

Revised Distribution Method for Intuitionistic Fuzzy Transportation Problem

A. Nagoor Gani¹ and S.Abbas²

¹P.G. & Research Dept. of Mathematics, Jamal Mohamed College (Autonomous)
Trichy - 620 020, Tamilnadu, India, Email: ganijmc@yahoo.co.in

²P.G. & Research Dept. of Mathematics, Khadir Mohideen College, Adirampattinam
Tamilnadu, India. Email: abbas.kmc@gmail.com

Received 27 June 2014; accepted 9 July 2014

Abstract. The aim of this paper is to obtain the optimum solution for an intuitionistic fuzzy transportation problem using Revised Distribution Method. Here a new type of intuitionistic triangular fuzzy number is used. The solution procedure is illustrated with numerical example.

Keywords: Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Transportation Problem, Revised Distribution Method

AMS mathematics Subject Classification (2010): 03E72, 03F55, 90B06

1. Introduction

Transportation problems are one of the powerful frame works which ensures efficient movement and timely availability of the raw materials and finished goods. This transportation problem is a linear programming problem obtained from a network structure consisting of a defined numbers of nodes and arcs attached to them. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem.

The objective of the FTP is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and fuzzy demand. In [4], Nagoor Gani *et al* presented a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Hadi Basirzadeh, present An Approach for Solving Fuzzy Transportation Problem using trapezoidal fuzzy numbers.

The concept of intuitionistic fuzzy sets (IFSs) proposed by Atanassov [1] in 1986 is found to be highly useful to deal with vagueness. The major advantage of IFS over fuzzy set is that IFSs separate the degree of membership (belongingness) and the degree of non-membership (non-belongings) of an element in the set. In this paper, we find the obtimum solution for an IFTP using Revised Distribution Method, here the supply and demand are triangular intuitionistic fuzzy numbers.

In this paper, we first review some basic concepts of intuitionistic fuzzy numbers in section 2. Section 3, the operations on Triangular Intuitionistic Fuzzy Numbers(TIFN).

Revised Distribution Method for Intuitionistic Fuzzy Transportation Problem

In section 4, we explain the method of ranking for TIFNs with example. In section 5, we define a intuitionistic fuzzy transportation problem. In section 6, we give the revised distribution algorithm to solve the intuitionistic fuzzy transportation problem. In section 7, a numerical example is given.

2. Preliminaries

Definition 2.1. (Intuitionistic fuzzy set)

An Intuitionistic fuzzy set (IFS) \tilde{A}^I in X is given by a set of ordered triples:

$\tilde{A}^I = \{ \langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \rangle / x \in X \}$, where $\mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} : X \rightarrow [0,1]$ are functions such that $0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$ for all $x \in X$. For each x the numbers $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ represent the degree of membership and degree of non-membership of the element $x \in X$ to $A \subset X$, respectively.

Definition 2.2. (Intuitionistic fuzzy number)

An intuitionistic fuzzy subset $\tilde{A}^I = \{ \langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \rangle / x \in X \}$, of the real line R is called an Intuitionistic Fuzzy Number (IFN) if the following holds:

- i) There exist $m \in R$, $\mu_{\tilde{A}^I}(m) = 1$ and $\nu_{\tilde{A}^I}(m) = 0$, (m is called the mean value of \tilde{A}^I).
- ii) $\mu_{\tilde{A}^I}$ is a continuous mapping from R to the closed interval $[0,1]$ and $\forall x \in R$, the relation $0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$ holds.

