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Semi Global Dominating Set in Fuzzy Graphs

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Abstract. In this paper, new types of fuzzy graphs are defined and discussed some properties of the defined graphs. Also we defined the semi global fuzzy dominating set and its number of fuzzy graphs. Some results and bounds on semi global fuzzy dominating number are derived, which is used in defense problems and bank transactions.

Keywords: Fuzzy graph, effective degree, semi complementary fuzzy graph, Semi-complete fuzzy graph, semi global fuzzy dominating set.

AMS Mathematics Subject Classification (2010): 03E72, 05C69, 05C72

1. Introduction

The study of domination was initiated by Ore and Berge. The domination number and Independent domination number are introduced by Cockayne and Hedetniemi [2]. Rosenfeld [8] introduced notion of fuzzy graph and several fuzzy Analogs of the graph theoretic concepts such as path, cycles and connectedness. Somasundaram and Somasundaram [10] discussed domination in Fuzzy graphs using effective edges. Nagoorgani and Chandrasekaran [4] discussed domination in Fuzzy graphs using Strong arcs[3] of the fuzzy graphs. Nagoorgani and Vadivel [5,6] discussed domination, Independent domination and Irredundant in fuzzy graph using strong arcs. The concept of Semi global domination in the crisp graphs was introduced by Siva Rama Raju and Kumar Addagarla [9]. On automorphisms of fuzzy graphs was studied by Bhutani and Rosenfeld in [1]. A lot of works have been done on fuzzy graphs, few of them are available in [11-14].

In this paper, we introduce new type of fuzzy graphs such as semi complementary fuzzy graph and semi complete fuzzy graph which are useful in the defense problems and Bank transactions. Also we discussed the semi global fuzzy domination set and its number in Fuzzy graphs, which is useful to solve fuzzy Transportation problems in more efficient way. Some bounds on semi global fuzzy domination number are established.

2. Preliminaries

Definition 2.1. A fuzzy subset of a nonempty set V is a mapping $\sigma: V \rightarrow [0,1]$. A fuzzy relation on V is a fuzzy subset of $V \times V$.

Definition 2.2. A fuzzy graph G = (σ , μ) is a pair of function $\sigma: V \rightarrow [0,1]$ and $\mu : V\times V \rightarrow [0,1]$ where $\mu(u, v) \leq \sigma(u) \Lambda \sigma(v)$ for u ,v ϵ V.

The underlying crisp graph of G= (σ , μ) is denoted by G*=(V,E) where V={u ϵ V :σ(u)>0}and E={(u , v) ϵ V× V : (u ,v)>0}.

Definition 2.3. The order p and size q of the fuzzy graph G = (σ , μ) are defined by $p = \sum_{v \in V} \sigma(v)$ and $q= \sum_{(u,v)\in E} \mu(u,v)$.

Definition 2.4. Let G be a fuzzy graph on V and S⊆V, then the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Definition 2.5. The complement of a fuzzy graph G=(σ , μ)is a fuzzy graph $G=(\sigma^c , \mu^c)$ where $\sigma^c = \sigma$ and $\mu^c(u ,v)=\sigma(u) \Lambda \sigma(v)-\mu(u ,v)$ for all u , v in V.

Definition 2.5. The strength of connectedness between two nodes u, v in a fuzzy graph G is $\mu\infty(u,v) = \sup \{ \mu^k(u,v); k= 1,2,3,\ldots\}$ where $\mu^k(u,v) = \sup\{ \mu(u, u_1) \Lambda \mu(u, u_2) \Lambda \ldots \Lambda \mu(u_{k-1}, v)\}$. An arc (u, v) is said to be strong arc if $\mu(u, v) \geq \mu\infty(u,v)$ and the node v is said to be a strong neighbor of u. If μ(u, v) = 0 for every v ∈ V, then u is called isolated node.

