

Some Characterization of Anti Fuzzy PS-ideals of PS-Algebras in Homomorphism and Cartesian Product

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Abstract. In this paper, we introduce the notion of anti fuzzy PS – Subalgebra along with PS-ideal in homomorphism and Cartesian product and discussed some of their properties.

Keywords: PS-algebra, fuzzy PS-algebra, anti fuzzy PS-subalgebra, anti fuzzy PS-ideal, homomorphism and Cartesian product

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1. Introduction

Iseki and Tanaka [2,3] introduced two classes of abstract algebras : BCK-algebras and BCI –algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Neggers et al. [4] introduced Q-algebras and d-algebras which is the generalization of BCK / BCI algebras and obtained several results. Prabpayak and Leerawat [5] introduced a new algebraic structure which is called KU-algebras and investigated some properties. The concept of fuzzy set was introduced by Zadeh in 1965 [12]. Since then these ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. Priya and Ramachandran [6,7] introduced the new algebraic structure, PS-algebras, which is an another generalization of BCI / BCK/Q /d/ KU algebras and investigated its properties related to fuzzy, fuzzy dot in detail. Biswas [1] introduced the concept of Anti fuzzy subgroups of groups. Modifying his idea, in this paper we apply the idea in PS-algebras. Especially, we introduce the notion of Anti fuzzy PS-ideals of PS-algebras and investigate how to deal with the homomorphism and Cartesian product, and obtain some of its results.

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel.

Definition 2.1. [2,9] A BCK- algebra is an algebra $(X, *, 0)$ of type $(2,0)$ satisfying the following conditions:

- i) $(x * y) * (x * z) \leq (z * y)$
- ii) $x * (x * y) \leq y$

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- iii) $x \leq x$
- iv) $x \leq y$ and $y \leq x \Rightarrow x=y$
- v) $0 \leq x \Rightarrow x=0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$.

Definition 2.2. [3,9] A BCI- algebra is an algebra $(X, *, 0)$ of type(2,0) satisfying the following conditions:

- i) $(x * y) * (x * z) \leq (z * y)$
- ii) $x * (x * y) \leq y$
- iii) $x \leq x$
- iv) $x \leq y$ and $y \leq x \Rightarrow x = y$
- v) $x \leq 0 \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$.

Definition 2.3. [5] A KU- algebra is an algebra $(X, *, 0)$ of type(2,0) satisfying the following conditions:

- i) $(x * y) * ((y * z) * (x * z)) = 0$
- ii) $x * 0 = 0$
- iii) $0 * x = x$
- iv) $x * y = 0$ and $y * x = 0$ imply $x = y$, for all $x, y, z \in X$.

Definition 2.4. [6,7] A nonempty set X with a constant 0 and a binary operation ‘ $*$ ’ is called PS – Algebra if it satisfies the following axioms.

1. $x * x = 0$
2. $x * 0 = 0$
3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y, \forall x, y \in X$.

In PS-algebras, where $x \leq y$ is defined by $y * x = 0$, for all $x, y \in X$.

Example 2.5. Let $X = \{ 0, a, b \}$ be the set with the following Cayley table.

*	0	a	b
0	0	a	b
a	0	0	0
b	0	b	0

Then $(X, *, 0)$ is a PS – Algebra.

Definition 2.6. [6] Let X be a PS-algebra and I be a subset of X , then I is called a PS-ideal of X if it satisfies following conditions:

1. $0 \in I$
2. $y * x \in I$ and $y \in I \Rightarrow x \in I$.

Definition 2.7. [12] Let X be a non-empty set. A fuzzy subset μ of the set X is a mapping $\mu : X \rightarrow [0,1]$.

Definition 2.8. Let X be a PS-algebra. A fuzzy set μ in X is called an anti fuzzy PS-ideal of X if it satisfies the following conditions.

- (i) $\mu(0) \leq \mu(x)$
- (ii) $\mu(x) \leq \max \{ \mu(y * x), \mu(y) \}$, for all $x, y \in X$

Definition 2.9. A fuzzy set μ in a PS-algebra X is called an anti fuzzy PS-subalgebra of X if $\mu(x * y) \leq \max \{ \mu(x), \mu(y) \}$, for all $x, y \in X$.