The membership and non-membership function of \tilde{A}^I is of the following form:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & \text{for } -\infty < x \leq m - \alpha \\ f_1(x) & \text{for } x \in [m - \alpha, m] \\ 1 & \text{for } x = m \\ h_1(x) & \text{for } x \in [m, m + \beta] \\ 0 & \text{for } m + \beta \leq x < \infty \end{cases}$$

where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function in $[m - \alpha, m]$ and $[m, m + \beta]$ respectively.

$$\nu_{\tilde{A}^I}(x) = \begin{cases} 1 & \text{for } -\infty < x \leq m - \alpha' \\ f_2(x) & \text{for } x \in [m - \alpha', m]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0 & \text{for } x = m \\ h_2(x) & \text{for } x \in [m, m + \beta']; 0 \leq h_1(x) + h_2(x) \leq 1 \\ 1 & \text{for } m + \beta' \leq x \leq \infty \end{cases}$$

Here m is the mean value of \tilde{A}^I . α and β are called left and right spreads of membership function $\mu_{\tilde{A}^I}(x)$ respectively. α' and β' represents left and right spreads of

non membership function $V_{\tilde{A}'}(x)$, respectively. Symbolically, the intuitionistic fuzzy number is represented as $\tilde{A}'_{IFN} = (m; \alpha, \beta; \alpha', \beta')$.

Definition 2.3. (Triangular intuitionistic fuzzy number (TIFN))

Type I : A (TIFN) \tilde{A}' is an intuitionistic fuzzy set in \mathbb{R} with the following membership function $\mu_{\tilde{A}'}(x)$ and non membership function $V_{\tilde{A}'}(x)$:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{Otherwise} \end{cases}$$

$$V_{\tilde{A}'}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1} & \text{for } a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2} & \text{for } a_2 \leq x \leq a_3' \\ 1 & \text{otherwise} \end{cases}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\mu_{\tilde{A}'}(x), V_{\tilde{A}'}(x) \leq 0.5$ for $\mu_{\tilde{A}'}(x) = V_{\tilde{A}'}(x) \forall x \in \mathbb{R}$. This TIFN is denoted by $\tilde{A}'_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$.

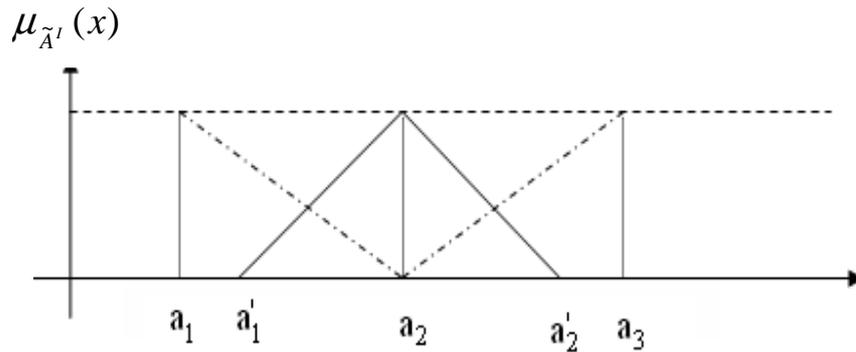


Figure 1:

Note: In this type of Triangular Intuitionistic Fuzzy Numbers, the membership value is smaller than to non membership value. That is $\mu_{\tilde{A}'}(x) < V_{\tilde{A}'}(x)$.

Type II :

A (TIFN) \tilde{A}' is an intuitionistic fuzzy set in \mathbb{R} with the following membership function $\mu_{\tilde{A}'}(x)$ and non membership function $V_{\tilde{A}'}(x)$:

Revised Distribution Method for Intuitionistic Fuzzy Transportation Problem

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{Otherwise} \end{cases}$$

$$V_{\tilde{A}'}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & \text{for } a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2} & \text{for } a_2 \leq x \leq a_3' \\ 1 & \text{otherwise} \end{cases}$$

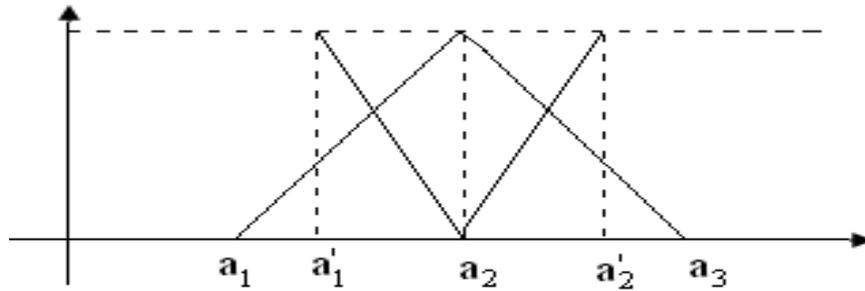


Figure 2:

Note: In this type of Triangular Intuitionistic Fuzzy Numbers , the membership value is bigger than the non membership value . That is $\mu_{\tilde{A}'}(x) > V_{\tilde{A}'}(x)$.