Definition 2.6. A fuzzy graph G =(σ, μ) is said to be bipartite if the vertex set V can be partitioned in to two nonempty sets $V_1$and V_2 such that μ(u, v) = 0 if u, v ∈V_1 or u, v∈ V_2.

Further if μ (u, v) = μ (u) Λ μ (v) for all u ∈ V_1 and v∈V_2 then G is called complete bipartite fuzzy graph and is denoted by $K_{\sigma1, \sigma2}$,where σ_1 and σ_2 are, respectively, the restrictions of σ to $V_1$and V_2.

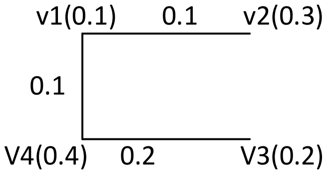
Definition 2.7. Let G=(σ , μ) be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every v ϵ V-D there exists u ϵD such that (u,v) is a strong arc. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by γ(G). The maximum scalar cardinality of a minimal domination set is called upper domination number and is denoted by the symbol Γ(G) .

Definition 2.8. Let G=(σ , μ) be a fuzzy graph. A subset D ⊆V is said to be total dominating set in G if every vertex in V is dominated by a node in D. The minimum cardinality of all total dominating sets is called total fuzzy domination number and denoted by $\gamma_t(G)$.

3. Semi-Complementary fuzzy graph and Semi Complete fuzzy graph

Definition 3.1. Let G =(σ , μ) be a fuzzy graph, then semi complementary fuzzy graph of G which is denoted by $G^{sc}= (\sigma^{sc}, \mu^{sc})$ defined as (i) $\sigma^{sc}(v) = \sigma (v)$ and
(ii) μ^{sc} = { uv∉μ and ∃w such that uw and wv in E then $\mu^{sc}(u,v) = \sigma(u) \Lambda \sigma(v)$}.

Example:



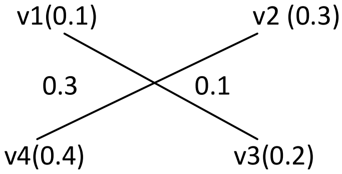


Figure 1. Fuzzy graph(G) **G^{sc}-Semi complementary fuzzy graph**

Observations:

(i) Let G be connected fuzzy graph, but G^{sc} need not be connected fuzzy graph..
(ii) $(G^c)^c = G$ but $(G^{sc})^{sc} \neq G$
(iii) G^{sc} is spanning sub graph of G and $| E(G^c)| \geq | E(G^{sc})|$
(iv) Every edge $\mu(u, v)$ in G^{sc} is not neighbor in G.
(v) G be a complete fuzzy graph then $G^{sc} = G^c$ = null graph
(vi) In G^{sc}, all the edges are effective edges.

Definition 3.2. Let G $=(\sigma , \mu)$ be a fuzzy graph with strongarcs which is said to be semi complete fuzzy graph , if every pair vertices have a common neighbor in G .

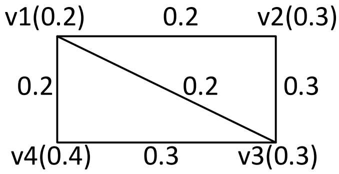


Figure 2. Semi complete fuzzy graph

Remark 3.3.

1. Every Complete fuzzy graph is semi complete fuzzy graph. But the converse is not true.
2. Every underlying graph of semi complete fuzzy graph has cycles.

Theorem 3.4. The necessary and sufficient condition for a connected fuzzy graph with strong arcs to be semi complete fuzzy graph is any pair vertices lie on the same triangle or lie on two different triangles have a common vertex.

Proof: Since by the definition of Semi complete fuzzy graph, every pair of vertices have a common neighbor, that is any pair of vertices lie on the same triangle. If not, they lie on two different triangles have a common vertex. Otherwise we are not getting the common neighbor for some pair of vertices.

