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On Varieties of Reverse Wiener Like Indices of a Graph

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Abstract. In this paper Reverse Wiener index, Reverse Detour Wiener index, Reverse Circular Wiener index Reverse Harary index, Reverse Detour Harary index, Reverse Circular Harary index, Reverse Reciprocal Wiener index, Reverse Detour Reciprocal Wiener index, Reverse Circular Reciprocal Wiener index, Reverse Hyper Wiener index, Reverse, Detour Hyper Wiener index, Reverse Circular Hyper Wiener index, Reverse Gutman Wiener index, Reverse Detour Gutman Wiener index, Reverse circular Gutman Wiener index are defined using the diameter and Detour diameter of a graph and the above indices have been estimated for the Cartesian product P_2 and C_n.

Keywords: Reverse circular Wiener index, Reverse circular Harary Wiener index, Reverse Circular Reciprocal Weiner index, Reverse Circular Hyper Wiener index, Reverse Circular Gutman Wiener index.

AMS Mathematics Subject Classification (2010): 05C10

1. Introduction

The field of graph theory is rich in its theoretical development as well as in finding application areas. One of the well-investigated notions in graph theory is the distance between vertices in a graph, especially when a graph is used in modeling real world problems. This notion of distance gave rise to the introduction of other concepts. The Wiener Index and Detour Index are the popular topological indices. From these indices Reverse Wiener Index and Reverse Detour Index are defined using the diameter and Detour diameter of a graph. In this paper using the above two indices the Reverse Harary index, Reverse Detour Harary index, Reverse Circular Harary index, Reverse Reciprocal Wiener index, Reverse Detour Reciprocal Wiener index, Reverse Circular Reciprocal Wiener index, Reverse Hyper Wiener index, Reverse,Detour Hyper Wiener index, Reverse Circular Hyper Wiener index, Reverse Gutman Wiener index, Reverse Detour Gutman Wiener index, Reverse circular Gutman Wiener index are defined using the diameter and Detour diameter of a graph and the above indices have been estimated for the Cartesian product P_2 and C_n.

2. Reverse Circular Index

Definition 2.1. Let d(u,v) denote the shortest distance between two vertices u,v $\in$ V(G). The Wiener polynomial of a graph G with q edges is denoted by WP(G:q) ad is defined as WP(G:q) = $\sum_{u,v \in V(G)} q^{d(u,v)}$. The Wiener index of G is WI = ΣWP′(G:1) where ‘′’ denotes the derivative of WP(G:q) with respect to ‘q’.

Definition 2.2. Let D(u,v) denote the longest distance between two vertices u,v $\in$ V(G). The Detour distance polynomial of a graph G with q edges is denoted by DP(G:q) = $\sum_{u,v \in V(G)} q^{D(u,v)}$. The Detour index is given by DI = ΣDP′(G:1) where ‘′’ denotes the derivative of DP(G:q) with respect to ‘q’.

Definition 2.3. Let $d^o(x, y)$ denote the circular distance between two vertices u,v$\in$V(G). The circular polynomial of a graph G with q edges is denoted by CP(G:q) and is defined as CP(G:q) = D(u,v)+d(u,v). The circular index of G is CI = CP′(G:q), where ′ denotes the derivative of CP(G:q) with respect to ‘q’.

Definition 2.4. The Wiener Matrix (WM) is a lower (or) upper triangular matrix whose elements are Wiener Distances d(u,v).

Definition 2.5. The Detour Matrix (DM) is a lower (or) upper triangular matrix whose elements are Detour Distances D(u,v).

Definition 2.6. A circular matrix of G is a lower or upper triangular matrix whose elements are $d^o(x, y)$.

Definition 2.7. The eccentricity $e_d(u)$ of a vertex ‘u’ is the distance to a vertex farthest from u.

