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Improved Vogel's Approximation Method to Solve Fuzzy Transshipment Problem

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Abstract.This paper deals with the large scale transshipment problem in Fuzzy Environment. Here we determined the efficient solutions for the large scale Fuzzy transshipment problem. Volgel'sApproximation Method (VAM) is a technique for finding a good initial feasible solution to an allocation problem. Here a improved version of Vogel's Approximation Method (IVAM) is used to find the efficient initial solution for the large scale transshipment problems. Performance of IVAM over VAM is discussed.

Keywords: Fuzzy transportation problem, fuzzy transshipment problem, Volgel's approximation method

AMS mathematics Subject Classification (2010): 03E72

1. Introduction

Orden [5] has extended transportation problem to include the case when transshipment is also allowed. A transportation problem allows only shipments that go directly from a supply point to a demand point. In many situations, shipments are allowed between supply points or between demand points. Sometimes there may also be points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Shipping problems with any or all of these characteristics are transshipment problems. Fortunately, the optimal solution to a transshipment problem can be found by solving a transportation problem. In what follows, we define a supply point to be a point that can send goods to another point but cannot receive goods from any other point. Similarly, a demand point is a point that can receive goods from other points but cannot send goods to any other point. A transshipment point is a point that can both receive goods from other points and send goods to other points.

The concept of fuzzy set was first introduced and investigated by Zadeh [9] and fuzzy numbers and arithmetic operations with these numbers introduced by Bellman & Zadeh and Kaufmann in [2]. In [3] Nagoor Gani et al. solving transportation problem using fuzzy number and in [4] Nagoor Gani et al solved the transshipment problem in fuzzy environment.

In this paper, section 2 deals with some preliminary definition for the fuzzy concept and fuzzy transshipment problem, section 3 explains the algorithms and section 4 a detail Numerical problem is discussed.

2. Preliminaries

2.1. Fuzzy set: A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A}$= {(x, μ_A(x)): x∈ A, μ_A(x)∈ [0,1]}. In the pair (x, μ_A(x)), the first element x belong to the classical set A, the second element μ_A(x), belong to the interval [0, 1], called Membership function.

2.2. Fuzzy number: A fuzzy set $\widetilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number,

(i) $\widetilde{A}$ must be a normal fuzzy set;

(ii) ${}^{\alpha}\widetilde{A}$ must be closed interval for every α∈[0,1]

(iii) the support of $\widetilde{A}$, ${}^{o+}\widetilde{A}$, must be bounded.

2.3. Triangular fuzzy number: It is a fuzzy number represented with three points as follows: $\widetilde{A}$ = (a_1, a_2, a_3). This representation is interpreted as membership functions and holds the following conditions

(i) a_1 to a_2 is increasing function

(ii) a_2 to a_3 is decreasing function

(iii) $a_1 \le a_2 \le a_3$.

2.4. Function principle operation of triangular fuzzy number

The following are the operations that can be performed on triangular fuzzy numbers: Let $\widetilde{A}$ = (a_1, a_2, a_3) and $\widetilde{B}$= (b_1, b_2, b_3) then,

(i) **Addition**: $\widetilde{A} + \widetilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$.

(ii) **Subtraction**: $\widetilde{A} - \widetilde{B} = (a_1-b_3, a_2-b_2, a_3-b_1)$.

(iii) **Multiplication:**$\widetilde{A} \times \widetilde{B}$ =

$$(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$$

2.5.Graded mean integration method

The graded mean integration method is used to defuzzify the triangular fuzzy number. The representation of triangular fuzzy number is $\widetilde{A}$= (a_1,a_2,a_3) and its defuzzified value is obtained by A=$\frac{a_1+4a_2+a_3}{6}$.

2.6. $\boldsymbol{\alpha}$-level set

The a-level set of the fuzzy number $\tilde{a}$ and $\tilde{b}$ is defined as the ordinary set $L_\alpha(\tilde{a}, \tilde{b})$for which the degree of their membership function exceeds the level $\alpha \in$[0,1].$L_\alpha(\tilde{a}, \tilde{b})$={a,b∈ R^m | $\mu_{\tilde{a}}(a_i, b_j) \geq \alpha, i = 1,2, \dots m, j = 1,2, \dots, n$}

2.7. Formulation of the fuzzy transshipment problem

The fuzzy transportation problem assumes that direct routs exist from each source to each destination. However, there are situations in which units may be shipped from one source to another or to other destinations before reaching their final destinations. This is called a fuzzy transshipment problem. The purpose of transshipment the distinction between a source and destination is dropped so that a transportation problem with m source and n destinations gives rise to a transshipment problem with m + n source and m + n destinations. The basic feasible solution to such a problem will involve [(m + n) + (m + n)-1] or 2m + 2n - 1 basic variables and if we omit the variables appearing in the (m + n) diagonal cells, we are left with m + n - 1 basic variables.

