***ii***) If $AS(\tilde{A}) < AS(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$
Case (***iii***) If $AS(\tilde{A}) = AS(\tilde{B})$ then comparison is not possible, then go to step 4.

Step 4: Examine w_1 and w_2.
Case (***i***) If $w_1 > w_2$ then $\tilde{A} \succ \tilde{B}$
Case (***ii***) If $w_1 < w_2$ then $\tilde{A} \prec \tilde{B}$
Case (***iii***) If $w_1 = w_2$then $\tilde{A} \sim \tilde{B}$

4. The Proposed Method

The assignment problem can be stated in the form of $n \times n$ cost matrix $[c_{ij}]$ of real numbers as given in the following table:

Persons	Jobs				
	1	2	3	….	n
1	c_{11}	c_{12}	c_{13}	….	c_{1n}
2	c_{21}	c_{22}	c_{23}	….	c_{2n}
⋮					
n	c_{n1}	c_{n2}	c_{n3}	….	c_{nn}

Mathematically assignment problem can be stated as

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2, \ldots n.$

$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2, \ldots n$

where $x_{ij} = 1, if\ the\ i^{th}\ person\ is\ assigned\ the\ j^{th}\ job$

$= 0, otherwise$ (4.1)

is the decision variable denoting the assignment of the person i to job *j*. c_{ij} is the cost of assigning the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one person).

When the costs or time $\tilde{c}_{ij}$ are fuzzy numbers, then the total cost becomes a fuzzy number $\tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$. Hence it cannot be minimized directly. For solving the problem, we defuzzify the fuzzy cost coefficients into crisp ones by the above fuzzy number ranking method.

For the assignment problem (1), with fuzzy objective function

$\min \tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$ we apply our ranking method to get the minimum objective value $\tilde{z}^*$ from the formulation

$$\Re(\tilde{z}^*) = minimize\ z = \sum_{i=1}^{n} \sum\nolimits_{j=1}^{n} \Re(\tilde{c}_{ij}) x_{ij}$$

Subject to

$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2, \ldots n.$

$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,2, \ldots n$

where $x_{ij} = 1, if\ the\ i^{th}\ person\ is\ assigned\ the\ j^{th}\ job$

$= 0, otherwise$ (4.2)

is the decision variable denoting the assignment of the person i to job *j*. $\tilde{c}_{ij}$ is the cost of assigning the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one person).

Since $\Re(\tilde{c}_{ij})$ are crisp values, this problem (4.2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional methods, namely the Hungarian method or the simplex method to solve the LPP form of the problem. Once the optimal solution x^* of model (4.2) is found, the fuzzy objective value of $\tilde{z}^*$ of the original can be calculated as $\tilde{z}^* = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}^*$.

5. Numerical Example

Example 5.1. Let us consider a Fuzzy assignment problem with rows representing 4 persons A,B,C,D and columns representing the 4 jobs Job1, Job2,Job3 and Job4. The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are trapezoidal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

Persons	Jobs			
	1	2	3	4
A	(3,5,6,7)	(5, 8, 11, 12)	(9, 10, 11, 15)	(5, 8, 10, 11)
B	(7, 8, 10, 11)	(3, 5, 6,7)	(6, 8, 10, 12)	(5, 8, 9, 10)
C	(2, 4, 5,6)	(5, 7, 10, 11)	(8, 11, 13, 15)	(4, 6, 7, 10)
D	(6, 8, 10, 12)	(2, 5, 6, 7)	(5, 7, 10, 11)	(2, 4, 5, 7)

Solution: In conformation to Model (2) the fuzzy assignment problem can be formulated in the following mathematical programming form:

Minimize

$[\Re(3,5,6,7)x_{11} + \Re(5,8,11,12)x_{12} + \Re(9,10,11,15)x_{13} +$
$\Re(5,8,10,11)x_{14} + \Re(7,8,10,11)x_{21} + \Re(3,5,6,7)x_{22} + \Re(6,8,10,12)x_{23} +$
$\Re(5,8,9,10)x_{24} + \Re(2,4,5,6)x_{31} + \Re(5,7,10,11)x_{32} + \Re(8,11,13,15)x_{33} +$
$\Re(4,6,7,10)x_{34}$
$+\Re(6,8,10,12)x_{31} + \Re(2,5,6,7)x_{32} + \Re(5,7,10,11)x_{33} + \Re(2,4,5,7)x_{34}]$

such that

$$x_{11} + x_{12} + x_{13} + x_{14} = 1, \quad x_{11} + x_{21} + x_{31} + x_{41} = 1,$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1, \quad x_{12} + x_{22} + x_{32} + x_{42} = 1,$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1, \quad x_{13} + x_{23} + x_{33} + x_{43} = 1,$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1, \quad x_{14} + x_{24} + x_{34} + x_{44} = 1. \tag{4.3}$$