3. Homomorphism of Anti Fuzzy PS-Ideals and Anti Fuzzy PS-Subalgebra

In this section, we discuss anti fuzzy PS-ideals and PS-subalgebra in PS-algebras under homomorphism and obtain some of its properties.

Definition 3.1. [10,11] Let $(X, *, 0)$ and $(Y, *, 0)$ be PS- algebras. A mapping $f: X \rightarrow Y$ is said to be a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

Remark: If $f: X \rightarrow Y$ is a homomorphism of PS-algebra, then $f(0) = 0$.

Definition 3.2. [8] Let $f: X \rightarrow X$ be an endomorphism and μ be a fuzzy set in X . We define a new fuzzy set in X by μ_f in X as $\mu_f(x) = \mu(f(x))$ for all $x \in X$.

Theorem 3.3. Let f be an endomorphism of PS- algebra X . If μ is an anti fuzzy PS-ideal of X , then so is μ_f .

Proof: Let μ be an anti fuzzy PS-ideal of X .

Now, $\mu_f(x) = \mu(f(x))$
 $\geq \mu(f(0)) = \mu_f(0)$, for all $x \in X$.

$\therefore \mu_f(0) \leq \mu_f(x)$

Let $x, y \in X$.

Then $\mu_f(x) = \mu(f(x))$
 $\leq \max \{ \mu(f(y) * f(x)), \mu(f(y)) \}$
 $= \max \{ \mu(f(y * x)), \mu(f(y)) \}$
 $= \max \{ \mu_f(y * x), \mu_f(y) \}$

$\therefore \mu_f(x) \leq \max \{ \mu_f(y * x), \mu_f(y) \}$

Hence μ_f is an anti fuzzy PS-ideal of X .

Theorem 3.4. Let $f: X \rightarrow Y$ be an epimorphism of PS- algebra. If μ_f is an anti fuzzy PS-ideal of X , then μ is an anti fuzzy PS-ideal of Y .

Proof: Let μ_f be an anti fuzzy PS-ideal of X .

Let $y \in Y$. Then there exists $x \in X$ such that $f(x) = y$.

Now, $\mu(0) = \mu(f(0))$
 $= \mu_f(0)$
 $\leq \mu_f(x) = \mu(f(x)) = \mu(y)$

$\therefore \mu(0) \leq \mu(y)$

Let $y_1, y_2 \in Y$.

$\mu(y_1) = \mu(f(x_1))$
 $= \mu_f(x_1)$
 $\leq \max \{ \mu_f(x_2 * x_1), \mu_f(x_2) \}$
 $= \max \{ \mu(f(x_2 * x_1)), \mu(f(x_2)) \}$

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$$\begin{aligned}
 &= \max \{ \mu (f (x_2) * f(x_1)) , \mu (f(x_2)) \} \\
 &= \max \{ \mu (y_2 * y_1) , \mu(y_2) \} \\
 \therefore \mu (y_1) &\leq \max \{ \mu (y_2 * y_1) , \mu(y_2) \} \\
 \Rightarrow \mu &\text{ is an anti fuzzy PS-ideal of } Y.
 \end{aligned}$$

Theorem 3.5. Let $f: X \rightarrow Y$ be a homomorphism of PS- algebra. If μ is an anti fuzzy PS-ideal of Y then μ_f is an anti fuzzy PS-ideal of X .

Proof: Let μ be an anti fuzzy PS-ideal of Y .

Let $x, y \in X$.

$$\begin{aligned}
 \mu_f (0) &= \mu (f(0)) \\
 &\leq \mu (f(x)) \\
 &= \mu_f (x) \\
 \Rightarrow \mu_f (0) &\leq \mu_f (x). \\
 \mu_f (x) &= \mu (f (x)) \\
 &\leq \max \{ \mu (f(y) * f(x)) , \mu (f (y)) \} \\
 &= \max \{ \mu (f (y * x)) , \mu (f (y)) \} \\
 &= \max \{ \mu_f (y * x) , \mu_f (y) \}
 \end{aligned}$$

$$\therefore \mu_f (x) \leq \max \{ \mu_f (y * x) , \mu_f (y) \}$$

Hence μ_f is an anti fuzzy PS-ideal of X .