Thus the fuzzy transshipment problem may be written as:
Minimize $\tilde{Z} = \sum_{i=1}^{m+n} \sum_{j=1,j\neq i}^{m+n} \tilde{c}_{ij}\tilde{x}_{ij}$
Subject to $\sum_{j=1,j\neq i}^{m+n} \tilde{x}_{ij} - \sum_{j=1,j\neq i}^{m+n} \tilde{x}_{ji} = \tilde{a}_i,$ i = 1,2,3,…,m
$\sum_{i=1,i\neq j}^{m+n} \tilde{x}_{ij} - \sum_{i=1,i\neq j}^{m+n} \tilde{x}_{ji} = \tilde{b}_j,$ j = m+1,m+2,m+3,…,m+n
where $\tilde{x}_{ij} \geq 0$, i, j=1,2,3,…,m+n, j≠i
where $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{m} \tilde{b}_j$ then the problem is balance otherwise unbalanced.
the above formulation is a fuzzy transshipment model, the transshipment model is reduced to transportation form as:
Minimize $\tilde{Z} = \sum_{i=1}^{m+n} \sum_{j=1,j\neq i}^{m+n} \tilde{c}_{ij}\tilde{x}_{ij}$
Subject to $\sum_{j=1}^{m+n} \tilde{x}_{ij} = \tilde{a}_i + T,$ i=1,2,3,…,m
$\sum_{j=1}^{m+n} \tilde{x}_{ij} = T,$ i=m+1,m+2,m+3,…,m+n
$\sum_{i=1}^{m+n} \tilde{x}_{ij} = T,$ j=1,2,3,…,m
$\sum_{i=1}^{m+n} \tilde{x}_{ij} = \tilde{b}_j + T,$ j=m+1,m+2,m+3,…,m+n
where $\tilde{x}_{ij} \geq 0$, i, j = 1,2,3,…,m + n, j ≠ i,
the above mathematical model represents a standard balanced transportation problem with (m+n) origins and (m+n) destinations. T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount of goods can be transshipped at each point, T should be large enough to take care of all transshipments. It is clear that the volume of goods transshipped at any point cannot exceed the amount produced or received and hence we take T= $\sum_{i=1}^{m} \tilde{a}_i \; or \sum_{j=1}^{m} \tilde{b}_j$.

3. Algorithm

Now the Transshipment table in fuzzy environment is look like fuzzy transportation table. Now we solve the fuzzy transshipment problem using Fuzzy Vogel's Approximation Method. Here we use a graded mean integration method to find the big or small fuzzy numbers. Here all the cost, supply and demand of the fuzzy transshipment problem are in fuzzy. So first we converting the fuzzy cost to α-level cost and then applying the Vogel's approximation method, we get the fuzzy initial solution and then we apply the improved Vogel's approximation method for the same problem.

3.1. Fuzzy Vogel's Approximation Method algorithm

1. Balance the given transshipment problem if either (total supply > total demand) or (total supply < total demand)
2. For each row of the fuzzy transshipment table identify the smallest and the next to smallest cost(don't consider the diagonal zeros). Determine the difference between them for each row and display them along the side of the fuzzy transshipment table by enclosing them in parenthesis against the respective rows. Similarly, compute the difference for each column and display it at the bottom of the table with parenthesis.
3. Identify a row or column with the largest number among all the difference displayed in side and bottom of the table with in the parenthesis. If tie occurs, use any arbitrary tie –breaking choice. Let the greatest difference correspond to i^{th} row and let $\tilde{c}_{ij}$ be the smallest cost in the i^{th} row. Allocate the maximum fuzzy feasible amount $\tilde{x}_{ij}$ = min $(a_i,b_j)^{th}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner (give first preference to diagonal zero).

4. Recomputed the column and row differences for the reduced fuzzy transshipment table and goto step (ii). Repeat the procedures until the entire rim requirements are satisfied.
5. Compute total fuzzy transportation cost for the feasible cost for the feasible allocations using the original balanced fuzzy transshipment cost matrix.

3.2. ImprovedfuzzyVogel's Approximation algorithm

VAM was improved by using total opportunity cost (TOC) matrix and regarding alternative allocation costs. The TOC matrix is obtained by adding the row opportunity cost matrix and column opportunity cost matrix.

Row opportunity cost matrix: For each row, the smallest cost of that row is subtracted from each element of the same row.