By our ranking method, we have

$\Re(3,5,6,7) = 2.096, \Re(5,8,11,12) = 3.608,$
$\Re(9,10,11,15) = 4.213, \Re(5,8,10,11) = 3.414$
$\Re(7,8,10,11) = 3.5, \Re(3,5,6,7) = 2.096$
$\Re(6,8,10,12) = 3.5, \Re(5,8,9,10) = 3.219$
$\Re(2,4,5,6) = 1.707, \Re(5,7,10,11) = 3.262$
$\Re(8,11,13,15) = 4.623, \Re(4,6,7,10) = 2.571$
$\Re(6,8,10,12) = 3.5, \Re(2,5,6,7) = 2.052$
$\Re(5,7,10,11) = 3.262, \Re(2,4,5,7) = 1.75$

We replace these values for their corresponding $\tilde{c}_{ij}$ in (4.3), which results in a conventional assignment problem in the LPP form. Solving it, we get the solution as

$x_{13}^* = x_{22}^* = x_{31}^* = x_{44}^* = \,,\; x_{11}^* = x_{12}^* = x_{14}^* = x_{21}^* = x_{23}^* = x_{24}^* = x_{32}^* = x_{33}^* = x_{34}^* = x_{41}^* = x_{42}^* = x_{43}^* = 0$

with the optimal objective value 9.766 which represents the optimal total cost. In other words the optimal assignment is $A \rightarrow 3, B \rightarrow 2, C \rightarrow 1, D \rightarrow 4$. The fuzzy optimal cost is calculated as

$\tilde{c}_{13} + \tilde{c}_{22} + \tilde{c}_{31} + \tilde{c}_{44} = (9, 10, 11, 15) + (3, 5, 6, 7)$+(2, 4, 5,6)+(2, 4, 5, 7)
= (16, 23, 27, 35).

Also $\Re(\tilde{z}^*) = \Re(16,23,27,35) = 9.766.$

Example 5.2. Let us consider a Fuzzy assignment problem with rows representing 3 persons A,B,C and columns representing the 3 jobs Job1, Job2 and Job3. The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are triangular or trapezoidal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

Persons	Jobs		
	1	2	3
A	(2,4,6)	(7, 8, 10, 12)	(4,5,6,8)
B	(8,10,12,13)	(10,13,15)	(5,7,10)
C	(2, 3, 5,6)	(6, 7, 9,10)	(9,11,12)

Solution: In conformation to Model (2) the fuzzy assignment problem can be formulated in the following mathematical programming form:

Minimize

$[\Re(2,4,6)x_{11} + \Re(7,8,10,12)x_{12} + \Re(4,5,6,8)x_{13} + \Re(8,10,12,13)x_{21} + \Re(10,13,15)x_{22} + \Re(5,7,10)x_{23} + \Re(2,3,5,6)x_{31} + \Re(6,7,9,10)x_{32} + \Re(9,11,125)x_{33}]$

such that

$$x_{11} + x_{12} + x_{13} = 1,\ \ x_{11} + x_{21} + x_{31} = 1,$$
$$x_{21} + x_{22} + x_{23} = 1,\ x_{12} + x_{22} + x_{32} = 1,$$
$$x_{31} + x_{32} + x_{33} = 1,\ x_{13} + x_{23} + x_{33} = 1, \qquad (4.4)$$

By our ranking method, we have

$\Re(2,4,6) = 1.556, \Re(7,8,10,12) = 3.543,$
$\Re(4,5,6,8) = 2.182, \Re(8,10,12,13) = 4.234$
$\Re(10,13,15) = 5.012, \Re(5,7,10) = 2.765$
$\Re(2,3,5,6) = 1.556, \Re(6,7,9,10) = 3.111$
$\Re(9,11,12) = 4.234$.

We replace these values for their corresponding $\tilde{c}_{ij}$ in (4.4), which results in a conventional assignment problem in the LPP form. Solving it, we get the solution as

$x_{11}^* = x_{23}^* = x_{32}^* = 1,\ x_{12}^* = x_{13}^* = x_{21}^* = x_{22}^* = x_{31}^* = x_{33}^* = 0$

with the optimal objective value 7.432 which represents the optimal total cost. In other words the optimal assignment is $A \rightarrow 1, B \rightarrow 3, C \rightarrow 2$. The fuzzy optimal cost is calculated as

$\tilde{c}_{11} + \tilde{c}_{23} + \tilde{c}_{32} = (2,4,6) + (5,7,10)$+(6,7,9,10) =(13,18,20,26).

Also $\Re(\tilde{z}^*) = \Re(13,18,20,26) = 7.432.$

In the above example it has been shown that the total optimal cost obtained by our method remains the same as that obtained by defuzzifying the total optimal cost by applying our present method.

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