Example 3.5. In Fig-2, v_1, v_2 are lie on the same triangle $v_1v_2v_3$ and so on. But the pair of vertices v_2,$v_4$are not lie on the same triangle but lie on two different triangles with common vertices v_1, v_3.

Theorem 3.6. Let G be the connected fuzzy graph , then $G^c = G^{sc}$if and only if the between every pair of non-adjacent vertices there must be two strong arcs.

Proof: Given G is connected fuzzy graph. Since G and G^{sc} have same vertex set. And $G^c = G^{sc}$ Implies $\mu(u, v) \in G^c$ if and only if $\mu(u, v)$ also belongs to G^{sc}.

Which implies every pair of non-adjacent vertices in G there must be two strong arcs.Similarly, let u and v are not adjacent in G with twostrong arcs between them then $G^c = G^{sc}$.

Theorem 3.7. Let G be semi complete fuzzy graph, Then $G^c = G^{sc}$.

Proof: Given G is semi complete fuzzy graph. Therefore between any pair of non adjacent vertices there must be two effective edges.If two vertices are adjacent in G then also there must be a path of two effective edges between them in G as it is a semi complete fuzzy graph. i.e., $G^c = G^{sc}$.

Remark 3.8. The converse of the above theorem is not true. i.e) If $G^{sc} = G^c$ then G need not be semi complete fuzzy graph.

Example 3.9.

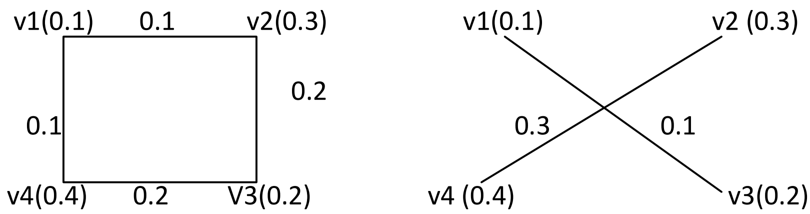


Figure 3. Fuzzy graph G and G^{sc}.

In the above Fuzzy graph, $G^{sc} = G^c$ but G is not semi complete Fuzzy graph.

Theorem 3.10. Let G =(σ , μ) be a fuzzy graph with strong arcs and G^{sc} is also connected fuzzy graph with effective edges then G is isomorphic to underlying cyclic graph.

Proof: Let uv∈E(G) which implies u, v are the vertices of G and G^{sc}.

Since G^{sc} is connected there is shortest uv path in G^{sc} . This induces a path P in G. Now P ∪ {uv} is a cycle in G. Thus G is cyclic.

Remark 3.11. The converse need not be true. That is, G is cyclic, and G^{sc} need not be connected.

Example 3.12. In Fig –3, G has path of fuzzy cycle, But G^{sc} is disconnected fuzzy graph.

Theorem 3.13. Let G =(σ , μ) be a connected fuzzy graph. Then $G \subseteq (G^{sc})^{sc}$ if and only if for each uv in G there is w in V such that between the vertices u, w and w, v there are two strong arcs.

Proof: Let us assume $G \subseteq (G^{sc})^{sc}$. Let uv∈E(G) implies uv∈ E($(G^{sc})^{sc}$)

That is, between u and v there is two strong arcsin G^{sc}.

Which implies there is w in V(G) such that uw, wv are in E(G^{sc})

=>between the vertices u, w and w, v there are two strong arcs in G

Conversely, assume uv∈E(G). Then by our assumption there is w in V such that between the vertices u, w and w, v there are two strong arcs in G.

=> uw, wv∈E(G^{sc}) and further uv∉ E(G^{sc}). That is, between u and v there are two strong arcs in G^{sc}.

=>uv∈E($(G^{sc})^{sc}$). Thus E(G) ⊆ E($(G^{sc})^{sc}$). Hence G⊆$(G^{sc})^{sc}$.