Theorem 3.6. Let $f: X \rightarrow X$ be an endomorphism on a PS-algebra X . If μ be an anti fuzzy PS- subalgebra of X , then μ_f is an anti fuzzy PS-subalgebra of X .

Proof: Let μ be an anti fuzzy PS- subalgebra of X . Let $x, y \in X$.

$$\begin{aligned}
 \text{Now , } \mu_f (x * y) &= \mu (f (x * y)) \\
 &= \mu (f (x) * f(y)) \\
 &\leq \max \{ \mu (f (x)) , \mu(f(y)) \} \\
 &= \max \{ \mu_f (x) , \mu_f (y) \}
 \end{aligned}$$

$\Rightarrow \mu_f$ is an anti fuzzy PS-subalgebra of X .

Theorem 3.7. Let $f: X \rightarrow Y$ be a homomorphism of a PS-algebra X into a PS-algebra Y . If μ is anti fuzzy PS- subalgebra of Y , then the pre- image of μ denoted by $f^{-1}(\mu)$, defined as $\{f^{-1}(\mu)\}(x) = \mu(f(x))$, $\forall x \in X$, is an anti fuzzy PS- subalgebra of X .

Proof: Let μ be an anti fuzzy PS- subalgebra of Y . Let $x, y \in X$.

$$\begin{aligned}
 \text{Now, } \{f^{-1}(\mu)\}(x * y) &= \mu (f (x * y)) \\
 &= \mu (f (x) * f(y)) \\
 &\leq \max \{ \mu (f (x)) , \mu(f(y)) \} \\
 &= \max \{ \{f^{-1}(\mu)\}(x) , \{f^{-1}(\mu)\}(y) \}
 \end{aligned}$$

$\Rightarrow f^{-1}(\mu)$ is an anti fuzzy PS-subalgebra of X .

4. Cartesian Product of Anti Fuzzy PS-ideals of PS-algebras

In this section, we introduce the concept of Cartesian product of anti fuzzy PS-ideals and anti fuzzy PS subalgebra of PS-algebras.

Definition 4.1. [9] Let μ and δ be the anti fuzzy sets in X . The Cartesian product $\mu \times \delta: X \times X \rightarrow [0, 1]$ is defined by $(\mu \times \delta)(x, y) = \max \{ \mu(x), \delta(y) \}$, for all $x, y \in X$.

Theorem 4.2. If μ and δ are anti fuzzy PS-ideals in a PS– algebra X , then $\mu \times \delta$ is an anti fuzzy PS-ideal in $X \times X$.

Proof: Let $(x_1, x_2) \in X \times X$.

$$\begin{aligned} (\mu \times \delta)(0,0) &= \max \{ \mu(0), \delta(0) \} \\ &\leq \max \{ \mu(x_1), \delta(x_2) \} \\ &= (\mu \times \delta)(x_1, x_2) \end{aligned}$$

Let $(x_1, x_2), (y_1, y_2) \in X \times X$.

Now,

$$\begin{aligned} (\mu \times \delta)(x_1, x_2) &= \max \{ \mu(x_1), \delta(x_2) \} \\ &\leq \max \{ \max \{ \mu(y_1 * x_1), \mu(y_1) \}, \max \{ \delta(y_2 * x_2), \delta(y_2) \} \} \\ &= \max \{ \max \{ \mu(y_1 * x_1), \delta(y_2 * x_2) \}, \max \{ \mu(y_1), \delta(y_2) \} \} \\ &= \max \{ (\mu \times \delta)((y_1, y_2) * (x_1, x_2)), (\mu \times \delta)(y_1, y_2) \} \end{aligned}$$

$\therefore (\mu \times \delta)(x_1, x_2) \leq \max \{ (\mu \times \delta)((y_1, y_2) * (x_1, x_2)), (\mu \times \delta)(y_1, y_2) \}$.