Definition 2.8. The radius r(G) of G is defined by r(G) = min{$e_d(u)$: u $\in$ V(G)}, and the diameter, d(G) of G is defined by d(G) = max{$e_d(u)$: u $\in$ V(G)}.

Definition 2.9. The detour eccentricity $e_D(u)$ of a vertex ‘u’ is the distance to a vertex farthest from u.

Definition 2.10. The detour radius R(G) is defined by R(G) = min{$e_D(u)$: u $\in$ V(G)}, and the detour diameter, D(G) of G is defined by D(G) = max{$e_D(u)$: u $\in$ V(G)}.

Definition 2.11. The reverse Wiener Index of a graph G denoted by $\wedge_d(G)$ is defined as $\wedge_d(G) = \sum_{i<j} r_{ij}$ where r_{ij} is obtained as $r_{ij} = \begin{cases} d - d_{ij} & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}$

where d is the diameter of the graph ‘G’ and d_{ij} is the length of the shortest path between vertices v_i and v_j.

Definition 2.12. The reverse Detour Index of a graph G denoted by $\wedge_D(G)$ is defined as $\wedge_D(G) = \sum_{i<j} R_{ij}$ where R_{ij} is obtained as $R_{ij} = \begin{cases} D_{ij} - d_D & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}$ where D is the Detour diameter of the graph 'G' and D_{ij} is the length of the longest path between the vertices v_i and v_j.

Definition 2.13 . The reverse circular index of a graph G is denoted by $\wedge_c(G)$ is defined as, $\wedge_c(G) = \sum_{i<j} (r_{ij} + R_{ij})$

Definition 2.14. Let d(u,v) denote the shortest distance between two vertices u,v $\in$ V(G). The Hyper Wiener polynomial of a graph G with q edges is denoted by WWP(G:q) ad is defined as WWP(G:q) = $\frac{1}{2}\sum_{u,v\in V(G)} q^{(d(u,v)^2 + d(u,v))}$. The Hyper Wiener index of G is WWI = ΣWWP′(G:1) where '′' denotes the derivative of WWP(G:q) with respect to 'q'.

Definition 2.17. Let d(u,v) denote the shortest distance between two vertices u,v $\in$ V(G). The Reciprocal Wiener polynomial of a graph G with q edges is denoted by RWP(G:q) ad is defined as RWP(G:q) = $\sum_{u,v\in V(G)} q^{\frac{1}{d(u,v)}}$. The Reciprocal Wiener index of G is RWI = ΣRWP′(G:1) where '′' denotes the derivative of RWP(G:q) with respect to 'q'.

Definition 2.20. Let d(u,v) denote the shortest distance between two vertices u,v $\in$ V(G). The Harary Wiener polynomial of a graph G with q edges is denoted by HWP(G:q) ad is defined as HWP(G:q) = $\sum_{u,v\in V(G)} q^{\frac{1}{(d(u,v))^2}}$. The Harary Wiener index of G is HWI = ΣHWP′(G:1) where '′' denotes the derivative of HWP(G:q) with respect to 'q'.

Definition 2.22. Let d(u,v) denote the shortest distance between two vertices u,v $\in$ V(G). The Gutman Wiener polynomial of a graph G with q edges is denoted by GWP(G:q) ad is defined as GWP(G:q) = $\sum_{u,v\in V(G)} q^{d(u)d(v)d(u,v)}$. The Gutman Wiener index of G is GWI = ΣGWP′(G:1) where '′' denotes the derivative of GWP(G:q) with respect to 'q'.

Definition 2.24. The Reverse Wiener indices and Reverse Detour indices can be calculated from the formula,

$$\Lambda(G) = \sum_{i<j} r_{ij} = \frac{1}{2}N(N-1)d - W(G)\,; \qquad \Lambda D(G) = \sum_{i<j} r_{ij} = D(G) - \frac{1}{2}N(N-1)d_D$$

where N denotes the number of vertices of the graph G.