Proposition 3.14. Let G =(σ , μ) be a connected fuzzy graph with vertex set V and S⊆V. Then S is an independent set of G and G^{sc} if and only if for every u, v ∈ S , between u and v there must be at least three strong arcs.
Proof: suppose u and v are neighbor in G implies one strong arc between them and u, v are neighbor in G^{sc} implies between u and v there must be two strong arcs.Now S is independent set of G and G^{sc} gives between u and v there must be atleast three strong arcs.

Theorem 3.15. Let G =(σ , μ) be a strong edge fuzzy bipartite graph with two vertex sets X and Y , then G^{sc} is disconnected fuzzy graph and is a union of two components.
Proof: Since G is strong edge fuzzy bipartite graph then, any vertex in X is neighbor of any vertex in Y and vice -versa.Also, between X and Y there are odd number strong arcs. So, by definition, in G^{sc} no vertex of X neighbor of a vertex in Y and vice versa.And, between any two vertices of X there are even number of strong arcs and in Y also.Hence there is a path between the vertices of X and similarly the vertices of Y in G^{sc}. Thus G^{sc} is disconnected and has components formed by graph in X and that of in Y.

Remark 3.16. The converse of the above theorem is not true.

4. Semi global fuzzy domination set in fuzzy graph
Definition 4.1. Let G =(σ , μ) be a fuzzy graph with strong arcs. The set D ⊆V is said to be Semi global fuzzy domination set (sgfd-set) of G if D is a Dominating set for both G and G^{sc} .
The minimum cardinality of all sgfd-sets of G is called Semi global fuzzy domination number and is denoted by $\gamma_{sg}(G)$. The maximum cardinality of sgfd-sets is called upper semi global fuzzy domination number and denoted by $\Gamma_{sg}(G)$
Example 4.2.

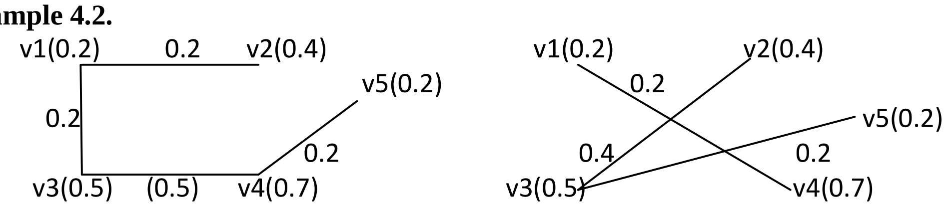


Figure 4. Fuzzy graph G with 5 vertices and G^{sc}.

Here, the sgfd-sets are {v_1,v_2,v_5}, {v_1,v_3,v_4} and { v_1, v_3,v_5}
Therefore, $\gamma_{sg}(G) = 0.8$ and $\Gamma_{sg}(G) = 1.4$

Proposition 4.3. The semi global fuzzy dominating set is not singleton.
Proof: Since sgfd-set containg dominating set for both G and G^{sc} then at least two vertices are in the set. i.e., The sgfd-set containing more than two vertices

Proposition 4.4. Let G=(σ , μ) be a complete fuzzy graph K_σ, for every u,v∈ σ* . then$\gamma_{sg}(K_\sigma)$ = p.
Proof: Since the semi complement of Complete fuzzy graph is isolated vertices.

i.e.) sgfd-set contains all the vertices of G. Therefore $\gamma_{sg}(K_\sigma) = p$.

Proposition 4.5. Let G be the complete bipartite fuzzy graph with strong arcs, then $\gamma_{sg}(K_{\sigma 1, \sigma 2}) = \text{Min}\{ \sigma (v_i)+ \sigma(v_j)\}$ where $v_i \in \sigma 1$ and $v_j \in \sigma 2$.

Proposition 4.6. Let G=(σ , μ) be the star fuzzy graph with strong neighbours, then $\gamma_{sg}(G) = \sigma (v_1) + \text{Min}\{ \sigma(v_j)\}$ where $v_1$is in G with more than one strong neighbours and v_j is in G are pendent vertices.