Type III:

A (TIFN) \tilde{A}' is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}'}(x)$ and non membership function $V_{\tilde{A}'}(x)$:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{Otherwise} \end{cases}$$

$$V_{\tilde{A}^I}(x) = \begin{cases} \frac{a_2 - x}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{x - a_2}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{otherwise} \end{cases}$$

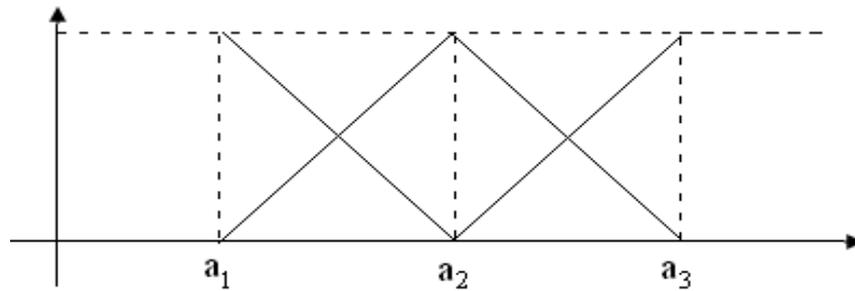


Figure 3:

Note: In this type of Triangular Intuitionistic Fuzzy Numbers , the membership value and the non membership value are same. That is $\mu_{\tilde{A}^I}(x) = V_{\tilde{A}^I}(x)$.

Definition 2.4. (Accuracy function)

We define $H(\tilde{A}^I) = \frac{(a_1 + 2a_2 + a_3) + (a_1' + 2a_2' + a_3')}{8}$, an accuracy function of \tilde{A}^I , to defuzzify the given number.

3. Operations on triangular intuitionistic fuzzy number

Let $\tilde{a}^I = (a_1, a_2, a_3 ; a_1', a_2, a_3')$ and $\tilde{b}^I = (b_1, b_2, b_3 ; b_1', b_2, b_3')$ two Triangular Intuitionistic Fuzzy Number then the arithmetic operations on \tilde{a}^I and \tilde{b}^I as follows :

Addition : $\tilde{a}^I \oplus \tilde{b}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3 ; a_1' + b_1', a_2 + b_2, a_3' + b_3')$

Subtraction : $\tilde{a}^I \ominus \tilde{b}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1 ; a_1' - b_3', a_2 - b_2, a_3' - b_1')$

Multiplication : $\tilde{a}^I \otimes \tilde{b}^I = (a_1 b_1, a_2 b_2, a_3 b_3 ; a_1' b_1', a_2 b_2, a_3' b_3')$

Scalar Multiplication : (i) If $k > 0$ then $k\tilde{a}^I = (ka_1, ka_2, ka_3 ; ka_1', ka_2, ka_3')$

(ii) If $k < 0$ then $k\tilde{a}^I = (ka_3, ka_2, ka_1 ; ka_3', ka_2, ka_1')$

4. Ranking of intuitionistic triangular fuzzy number

Let $\tilde{A}^I = (a, b, c, e, b, f)$. Then the Graded Mean Integration Representation (GMIR) of membership function of \tilde{A}^I is,

$R_\mu(\tilde{A}^I) = \frac{(a - c) + 2c + 4b}{6}$ and non membership value of \tilde{A}^I is

Revised Distribution Method for Intuitionistic Fuzzy Transportation Problem

$$R_v(\tilde{A}^I) = \frac{2(e-f) + 2b + 4f}{6}$$

5. Numerical example

In this section, we illustrate the new ranking procedure with a numerical example.