Theorem 2.25. The Reverse Wiener index, Reverse detour index and Reverse circular index of Cartesian product P_2 and C_n are respectively.

$$\Lambda(P_2 \text{ X } C_n) = \begin{cases} \dfrac{N(N-1)(n-1)}{2} - \dfrac{n^3+2n^2-n}{2}, & n \text{ is odd} \\ \dfrac{N(N-1)(n+2)}{4} - \dfrac{n^3+2n^2}{2}, & n \text{ is even} \end{cases}$$

$$\Lambda D(P_2 \text{ X } C_n) = \begin{cases} n(2n-1)^2 - \dfrac{N(N-1)(2n-1)}{2}, & n \text{ is odd} \\ 4n^3 - 5n^2 + 2n - \dfrac{N(N-1)(2n-1)}{2}, & n \text{ is even} \end{cases}$$

$$\Lambda c(P_2 \text{ X } C_n) = \begin{cases} \dfrac{nN(1-N)}{2} + \dfrac{7n^3 - 10n^2 + 3n}{2}, & n \text{ is odd} \\ \dfrac{N(N-1)(6-3n)}{4} + \dfrac{7n^3 - 12n^2 + 4n}{2}, & n \text{ is even} \end{cases}$$

Proof: Let $V(P_2 \text{ X } C_n)=\{u_i/1\leq i\leq n\}$ and $E(P_2 \text{ X } C_n)=\{u_iu_{i+1}/1\leq i\leq n-1\} \cup \{u_nu_1\}$ be the vertex set and edge set of P_2 X C_n respectively.

Case (i): *n* is odd, $n \geq 5$

The Wiener-Detour matrices and circular matrices of P_2 XC_5 are respectively given in Fig. 2 and Fig. 3.

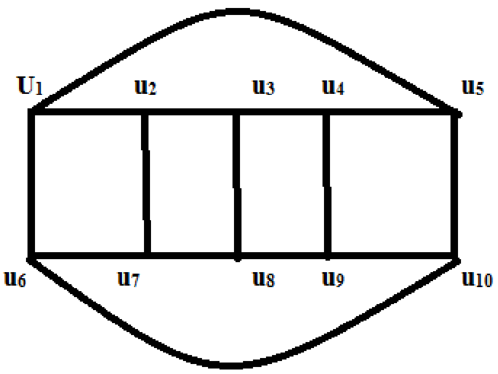

Figure 1:

WDM(P_2 XC_5)

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
u_1	0	9	9	9	9	9	9	9	9	9
u_2	1	0	9	9	9	9	9	9	9	9
u_3	2	1	0	9	9	9	9	9	9	9
u_4	2	2	1	0	9	9	9	9	9	9
u_5	1	2	2	1	0	9	9	9	9	9
u_6	1	2	3	3	2	0	9	9	9	9
u_7	2	1	2	3	3	1	0	9	9	9
u_8	3	2	1	2	3	2	1	0	9	9
u_9	3	3	2	1	2	2	2	1	0	9
u_{10}	2	3	3	2	1	1	2	2	1	0

Figure 2:

CM(P_2XC_5)

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
u_1	0	–	–	–	–	–	–	–	–	–
u_2	10	0	–	–	–	–	–	–	–	–
u_3	11	10	0	–	–	–	–	–	–	–
u_4	11	11	10	0	–	–	–	–	–	–
u_5	10	11	11	10	0	–	–	–	–	–
u_6	10	11	12	12	11	0	–	–	–	–
u_7	11	10	11	12	12	10	0	–	–	–
u_8	12	11	10	11	12	11	10	0	–	–
u_9	12	12	11	10	11	11	11	10	0	–
u_{10}	11	12	12	11	10	10	11	11	10	0

Figure 3:

WP(P_2 XC_5:q) = 15q+20q^2+10q^3 ; WP′(P_2 XC_5:1) = 85; DP(P_2 XC_5:q) = 45q^9
DP′(P_2 XC_5:1) =405; CP(P_2 XC_5:q) = 15q^{10}+20q^{11}+10q^{12} ; CP′(P_2 XC_5:1) =490
In general, when n is odd, n≥5,

$$WI(\mathrm{P}_2 \text{ X } \mathrm{C}_n) = \frac{n^3 + 2n^2 - n}{2} \quad ; \quad DI(\mathrm{P}_2 \text{ X } \mathrm{C}_n) = n(2n-1)^2 ;$$

$$CI(\mathrm{P}_2 \text{ X } \mathrm{C}_n) = \frac{7n^3 - 10n^2 + 3n}{2} .$$

The Reverse Wiener index, Reverse Detour index, Reverse Circular index can be calculated from the formula

$$\Lambda(G) = \sum_{i<j} r_{ij} = \frac{1}{2} N(N-1)d - W(G) \quad ; \quad \Lambda D(G) = \sum_{i<j} R_{ij} = D(G) - \frac{1}{2} N(N-1)d$$

We get,

$$\Lambda(\mathrm{P}_2 \text{ X} \mathrm{C}_n) = \frac{N(N-1)(n-1)}{2} - \frac{n^3 + 2n^2 - n}{2} \;;\; \Lambda D(\mathrm{P}_2 \text{ X} \mathrm{C}_n) = n(2n-1)^2 - \frac{N(N-1)(2n-1)}{2} \;;$$

$$\Lambda C(\mathrm{P}_2 \text{ X} \mathrm{C}_n) = \frac{nN(1-N)}{2} - \frac{7n^3 - 10n^2 + 3n}{2}$$

Therefore RWI(P_2 XC_5) = 95; RDI(P_2 XC_5) = 0; RCI(P_2 XC_5) = 95.

Case (ii): *n* is even
The Wiener-Detour matrices and circular matrices of P_2 XC_6 are respectively given in Fig. 5 and Fig. 6.

WDM(P_2 X C_6)

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
u_1	0	11	10	11	10	11	11	10	11	10	11	10
u_2	1	0	11	10	11	10	10	11	10	11	10	11
u_3	2	1	0	11	10	11	11	10	11	10	11	10
u_4	3	2	1	0	11	10	10	11	10	11	10	11
u_5	2	3	2	1	0	11	11	10	11	10	11	10
u_6	1	2	3	2	1	0	10	11	10	11	10	11
u_7	1	2	3	4	3	2	0	11	10	11	10	11
u_8	2	1	2	3	4	3	1	0	11	10	11	10
u_9	3	2	1	2	3	4	2	1	0	11	10	11
u_{10}	4	3	2	1	2	3	3	2	1	0	11	10
u_{11}	3	4	3	2	1	2	2	3	2	1	0	11
u_{12}	2	3	4	3	2	1	1	2	3	2	1	0

Figure 5:

CM(P_2XC_6)

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
u_1	0	=	=	=	=	=	=	=	=	=	=	=
u_2	12	0	=	=	=	=	=	=	=	=	=	=
u_3	12	12	0	=	=	=	=	=	=	=	=	=
u_4	14	12	12	0	=	=	=	=	=	=	=	=
u_5	12	14	12	12	0	=	=	=	=	=	=	=
u_6	12	12	14	12	12	0	=	=	=	=	=	=
u_7	12	12	14	14	14	12	0	=	=	=	=	=
u_8	12	12	12	14	14	14	12	0	=	=	=	=
u_9	14	12	12	12	14	14	12	12	0	=	=	=
u_{10}	14	14	12	12	12	14	14	12	12	0	=	=
u_{11}	14	14	14	12	12	12	12	14	12	12	0	=
u_{12}	12	14	14	14	12	12	12	12	14	12	12	0

Figure 6:

WP(P_2XC_6:q)=6q0+18q^3+24q^2+18q WP′(P_2XC_6:1)=144 DP(P_2 XC_6:q)=36q^{11}+30q^{10}
DP′(P_2 XC_6:1)=696 CP(P_2 XC_6:q) = 42q^{12}+24q^{14} CP′(P_2X C_6:1)=840
In general, when n is even, n≥4,

$$WI(\mathrm{P}_2 \text{ X } \mathrm{C}_n) = \frac{n^3 + 2n^2}{2} \quad ; \quad DI(\mathrm{P}_2 \text{ X } \mathrm{C}_n) = 4n^3 - 5n^2 + 2n ;$$

$$CI(\mathrm{P}_2 \text{ X } \mathrm{C}_n) = \frac{7n^3 - 12n^2 + 4n}{2} .$$

The Reverse Wiener index, Reverse Detour index, Reverse Circular index can be calculated from the formula

$$\Lambda(G) = \sum_{i<j} r_{ij} = \frac{1}{2}N(N-1)d - W(G) \quad ; \qquad \Lambda D(G) = \sum_{i<j} R_{ij} = D(G) - \frac{1}{2}N(N-1)d$$

We get,

$$\Lambda(\mathrm{P}_2\ \mathrm{X}\,\mathrm{C}_\mathrm{n}) = \frac{N(N-1)(n+2)}{4} - \frac{n^3+2n^2}{2} \ ; \ \Lambda D(\mathrm{P}_2\ \mathrm{X}\,\mathrm{C}_\mathrm{n}) = 4n^3 - 5n^2 + 2n - \frac{N(N-1)(2n-1)}{2} \quad ;$$

$$\Lambda C(\mathrm{P}_2\ \mathrm{X}\,\mathrm{C}_\mathrm{n}) = \frac{(6-3n)N(N-1)}{2} + \frac{7n^3 - 12n^2 + 4n}{2}$$

Therefore, RWI(P_2 XC_6) = 120, RDI(P_2 XC_6) = 36, RCI(P_2 XC_6) =156.

Theorem 2.26. The Reverse Hyper Wiener index, Reverse detour Hyper Wiener index and Reverse circular Hyper Wiener index of Cartesian product graph P_2 XC_n are respectively.

$$\Lambda WW(\mathrm{P}_2\ \mathrm{X}\ \mathrm{C}_\mathrm{n}) = \begin{cases} \frac{N(N-1)(n-1)}{2} - \frac{5n^3+30n^2-309n+630}{4}, & n \text{ is odd} \\ \frac{N(N-1)(n+2)}{4} - \frac{17n^3-135n^2+616n-960}{6}, & n \text{ is even} \end{cases}$$

$$\Lambda DD(\mathrm{P}_2\ \mathrm{X}\ \mathrm{C}_\mathrm{n}) = \begin{cases} 92n^3 - 823n^2 + 2976n - 3780 - \frac{N(N-1)(2n-1)}{2}, & n \text{ is odd} \\ \frac{1681n^3-24035n^2+125898n-221016}{8} - \frac{N(N-1)(2n-1)}{2}, & n \text{ is even} \end{cases}$$

$$\Lambda CC(\mathrm{P}_2\ \mathrm{X}\ \mathrm{C}_\mathrm{n}) = \begin{cases} \frac{N(N-1)(4-3n)}{2} + \frac{5111n^3-72645n^2+380158n-666888}{24}, & n \text{ is odd} \\ \frac{nN(1-N)}{2} + \frac{363n^3-3262n^2+11595n-14490}{4} & \text{is even} \end{cases}$$