Hence, $\mu \times \delta$ is an anti fuzzy PS- ideal in $X \times X$.

Theorem 4.3. Let μ and δ be fuzzy sets in PS-algebra X such that $\mu \times \delta$ is an anti fuzzy PS-ideal of $X \times X$. Then

- (i) Either $\mu(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$ for all $x \in X$.
- (ii) If $\mu(0) \leq \mu(x)$ for all $x \in X$, then either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$
- (iii) If $\delta(0) \leq \delta(x)$ for all $x \in X$, then either $\mu(0) \leq \mu(x)$ (or) $\mu(0) \leq \delta(x)$

Proof: Let $\mu \times \delta$ be an anti fuzzy PS-ideal of $X \times X$.

- (i) Suppose that $\mu(0) > \mu(x)$ and $\delta(0) > \delta(x)$ for some $x, y \in X$.

$$\begin{aligned} \text{Then } (\mu \times \delta)(x, y) &= \max \{ \mu(x), \delta(y) \} \\ &< \max \{ \mu(0), \delta(0) \} = (\mu \times \delta)(0,0), \text{ Which is a contradiction.} \end{aligned}$$

Therefore $\mu(0) \leq \mu(x)$ or $\delta(0) \leq \delta(x)$ for all $x \in X$.

- (ii) Assume that there exists $x, y \in X$ such that $\delta(0) > \mu(x)$ and $\delta(0) > \delta(y)$.

Then $(\mu \times \delta)(0,0) = \max \{ \mu(0), \delta(0) \} = \delta(0)$ and hence

$$(\mu \times \delta)(x, y) = \max \{ \mu(x), \delta(y) \} < \delta(0) = (\mu \times \delta)(0,0) \text{ Which is a contradiction.}$$

Hence, if $\mu(0) \leq \mu(x)$ for all $x \in X$, then either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$.

Similarly, we can prove that if $\delta(0) \leq \delta(x)$ for all $x \in X$, then either $\mu(0) \leq \mu(x)$ (or) $\mu(0) \leq \delta(x)$, which yields (iii).

Theorem 4.4. Let μ and δ be fuzzy sets in a PS-algebra X such that $\mu \times \delta$ is an anti fuzzy PS-ideal of $X \times X$. Then either μ or δ is an anti fuzzy PS-ideal of X .

Proof: Let $\mu \times \delta$ be an anti fuzzy PS-ideal of $X \times X$.

First we prove that δ is an anti fuzzy PS-ideal of X .

Since by 4.3(i) either $\mu(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$ for all $x \in X$.

Assume that $\delta(0) \leq \delta(x)$ for all $x \in X$.

It follows from 4.3(iii) that either $\mu(0) \leq \mu(x)$ (or) $\mu(0) \leq \delta(x)$.

If $\mu(0) \leq \delta(x)$, for any $x \in X$, then $\delta(x) = \max \{ \mu(0), \delta(x) \} = (\mu \times \delta)((0, x)$

$$\begin{aligned} \delta(x) &= (\mu \times \delta)(0, x) \\ &\leq \max \{ (\mu \times \delta)((0, y) * (0, x)), (\mu \times \delta)(0, y) \} \end{aligned}$$

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$$\begin{aligned} &= \max \{ (\mu \times \delta) ((0^*0), (y^*x)), (\mu \times \delta) (0, y) \} \\ &= \max \{ (\mu \times \delta) (0, (y^*x)), (\mu \times \delta) (0, y) \} \\ &= \max \{ \delta(y^*x), \delta(y) \} \end{aligned}$$

Hence δ is an anti fuzzy PS-ideal of X .

Next we will prove that μ is an anti fuzzy PS-ideal of X .

Let $\mu(0) \leq \mu(x)$

Since by theorem 4.3(ii), either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$.