Column opportunity cost matrix: For each column of the original transshipment cost matrix the smallest cost of that column is subtracted from each element of the same column.

(Note: while selecting the smallest cost don't consider the diagonal zero)

Proposed algorithm is applied on the TOC matrix. Detailed processes are given below:

3.2.1. Improved Vogel's Approximation algorithm

1. Balance the given transshipment problem if either (total supply > total demand) or (total supply < total demand)
2. Obtain the TOC matrix
3. Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.
4. Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column.
5. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.
6. If the penalties corresponding to two or more rows or columns are equal, select the top most row and the extreme left column.
7. No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two and the remaining row or column is assigned zero supply (or demand).
8. Calculate fresh penalty cost for the remaining sub-matrix as step 3 and allocate following the procedure of previous step. Continue the process until all rows and columns are satisfied.
9. Compute total fuzzy transportation cost for the feasible cost for the feasible allocations using the original balanced fuzzy transshipment cost matrix.

4. Numerical example

Consider the following transshipment problem with two origins and three destinations

	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	
$\tilde{O}_1$	(6,7,8)	(7,8,9)	(8,9,10)	(150,200,250)
$\tilde{O}_2$	(4,5,6)	(3,4,5)	(2,3,4)	(250,300,350)
	(50,100,150)	(125,150,175)	(225,250,275)	(400,500,600)

	$\tilde{O}_1$	$\tilde{O}_2$
$\tilde{O}_1$	(0,0,0)	(7,8,9)
$\tilde{O}_2$	(5,6,7)	(0,0,0)

	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$
$\tilde{D}_1$	(0,0,0)	(4,5,6)	(0.5,1,1.5)
$\tilde{D}_2$	(0.5,1,1.5)	(0,0,0)	(3,4,5)
$\tilde{D}_3$	(6,7,8)	(7,8,9)	(0,0,0)

	$\tilde{O}_1$	$\tilde{O}_2$
$\tilde{D}_1$	(6,7,8)	(1,2,3)
$\tilde{D}_2$	(0.5,1,1.5)	(4,5,6)
$\tilde{D}_3$	(7,8,9)	(8,9,10)

Solution: Now forming the transformed transportation problem

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	
$\tilde{O}_1$	(0,0,0)	(7,8,9)	(6,7,8)	(7,8,9)	(8,9,10)	(550,700,850)
$\tilde{O}_2$	(5,6,7)	(0,0,0)	(4,5,6)	(3,4,5)	(2,3,4)	(650,800,950)
$\tilde{D}_1$	(6,7,8)	(1,2,3)	(0,0,0)	(4,5,6)	(0.5,1,1.5)	(400,500,600)
$\tilde{D}_2$	(0.5,1,1.5)	(4,5,6)	(0.5,1,1.5)	(0,0,0)	(3,4,5)	(400,500,600)
$\tilde{D}_3$	(7,8,9)	(8,9,10)	(6,7,8)	(7,8,9)	(0,0,0)	(400,500,600)
	(400,500,600)	(400,500,600)	(450,600,750)	(525,650,775)	(625,750,875)	2400,3000,3600

Now applying the α −level to the fuzzy costsat $\alpha = 0.5$.Then for (0,0,0)= $0 \leq x \leq 0$ we select 0, (7,8,9) = 7.5$\leq x \leq 8.5$ we select 8 (6,7,8)) = 6.5$\leq x \leq 7.5$ we select 7 similarly for triangular fuzzy number at α =0.5 we select its middle range. So the new transportation table is

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	
$\tilde{O}_1$	0	8	7	8	9	(550,700,850)
$\tilde{O}_2$	6	0	5	4	3	(650,800,950)
$\tilde{D}_1$	7	2	0	5	1	(400,500,600)
$\tilde{D}_2$	1	5	1	0	4	(400,500,600)
$\tilde{D}_3$	8	9	7	8	0	(400,500,600)
	(400,500,600)	(400,500,600)	(450,600,750)	(525,650,775)	(625,750,875)	

Hereafter we apply the VAM algorithm

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	
$\tilde{O}_1$	0 (400,500,600)	8	7	8 (-50,200,450)	9	(550,700,850) (-50,200,450)
$\tilde{O}_2$	6	0 (400,500,600)	5	4 (-425,50,575)	3 (-25,250,475)	(650,800,950) (50,300,550) (-425,50,575)
$\tilde{D}_1$	7	2	0 (400,500,600)	5	1	(400,500,600)
$\tilde{D}_2$	1	5	1 (-150,100,350)	0 (50,400,750)	4	(400,500,600) (50,400,750)
$\tilde{D}_3$	8	9	7	8	0 (400,500,600)	(400,500,600)
	(400,500,600)	(400,500,600)	(450,600,750) (-150,100,350)	(525,650,775) (-225,250,725) (-800,200,1150)	(625,750,875) (-25,250,475)	