Proof: Case (i): *n* is odd, $n \geq 5$

From the Fig. 2 matrix, we generate the below matrices,

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
u_1	0	66	55	66	55	66	66	55	66	55	66	55
u_2	1	0	66	55	66	55	55	66	55	66	55	66
u_3	3	1	0	66	55	66	66	55	66	55	66	55
u_4	6	3	1	0	66	55	55	66	55	66	55	66
u_5	3	6	3	1	0	66	66	55	66	55	66	55
u_6	1	3	6	3	1	0	55	66	55	66	55	66
u_7	1	3	6	10	3	3	0	66	55	66	55	66
u_8	3	1	3	6	10	3	1	0	66	55	66	55
u_9	6	3	1	3	6	10	3	1	0	66	55	66
u_{10}	10	6	3	1	3	6	6	3	1	0	66	55
u_{11}	6	10	6	3	1	3	3	6	3	1	0	66
u_{12}	3	6	10	6	3	1	1	3	6	3	1	0

Figure 7:

We get,

$$WWI(\mathrm{P}_2\ \mathrm{X}\,\mathrm{C}_\mathrm{n}) = \frac{5n^3+30n^2-309n+630}{4} \quad ; \quad DDI(\mathrm{P}_2\ \mathrm{X}\,\mathrm{C}_\mathrm{n}) = 92n^3 - 823n^2 + 2976n - 3780$$

$$CCI(\mathrm{P}_2\ \mathrm{X}\,\mathrm{C}_\mathrm{n}) = \frac{373n^3+1404n^2-18181n+34860}{12}$$

$$\Lambda WW(P_2 \text{ X } C_n) = \frac{N(N-1)(n-1)}{2} - \frac{5n^3+30n^2-309n+630}{4};$$

$$\Lambda DD(P_2 \text{ X } C_n) = 92n^3-823n^2+2976n-3780-\frac{N(N-1)(2n-1)}{2}$$

$$\Lambda CC(P_2 \text{ X } C_n) = \frac{N(N-1)(4-3n)}{2}+\frac{5111n^3-72645n^2+380158n-666888}{24}$$

Therefore RWWI(P_2 XC_5) =65 RDDI(P_2 XC_5) =1620 RCCI(P_2 XC_5) =3205/2

Case (ii): *n* is even

We get,

$$WWI(P_2 \text{ X} C_n) = \frac{17n^3-135n^2+616n-960}{6} \quad ; \quad DDI(P_2 \text{ X} C_n) = \frac{1681n^3-24035n^2+125898n-221016}{8}$$

$$CCI(P_2 \text{ X} C_n) = \frac{653n^3-6927n^2+30178n-47040}{6}$$

$$\Lambda H(P_2 \text{ X } C_n) = \frac{N(N-1)(n+2)}{4} - \frac{17n^3-135n^2+616n-960}{6};$$

$$\Lambda HDD(P_2 \text{ X } C_n) = \frac{1681n^3-24035n^2+125898n-221016}{8} - \frac{N(N-1)(2n-1)}{2}$$

$$\Lambda HCC(P_2 \text{ X } C_n) = \frac{nN(1-N)}{2} + \frac{363n^3-3262n^2+11595n-14490}{4}$$

Therefore RWWI(P_2 X C_6) =6 RDDI(P_2 X C_6) =3300 RCCI(P_2 X C_6) =3360

Theorem 2.27. The Reverse Reciprocal Wiener index, Reverse detour Reciprocal Wiener index and Reverse circular Reciprocal Wiener index of Cartesian product graph P_2 XC_n are respectively.

$$\Lambda REW(P_2 XC_n) = \begin{cases} \dfrac{N(N-1)(n-1)}{2} + \dfrac{9n^3-325n^2-1761n+2205}{480}, & n \text{ is odd} \\ \dfrac{N(N-1)(n+2)}{4} + \dfrac{19n^3-810n^2-6424n-9600}{1440}, & n \text{ is even} \end{cases}$$

$$\Lambda RED(P_2 XC_n) = \begin{cases} n - \dfrac{N(N-1)(2n-1)}{2}, & n \text{ is odd} \\ \dfrac{29n^3-484n^2+142172n+39120}{4} - \dfrac{N(N-1)(2n-1)}{2}, & n \text{ is even} \end{cases}$$

$$\Lambda REC(P_2 XC_n) = \begin{cases} \dfrac{nN(1-N)}{2} + \dfrac{9n^3-325n^2-1281n+2205}{480}, & n \text{ is odd} \\ \dfrac{N(N-1)(4-3n)}{2} + \dfrac{10459n^3-175050n^2+51175496n+14073600}{1440} & \text{is even} \end{cases}$$