Let $\tilde{A}^I = (2,3,5 ; 1,3,6)$ and $\tilde{B}^I = (5,6,9 ; 1,6,13)$.

Then $R_\mu(\tilde{A}^I) = 3.16$, $R_v(\tilde{A}^I) = 3.33$ and $R_\mu(\tilde{B}^I) = 6.33$, $R_v(\tilde{B}^I) = 6.66$.

So $R_\mu(\tilde{A}^I) < R_\mu(\tilde{B}^I)$ and $R_v(\tilde{A}^I) < R_v(\tilde{B}^I)$.

Therefore $\tilde{A}^I < \tilde{B}^I$.

6. Intuitionistic fuzzy transportation problem (IFTP)

Consider a transportation with m Intuitionistic Fuzzy (IF) origins and n IF destination. Let C_{ij} ($i=1,2,\dots,m, j=1,2,\dots,n$) be the cost of transporting one unit of the product from i^{th} origin to j^{th} destination. Let \tilde{a}^I_i ($i=1,2,\dots,m$) be the quantity of commodity available at IF origin i . Let \tilde{b}^I_j ($j=1,2,\dots,n$) be the quantity of commodity needed of IF destination j . Let X_{ij} ($i=1,2,\dots,m, j=1,2,\dots,n$) is quantity transported from i^{th} IF origin to j^{th} destination.

Mathematical Model of Intuitionistic Fuzzy Transportation Problem is

$$\begin{aligned} \text{Minimize } \tilde{Z}^I &= \sum_{i=1}^m \sum_{j=1}^n \tilde{x}_{ij}^I \tilde{c}_{ij}^I \\ \sum_{j=1}^n \tilde{x}_{ij}^I &= \tilde{a}_i^I, \quad i = 1, 2, \dots, m \\ \sum_{i=1}^m \tilde{x}_{ij}^I &= \tilde{b}_j^I, \quad j = 1, 2, \dots, n \\ \tilde{x}_{ij}^I &\geq 0, \text{ for all } i \text{ and } j. \end{aligned}$$

The above IFTP can be stated in the below tabular form.

	D₁	D₂	...	D_n	IF Capacity
O₁	C_{11} X_{11}	C_{12} X_{12}	...	C_{1n} X_{1n}	\tilde{a}^I_1
O₂	C_{21} X_{21}	C_{22} X_{22}	...	C_{2n} X_{2n}	\tilde{a}^I_2
	\vdots	\vdots		\vdots	
O_m	C_{m1} X_{m1}	C_{m2} X_{m2}		C_{mn} X_{mn}	\tilde{a}^I_m
IF Demand	\tilde{b}^I_1	\tilde{b}^I_2		\tilde{b}^I_n	$\sum \tilde{a}^I_i$ $= \sum \tilde{b}^I_j$

7. Algorithm for revised distribution method

A. Nagoor Gani and S.Abbas

This section presents a Revised Distribution Method method to solve the intuitionistic fuzzy transportation problem which is different from the preceding method. It making allocation by minimum demand and supply.

Step (i): Start with the minimum value in the supply column and demand row. If tie occurs, then select the demand or supply value with least cost.

Step (ii): Compare the figure of available supply (capacity) in the row and demand in the column and allocate the units equal to capacity or demand whichever is less.

Step (iii): If the demand in the column is satisfied, move to the next minimum value in the Demand row and supply column.

Step (iv): Repeat Steps (ii) and (iii) until capacity condition of all the plants demand conditions of all ware houses have been satisfied.

8. An illustrate example

Consider the 3X3 IFTP.

	D ₁	D ₂	D ₃	IF Supply
S ₁	(2,5,8;2,5,8)	(11,16,20;11,16,20)	(6,8,10;6,8,10)	(5,8,11 ; 5,8,11)
S ₂	(10,13,16;10,13,16)	(4,7,10;4,7,10)	(2,6,9;2,6,9)	(9,12,14;9,12,14)
S ₃	(2,4,7;2,4,7)	(7,9,12;7,9,12)	(8,10,13;8,10,13)	(2,5,10;2,5,10)
IF Demand	(12,15,20;12,15,20)	(1,4,7;1,4,7)	(3,6,8;3,6,8)	

Since $\sum \tilde{a}'_i = \sum \tilde{b}'_j = (16,25,35;16,25,35)$ the problem is balanced transportation problem. Using the above algorithm we get the optimum solution. The solution table is given below.