Definition 4.7. Let G =(σ , μ) be connected strong fuzzy graph . The set D ⊆V is said to be Global fuzzy domination set(gfd-set) of G if D is a Dominating set for both G and G^c .

The minimum cardinality of all gfd-sets of G is called Global fuzzy domination number and is denoted by $\gamma_g(G)$. The maximum cardinality of all gfd-sets of G is called Upper Global fuzzy domination number and denoted by $\Gamma_g(G)$.

Example 4.8.

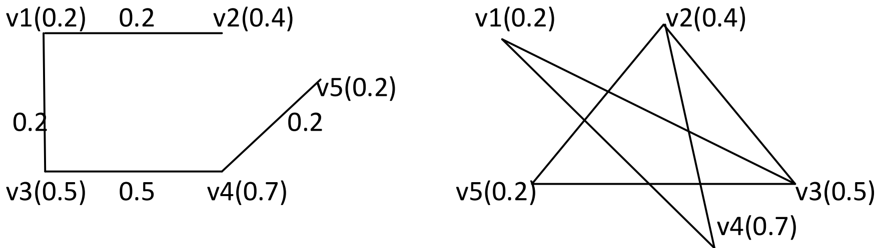


Figure 5. Fuzzy graph (G) with 5 vertices and G^c.

Here in G^c, $\mu^c(u, v) = \sigma (u) \Lambda \sigma (v)- \mu(u,v)$, for every u, v ЄV.
In the above example, gfd-set is { v_1, v_5}. Therefore, $\gamma_g(G) = 0.7 = \Gamma_g(G)$.

Remark 4.9. The sgfd-set is also gfd-set of G That is ,$\gamma_g(G) \leq \gamma_{sg}(G)$

Proposition 4.10. Let G be a semi complete fuzzy graph, D⊂V, Then D is sgfd-set in G if and only if D is a global fuzzy domination set in G.

Example 4.11. In Fig-2, the sgfd-set is {v_1, v_2, v_3) and is also gfd-set.
i.e.) $\gamma_{sg}(G) = 0.8 = \gamma_g(G)$.

Theorem 4.12. Let G =(σ, μ) be connected strong fuzzy graph, then $\text{Min}\{ \sigma (Vi)+ \sigma (Vj)\} \leq \gamma_{sg}(G) \leq p$, $i \neq j$ and for every vi, vj Є V.
Proof: We know that Semi global fuzzy dominating set has at least two vertices.
Let {vi, vj} are the vertices, then $\text{Min}\{ \sigma (Vi)+ \sigma (Vj)\} = \gamma_{sg}(G)$
If the set contains other than {vi, vj} then $\text{Min}\{ \sigma (Vi)+ \sigma (Vj)\} <\gamma_{sg}(G)$, $i \neq j$
If the given G is complete fuzzy graph, then sgfd-set contains all the vertices of the G, that is $\gamma sg(G)\leq O(G) = p$

We get, $\text{Min}\{ \sigma (Vi)+ \sigma (Vj)\} \leq \gamma_{sg}(G) \leq p = O(G)$.

Theorem 4.13. Let G be a semi complete fuzzy graph, Then $\gamma_{sg}(G) \geq$ Min{ $\sigma(V_i)+\sigma(V_j)+\sigma(V_k)$} , $i \neq j \neq k$

Proof: It is enough to prove sgfd-set contains at least three vertices.

Suppose sgfd-set contains less than three vertices

We know that sgfd-set not a singleton. i.e) sgfd-set contains at least two vertices

Let D = { v_1, v_2} be a sgfd-set in G

Case 1: <D> is connected in G

Then v_1v_2 is an effective edge in G. By the definition of semi complete, there is a v_3 in G such that <$v_1v_2v_3$> is triangle in G, i.e., D is not a fuzzy domination set in G^{sc}. Which is contradiction to D is a sgfd-set in G.