Proof: Case (i): *n* is odd

From the Fig. 2, we generate the following matrices

We get,

$$REWI(P_2 XC_n) = \frac{9n^3-325n^2-1761n+2205}{480}; \qquad REDI(P_2 XC_n) = n$$

$$RECI(P_2 XC_n) = \frac{-9n^3+325n^2+2241n-2205}{480}$$

Therefore RREWI(P_2 XC_5)=455/3 RREDI(P_2 XC_5)=-400 RRECI(P_2 XC_5)= -745/3.

Case (ii): *n* is even

Weget,

$$REWI(P_2XC_n)=\frac{19n^3-810n^2-6424n-9600}{1440} \quad ; \qquad REDI(P_2XC_n)=\frac{29n^3-484n^2+142172n+39120}{4}$$

$$RECI(P_2XC_n)=\frac{-27989n^3+1190310n^2+11453144n-13307520}{2106720}$$

$$\begin{array}{c|cccccccccc}
 & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10} \\
\hline
u_1 & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_2 & \frac{1}{1} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_3 & \frac{1}{2} & \frac{1}{1} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_4 & \frac{1}{2} & \frac{1}{2} & \frac{1}{1} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_5 & \frac{1}{1} & \frac{1}{2} & \frac{1}{2} & \frac{1}{1} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_6 & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_7 & \frac{1}{2} & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{1} & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
u_8 & \frac{1}{3} & \frac{1}{2} & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{1} & 0 & \frac{1}{9} & \frac{1}{9} \\
u_9 & \frac{1}{3} & \frac{1}{3} & \frac{1}{2} & \frac{1}{1} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{1} & 0 & \frac{1}{9} \\
u_{10} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{2} & \frac{1}{1} & \frac{1}{1} & \frac{1}{2} & \frac{1}{2} & \frac{1}{1} & 0
\end{array}$$

Figure 9:

Therefore, $RREWI(P_2XC_6)=1279/6$, $RREDI(P_2XC_6)=219522$, $RRECI(P_2XC_6)=1310095/12$.

Theorem 2.28. The Reverse Harary Wiener index, Reverse detour Harary Wiener index and Reverse circular Harary Wiener index of Cartesian product graph $P_2\ XC_n$ are respectively.

$$\Lambda HW(P_2XC_n)=\begin{cases}\dfrac{N(N-1)(n-1)}{2}+\dfrac{1443n^3-22595n^2+42747n+9765}{28800}, & n \text{ is odd}\\[2ex] \dfrac{N(N-1)(n+2)}{4}-\dfrac{17957n^3-322470n^2+2332168n-3984000}{86400}, & n \text{ is even}\end{cases}$$

$$\Lambda HD(P_2XC_n)=\begin{cases}\dfrac{-8098021n^3+225107124n^2-2212072220n+17967572400}{184927881600}-\dfrac{N(N-1)(2n-1)}{2}, & n \text{ is odd}\\[2ex] \dfrac{-4n^3+94n^2-777n+7560}{9945}-\dfrac{N(N-1)(2n-1)}{2}, & n \text{ is even}\end{cases}$$

$$\Lambda HC(P_2XC_n)=\begin{cases}\dfrac{nN(1-N)}{2}+\dfrac{316343n^3-4933335n^2+8949807n+6996465}{6364800}, & n \text{ is odd}\\[2ex] \dfrac{N(N-1)(4-3n)}{2}-\dfrac{38353625920n^3-6879537200n^2+49695793600n-85092625200}{184927881600}, & n \text{ is even}\end{cases}$$