Assume that $\delta(0) \leq \mu(x)$, then

$$\begin{aligned} \mu(x) &= \max \{ \mu(x), \delta(0) \} = (\mu \times \delta) (x, 0) \\ \mu(x) &= (\mu \times \delta) (x, 0) \\ &\leq \max \{ (\mu \times \delta) ((y, 0) * (x, 0)), (\mu \times \delta) (y, 0) \} \\ &= \max \{ (\mu \times \delta) ((y^*x), (0^*0)), (\mu \times \delta) (y, 0) \} \\ &= \max \{ (\mu \times \delta) ((y^*x), 0), (\mu \times \delta) (y, 0) \} \\ &= \max \{ \mu(y^*x), \mu(y) \} \end{aligned}$$

Hence, μ is an anti fuzzy PS-ideal of X .

Theorem 4.5. If λ and μ are anti fuzzy PS-subalgebras of a PS-algebra X , then $\lambda \times \mu$ is also an anti fuzzy PS-subalgebra of $X \times X$.

Proof : For any $x_1, x_2, y_1, y_2 \in X$.

$$\begin{aligned} \text{Then } (\lambda \times \mu) ((x_1, y_1) * (x_2, y_2)) &= (\lambda \times \mu) (x_1 * x_2, y_1 * y_2) \\ &= \max \{ \lambda (x_1 * x_2), \mu (y_1 * y_2) \} \\ &\leq \max \{ \max \{ \lambda(x_1), \lambda(x_2) \}, \max \{ \mu(y_1), \mu(y_2) \} \} \\ &= \max \{ \max \{ \lambda(x_1), \mu(y_1) \}, \max \{ \lambda(x_2), \mu(y_2) \} \} \\ &= \max \{ (\lambda \times \mu) (x_1, y_1), (\lambda \times \mu) (x_2, y_2) \} \end{aligned}$$

This completes the proof.

Definition 4.6. Let β be a fuzzy subset of X . The strongest anti fuzzy β -relation on PS-algebra X is the fuzzy subset μ_β of $X \times X$ given by $\mu_\beta (x, y) = \max \{ \beta(x), \beta(y) \}$, for all $x, y \in X$.

Theorem 4.7. Let μ_β be the strongest anti fuzzy β -relation on PS-algebra X , where β is a fuzzy set of a PS-algebra X . If β is an anti fuzzy PS-ideal of a PS-algebra X , then μ_β is an anti fuzzy PS-ideal of $X \times X$.

Proof : Let β be an anti fuzzy PS-ideal of a PS-algebra X .

let $(x, y) \in X \times X$.

$$\begin{aligned} \mu_\beta (0, 0) &= \max \{ \beta(0), \beta(0) \} \\ &\leq \max \{ \beta(x), \beta(y) \} \\ &= \mu_\beta (x, y) \end{aligned}$$

let $(x_1, x_2), (y_1, y_2) \in X \times X$.

$$\begin{aligned} \text{Then } \mu_\beta (x_1, x_2) &= \max \{ \beta(x_1), \beta(x_2) \} \\ &\leq \max \{ \max \{ \beta(y_1 * x_1), \beta(y_1) \}, \max \{ \beta(y_2 * x_2), \beta(y_2) \} \} \\ &= \max \{ \max \{ \beta(y_1 * x_1), \beta(y_2 * x_2) \}, \max \{ \beta(y_1), \beta(y_2) \} \} \\ &= \max \{ \mu_\beta ((y_1 * x_1), (y_2 * x_2)), \mu_\beta (y_1, y_2) \} \end{aligned}$$

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$$= \max \{ \mu_{\beta} ((y_1, y_2) * (x_1, x_2)), \mu_{\beta} (y_1, y_2) \}$$

Therefore μ_{β} is an anti fuzzy PS-ideal of $X \times X$.

Theorem 4.8. Let μ_{β} be the strongest anti fuzzy β -relation on PS-algebra X , where β is a fuzzy set of a PS-algebra X . If μ_{β} is an anti fuzzy PS-ideal of $X \times X$, then β is an anti fuzzy PS-ideal of a PS-algebra X .