Case 2: <D> is disconnected in G

Since G is semi complete fuzzy graph, Then there is v_3 in G such that $v_1v_3$and v_3v_2 are the effective edges in G. Therefore, In G^{sc}, v_3 is not dominated by a vertex in D. This implies, D is not a sgfd-set in G, which is a contradiction.

Therefore we get, $\gamma_{sg}(G) \geq$ Min{ $\sigma(V_i)+\sigma(V_j)+\sigma(V_k)$} , $i \neq j \neq k$ for semi complete fuzzy graph.

Theorem 4.14. Let G =(σ , μ) be the fuzzy graph with strong arcs .Then,

$\gamma_{sg}(G)$ =Min{ σ (Vi)+ σ (Vj) } , i $\neq$j if and only if there is an strong arc uv in G such that each vertex in V – {u, v} is adjacent to u or v but not both.

Proof: Suppose $\gamma_{sg}(G)$ = Min{ σ (Vi)+ σ (Vj) } , i $\neq$ j .

We assume D = {u, v} be the sgfd-set in G.Let<D>is connected in G, then uv is an strong arc in G.If any vertex w in V-{u,v} is adjacent to both u and v, Which implies D is not a dominating set for G^{sc}, which is a contradiction i.e., strong arc uv in G such that each vertex in V – {u,v} is adjacent to u or v but not both.

Conversely, each vertex in V – {u,v} is adjacent to u or v but not both, then

$\gamma_{sg}(G)$ = Min{ σ (Vi)+ σ (Vj) } , i $\neq$ j.

Theorem 4.15. The set D$\subset$V is a sgfd-set in the strong fuzzy graph G if and only if each vertex in V – D lies on an effective edge whose end vertices are totally dominated by distinct vertices in D.

Proof: Let us assume D is a sgfd-set in the strong fuzzy graph G. Let v_1 Є {V– D}. Then there exist distinct vertices v_2, v_3 in D such that v_1v_2 in E(G) and v_1v_3 in E(G^{sc}),

Since v_1v_3 is in E(G^{sc}), there exist v_4 in V such that v_1v_4 and v_4v_3 are effective edges in G.

Case 1: Suppose $v_4 = v_2$, Then, v_1v_2 and v_2v_3 are effectives edges in G which implies v_1 lies on the edge $v_1 v_4$ and v_1, v_4 are dominated by v_2 and v_3 respectively from D – {v_1 } and D– { v_1,v_2}

Case 2: Suppose $v_4 \neq v_2$, Then <$v_2v_1 v_4 v_3$> is path in G which implies v_1 lies on the edge $v_1 v_4$ and v_1, v_4 are dominated by v_2 and v_3 respectively from D- { v_1,v_4}

i.e., the end points are totally dominated by distinct vertices in D.

Conversely, assume v_1 Є {V–D}. By our assumption there is an edge $v_1 v_2$ in G such that $v_1 v_3$, $v_1 v_4$ are effective edges in G and {v_3, v_4} are in G ($v_3 \neq v_4$)

If $v_3 = v_2$ then < $v_1 v_2 v_4$> is a path in G and $v_1 v_2$ is G, $v_1 v_4$ is in G^{sc}.

If $v_3 \neq v_2$ then < $v_3v_1 v_2 v_4$> is a path in G, which implies $v_1 v_3$ is in G and $v_1 v_4$ is in G^{sc}.

Hence, we have D is sgfd-set in G.

5. Conclusion

Here, new type of fuzzy graphs such as semi complementary fuzzy graph and semi complete fuzzy graph are introduced. Some results on Semi complementary and Semi complete fuzzy graphs are derived. Also the semi global fuzzy domination set and its number of fuzzy graphs are discussed, which is useful to solve fuzzy Transportation problems in more efficient way. Some bounds on semi global fuzzy domination number are established. Further some other domination parameters will be introduced in fuzzy graphs.

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