Proof: Case (i): *n* is odd

From the Fig. 2 matrix, we generate the Harary matrices, we get,

$$HWI(P_2XC_n) = \frac{1443n^3 - 22595n^2 + 42747n + 9765}{28800}$$

$$HDI(P_2XC_n) = \frac{-8098021n^3 + 225107124n^2 - 2212072220n + 1796757240\,0}{1849278816\,0}$$

$$HCI(P_2XC_n) = \frac{-321463n^3 + 5053655n^2 + 8949807n + 6996465}{6364800}$$

Therefore RHWI(P_2 XC_5) =25115/144 RHDI(P_2 XC_5) = -3640/9 RHCI(P_2 XC_5) = -33125/144.

Case (ii): *n* is even

We get,

$$HWI(P_2XC_n) = \frac{17957n^3 - 322470n^2 + 2332168n - 3984000}{86400};$$

$$HDI(P_2XC_n) = \frac{-4n^3 + 94n^2 - 777n + 7560}{9945}$$

$$HCI(P_2XC_n) = \frac{239710162n^3 - 4299710751n^2 + 31059871050n - 52172214830}{1155799260}$$

Therefore RHWI(P_2 X C_6) =1901/8 RHDI (P_2 X C_6) = 723/1210 RHCI(P_2 X C_6) = 109812150500/115579801.

Theorem 2.29. The Reverse Gutman Wiener index, Reverse detour Gutman Wiener index and Reverse circular Gutman Wiener index of Cartesian product graph P_2 XC_n are respectively.

$$\Lambda GW(P_2XC_n) = \begin{cases} \frac{N(N-1)(n-1)}{2} - \frac{9n^3 + 18n^2 - 9n}{2}, & n \text{ is odd} \\ \frac{N(N-1)(n+2)}{4} - \frac{9n^3 + 18n^2}{2}, & n \text{ is even} \end{cases}$$

$$\Lambda GD(P_2XC_n) = \begin{cases} \frac{N(N-1)(n-1)}{2} - \frac{9n^3 + 18n^2 - 9n}{2}, & n \text{ is odd} \\ \frac{567n^3 - 540n^2 - 828n + 2160}{16} - \frac{N(N-1)(2n-1)}{2}, & n \text{ is even} \end{cases}$$

$$\Lambda GC(P_2XC_n) = \begin{cases} \frac{nN(1-N)}{2} + \frac{63n^3 - 90n^2 + 27n}{2}, & n \text{ is odd} \\ \frac{N(N-1)(4-3n)}{2} - \frac{639n^3 - 396n^2 - 828n + 2160}{16} & \text{is even} \end{cases}$$

Proof: Case (i): *n* is odd, $n \geq 5$

From the Fig. 2 matrix, we generate the Gutman matrices, we get,

$$GWI(P_2XC_n) = \frac{9n^3 + 18n^2 - 9n}{2}; \quad GDI(P_2XC_n) = \frac{9n^3 + 18n^2 - 9n}{2}; \quad GCI(P_2XC_n) = \frac{81n^3 - 54n^2 + 9n}{2}$$

Therefore RGWI(P_2 XC_5) =-585 RGDI(P_2 XC_5) = 3240 RGCI(P_2 XC_5) = 2655.

Case (ii): *n* is even
We get

$$GWI\,(P_2 XC_n) = \frac{9n^3 + 18n^2}{2} \quad ; \quad GDI\,(P_2 XC_n) = \frac{567n^3 - 540n^2 - 828n + 2160}{16} \quad ;$$

$$GCI\,(P_2 XC_n) = \frac{731n^3 - 7614n^2 + 42472n - 78144}{8}$$

Therefore, RGWI(P_2 X C_6) = -1032, RGDI(P_2XC_6) = 5538, RGCI(P_2 X C_6) = 6636.

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