Number of fuzzy unitstransported from origin to destinations
$\tilde{O}_1$ to $\tilde{D}_2$ is (-50,200,450), $\tilde{O}_2$ to $\tilde{D}_2$ is (-425,50,575), $\tilde{O}_2$ to $\tilde{D}_3$ is (-25,250,475), $\tilde{D}_2$ to $\tilde{D}_1$ is (-150,100,350) and their corresponding fuzzy cost are (from the original transformed transportation table) $\tilde{O}_1$ to $\tilde{D}_2$ is (7,8,9), $\tilde{O}_2$ to $\tilde{D}_2$ is (3,4,5), $\tilde{O}_2$ to $\tilde{D}_3$ is (2,3,4), $\tilde{D}_2$ to $\tilde{D}_1$ is (0.5,1,1.5).

Therefore the initial basic feasible fuzzytransportation cost is

$$= (7,8,9)*(-50,200,450)+ (3,4,5)*(-425,50,575)+ (2,3,4)*(-25,250,475) + (0.5,1,1.5)*(-150,100,350)$$
$$= (-450,1600,4050)+(-2125,200,2875)+(-100,750,1900)+(-225,100,525)$$
$$= (-2900,2650,9350)=2841.66.$$

Now we apply the Improved VAM for the same problem

First find the TOC table

Row opportunity cost table is

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$
$\tilde{O}_1$	0	1	0	1	2
$\tilde{O}_2$	3	0	2	1	0
$\tilde{D}_1$	6	1	0	4	0
$\tilde{D}_2$	0	4	0	0	3
$\tilde{D}_3$	1	2	0	1	0

Column opportunity cost table is

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$
$\tilde{O}_1$	0	6	6	4	8
$\tilde{O}_2$	5	0	4	0	2
$\tilde{D}_1$	6	0	0	1	0
$\tilde{D}_2$	0	3	0	0	3
$\tilde{D}_3$	7	7	6	4	0

Now transportation table with TOC cost is and solved it using IVA method

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	
$\tilde{O}_1$	0 (400,500,600)	7	6 (-150,100,350)	5 (-400,100,500)	12	(550,700,850) (-50,200,450) (-400,100,500)
$\tilde{O}_2$	8	0 (400,500,600)	6	1 (-425,50,575)	2 (-25,250,475)	(650,800,950) (50,300,550) (-425,50,575)
$\tilde{D}_1$	12	1	0 (400,500,600)	5	0	(400,500,600)
$\tilde{D}_2$	0	7	0	0 (400,500,600)	6	(400,500,600)
$\tilde{D}_3$	8	9	6	5	0 (400,500,600)	(400,500,600)
	(400,500,600)	(400,500,600)	(450,600,750) (-150,100,350)	(525,650,775) (-75,150,375) (-650,100,800)	(625,750,875) (-25,250,475)	

Number of fuzzy unitstransported from origin to destinations $\tilde{O}_1$ to $\tilde{D}_1$ is (-150,100,350), $\tilde{O}_1$ to $\tilde{D}_2$ is (-400,100,500), $\tilde{O}_2$ to $\tilde{D}_2$ is (-425,50,575), $\tilde{O}_2$ to $\tilde{D}_3$ is (-25,250,475) and their corresponding fuzzy cost are (from the original transformed transportation table) $\tilde{O}_1$ to $\tilde{D}_1$ is (6,7,8), $\tilde{O}_1$ to $\tilde{D}_2$ is (7,8,9), $\tilde{O}_2$ to $\tilde{D}_2$ is (3,4,5), $\tilde{O}_2$ to $\tilde{D}_3$ is (2,3,4).

Therefore the initial basic feasible fuzzytransportation cost is

$$= (6,7,8)\times(-150,100,350) + (7,8,9)\times(-400,100,500) + (3,4,5)\times(-425,50,575) + (2,3,4)\times(-25,250,475)$$
$$= (-1200,700,2800) + (-3600,800,4500) + (-2125,200,2875) + (-100,750,1900)$$
$$= (-7025,2450,12075) = 2475.$$

5. Conclusion

This study is focused on obtaining the optimal solution for the fuzzy transshipment problem. Here we use improved Vogel's approximation method to obtain the solution for the fuzzy transshipment problem. And we obtain an efficient solution for a problem using IVAM when comparing VAM.

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