	D ₁	D ₂	D ₃	IF Supply
S ₁	(2,5,8;2,5,8) (5,8,11 ; 5,8,11)	(11,16,20;11,16,20)	(6,8,10;6,8,10)	(5,8,11 ; 5,8,11)
S ₂	(10,13,16;10,13,16) (-6,2,10;-6,2,10)	(4,7,10;4,7,10) (1,4,7;1,4,7)	(2,6,9;2,6,9) (3,6,8;3,6,8)	(9,12,14;9,12,14)
S ₃	(2,4,7;2,4,7) (2,5,10;2,5,10)	(7,9,12;7,9,12)	(8,10,13;8,10,13)	(2,5,10;2,5,10)
IF Demand	(12,15,20;12,15,20)	(1,4,7;1,4,7)	(3,6,8;3,6,8)	

The optimum intuitionistic fuzzy transportation cost is

$$\begin{aligned}
 Z &= (2,5,8;2,5,8) (5,8,11 ; 5,8,11) + (10,13,16;10,13,16) (-6,2,10;-6,2,10) \\
 &\quad + (4,7,10;4,7,10) (1,4,7;1,4,7) + (2,6,9;2,6,9) (3,6,8;3,6,8) \\
 &\quad + (2,4,7;2,4,7) (2,5,10;2,5,10) \\
 Z &= (-36,150,460;-36,150,460)
 \end{aligned}$$

Revised Distribution Method for Intuitionistic Fuzzy Transportation Problem

The intuitionistic optimal value = (170.67, 191.33)
Therefore the Minimum Transportation Cost = 181.

9. Conclusion

In this paper, the revised distribution method is used for solving intuitionistic fuzzy transportation problem. This method can be used for all kinds of intuitionistic fuzzy transportation problems, whether maximize or minimize objective function. This method is based on the allocation of demand and supply items in the transportation matrix, and finds an optimal solution in terms of the ones.

REFERENCES

1. K.T.Atanssov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1) (1986) 87-96.
2. S.Aramuthakannan and P.R.Kandasamy, Revised distribution method of finding optimal solution for transportation problems, *IOSR Journal of Mathematics*, 4(5) (2013) 39-42.
3. H.Basirzadeh, Ones assignment method for solving assignment problems, *Applied Mathematical Sciences*, 6(47) (2012) 2345 – 2355.
4. G.S.Mahapatra and T.K.Roy, Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations, *World Academy of science, Engineering and Technology*, 50 (2009) 574-581.
5. A.Nagoor Gani and S.Abbas, Solving intuitionistic fuzzy transportation problem using zero suffix algorithm, *IJMSEA*, 6(III) (2012) 73-82.
6. A.Nagoor Gani and K.Abdul Razak, Solving transportation problem using fuzzy number, *Bulletin of Pure and Applied Science* 23E(2) (2004) 281-289.
7. A.Nagoor Gani and V.N.Mohamed, Solution of a fuzzy assignment problem by using a new ranking method, *Intern. J. Fuzzy Mathematical Archive*, 2(2013) 8-16.
8. A.Nagoor Gani and G.Sabarinathan, Optimal policy for vendor-buyer integrated inventory model within just in time in fuzzy environment, *Intern. J. Fuzzy Mathematical Archive*, 2(2013) 26-35.
9. A.Nagoor Gani and K.Ponnalagu, A new approach on solving intuitionistic fuzzy linear programming problem, *Applied Mathematical Sciences*, 6(70) (2012) 3467-3474
10. Z.Wenyi and H.Li., Note on some operations on intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 157 (2006) 990 – 991.
11. L.A.Zadeh, Fuzzy sets, *Information and Computation*, 8 (1965) 338-353.