Proof: Let μ_{β} is an anti fuzzy PS-ideal of $X \times X$. Then

For all $(x, y) \in X \times X$.

$$\Rightarrow \max \{ \beta(0), \beta(0) \} = \mu_{\beta} (0,0) \leq \mu_{\beta} (x,y) = \max \{ \beta(x), \beta(y) \}$$

$$\Rightarrow \beta(0) \leq \beta(x) \text{ or } \beta(0) \leq \beta(y).$$

$$\begin{aligned} \max \{ \beta(x_1), \beta(x_2) \} &= \mu_{\beta} (x_1, x_2) \\ &\leq \max \{ \mu_{\beta} ((y_1, y_2) * (x_1, x_2)), \mu_{\beta} (y_1, y_2) \} \\ &= \max \{ \mu_{\beta} ((y_1 * x_1), (y_2 * x_2)), \mu_{\beta} (y_1, y_2) \} \\ &= \max \{ \max \{ \beta(y_1 * x_1), \beta(y_2 * x_2) \}, \max \{ \beta(y_1), \beta(y_2) \} \} \\ &= \max \{ \max \{ \beta(y_1 * x_1), \beta(y_1) \}, \max \{ \beta(y_2 * x_2), \beta(y_2) \} \} \end{aligned}$$

Put $x_2 = y_2 = 0$, we get

$$\beta(x_1) \leq \max \{ \beta(y_1 * x_1), \beta(y_1) \}.$$

Hence β is an anti fuzzy PS-ideal of a PS-algebra X .

Theorem 4.9. Let μ_{β} be the strongest anti fuzzy β -relation on PS-algebra X , where β is a fuzzy set of a PS-algebra X . If β is an anti fuzzy PS-subalgebra of a PS-algebra X , then μ_{β} is an anti fuzzy PS-subalgebra of $X \times X$.

Proof : Let β be an anti fuzzy PS-subalgebra of a PS-algebra X .

Let $x_1, x_2, y_1, y_2 \in X$.

$$\begin{aligned} \text{Then } \mu_{\beta} ((x_1, y_1) * (x_2, y_2)) &= \mu_{\beta} (x_1 * x_2, y_1 * y_2) \\ &= \max \{ \beta(x_1 * x_2), \beta(y_1 * y_2) \} \\ &\leq \max \{ \max \{ \beta(x_1), \beta(x_2) \}, \max \{ \beta(y_1), \beta(y_2) \} \} \\ &= \max \{ \max \{ \beta(x_1), \beta(y_1) \}, \max \{ \beta(x_2), \beta(y_2) \} \} \\ &= \max \{ \mu_{\beta} (x_1, y_1), \mu_{\beta} (x_2, y_2) \} \end{aligned}$$

Therefore μ_{β} is an anti fuzzy PS-subalgebra of $X \times X$.

Theorem 4.10. Let μ_{β} be the strongest anti fuzzy β -relation on PS-algebra X , where β is a fuzzy set of a PS-algebra X . If μ_{β} is an anti fuzzy PS-subalgebra of $X \times X$, then β is an anti fuzzy PS-subalgebra of a PS-algebra X .

Proof: Let $x, y \in X$.

$$\begin{aligned} \text{Now, } \beta(x * y) &= \max \{ \beta(x * y), \beta(x * y) \} \\ &= \mu_{\beta} ((x * y) * (x * y)) \\ &\leq \max \{ \mu_{\beta} (x * y), \mu_{\beta} (x * y) \} \\ &= \max \{ \max \{ \beta(x), \beta(y) \}, \max \{ \beta(x), \beta(y) \} \} \\ &= \max \{ \beta(x), \beta(y) \} \end{aligned}$$

$\Rightarrow \beta(x * y) \leq \max \{ \beta(x), \beta(y) \}$, which completes the proof.

5. Conclusion

In this article authors have been discussed anti fuzzy PS-ideals and anti fuzzy PS-subalgebras in PS-algebras under homomorphism and Cartesian product. It has been

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observed that PS-algebras as an another generalization of BCK/BCI/Q/d/TM/KU-algebras. Interestingly, the strongest anti fuzzy β -relation on PS-algebras concept has been discussed in Cartesian product and it adds an another dimension to the defined PS-algebras. This concept can further be generalized to intuitionistic fuzzy set, interval valued fuzzy sets for new results in our future work.

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